Jie Hou

Education

Rice University 09/2012 – Present

Ph.D. Candidate in **Geophysics**, Earth Science Department

China University of Petroleum (East China) 2008 – 2012

B.S. in **Exploration Geophysics**

Research Interests

- True Amplitude Imaging (RTM)
- Acceleration of linear/nonlinear waveform inversion
- Inversion Velocity Analysis

Inversion Velocity Analysis

with approximate Born inversion

Jie Hou

TRIP 2015 Review Meeting

Apr 25, 2016

FWI

Full Waveform Inversion (FWI) finds velocity model to minimize the data misfit:

$$J_{\text{FWI}}[\mathbf{m}] = \frac{1}{2}||\mathcal{F}[\mathbf{m}] - \mathbf{d}_{obs}||^2$$

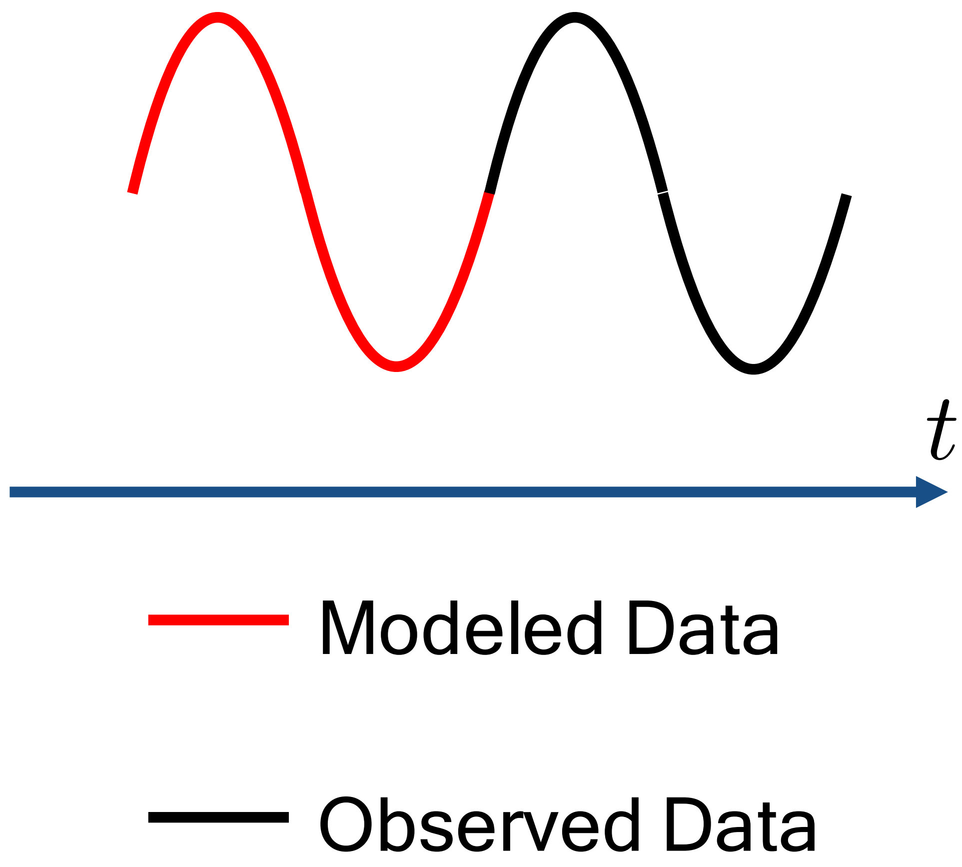
$\mathbf{m}$ velocity model $\mathbf{d}_{obs}$ observed data

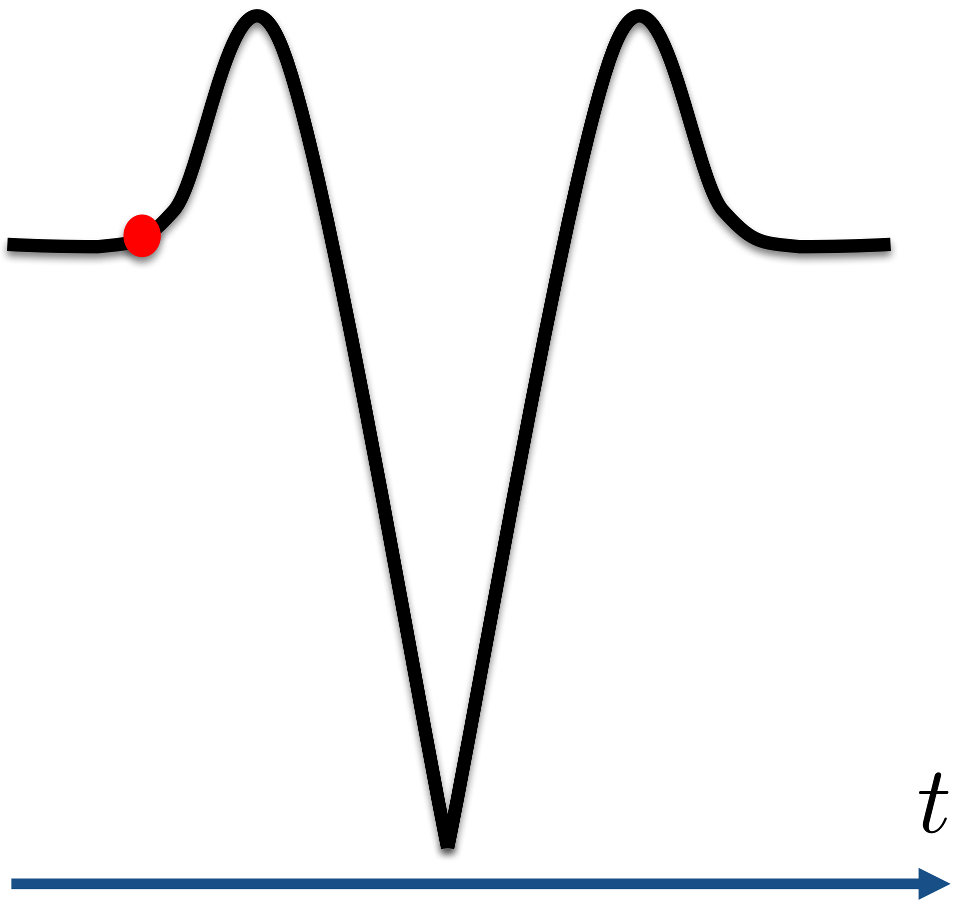
$\mathcal{F}$ forward modeling operator

- Extends into any modeling physics, data geometry
- Large scale, nonlinear, ill-posed problem

Behavior of Objective Function

Simple Example:





Objective Function

Behavior of Objective Function

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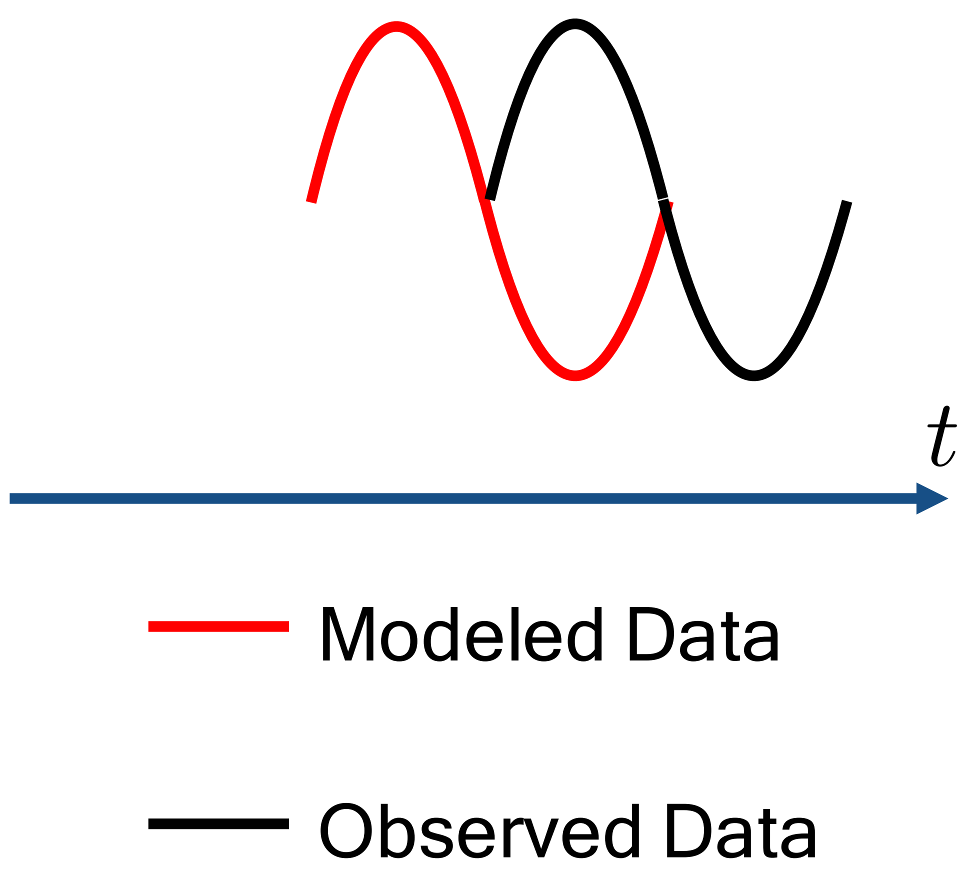

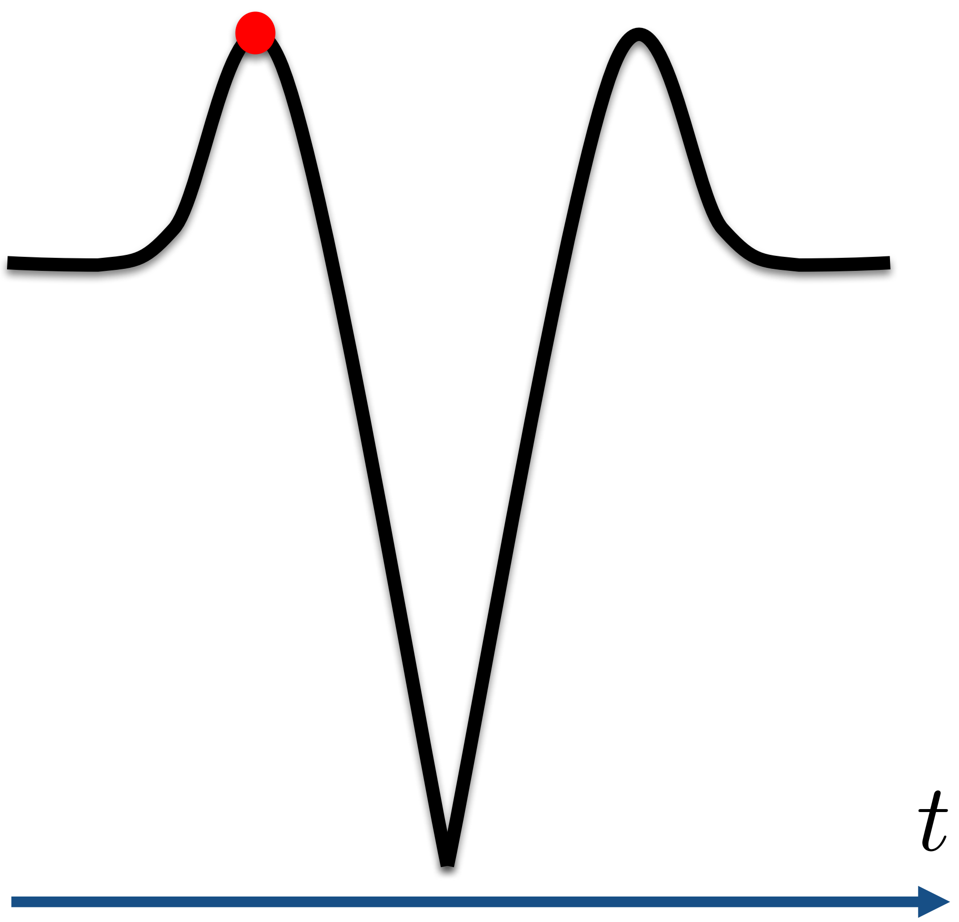

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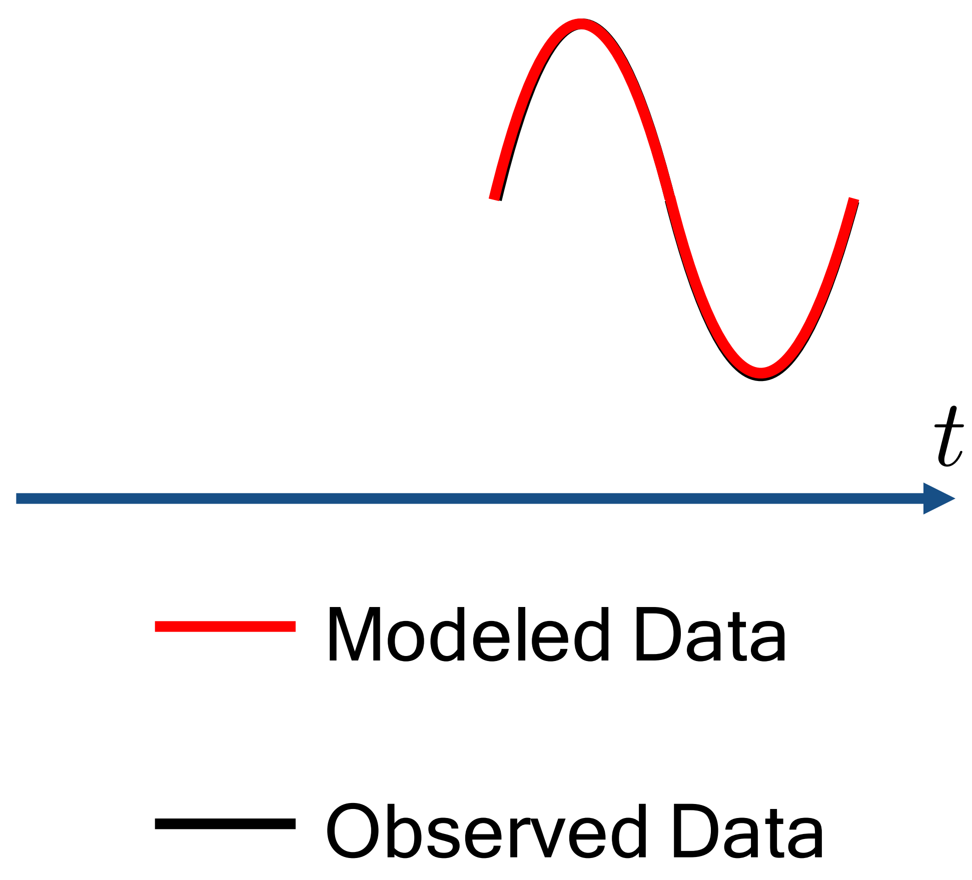

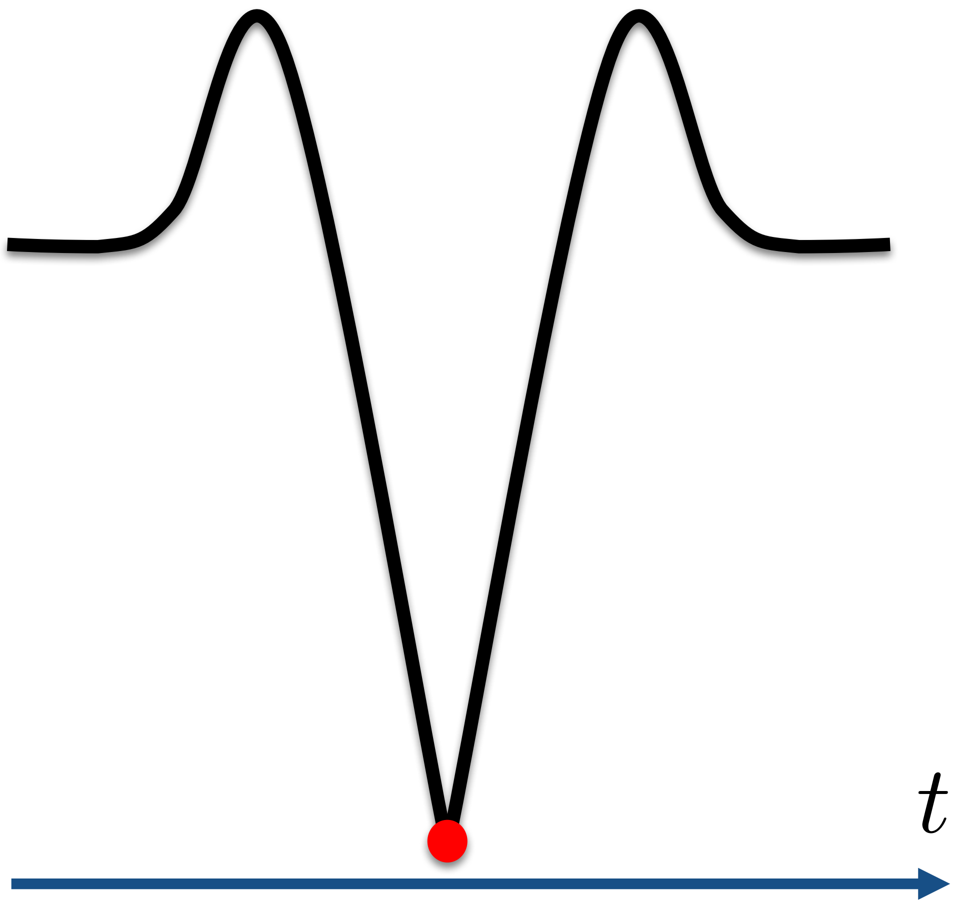

Objective Function

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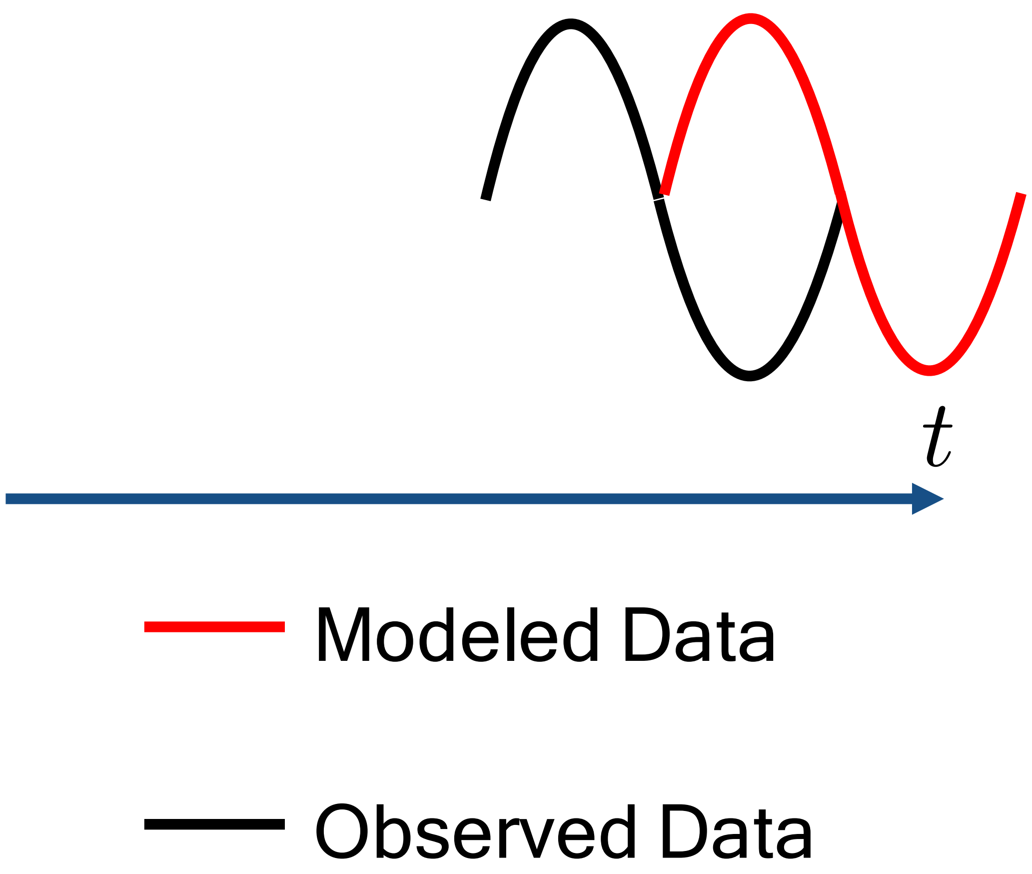

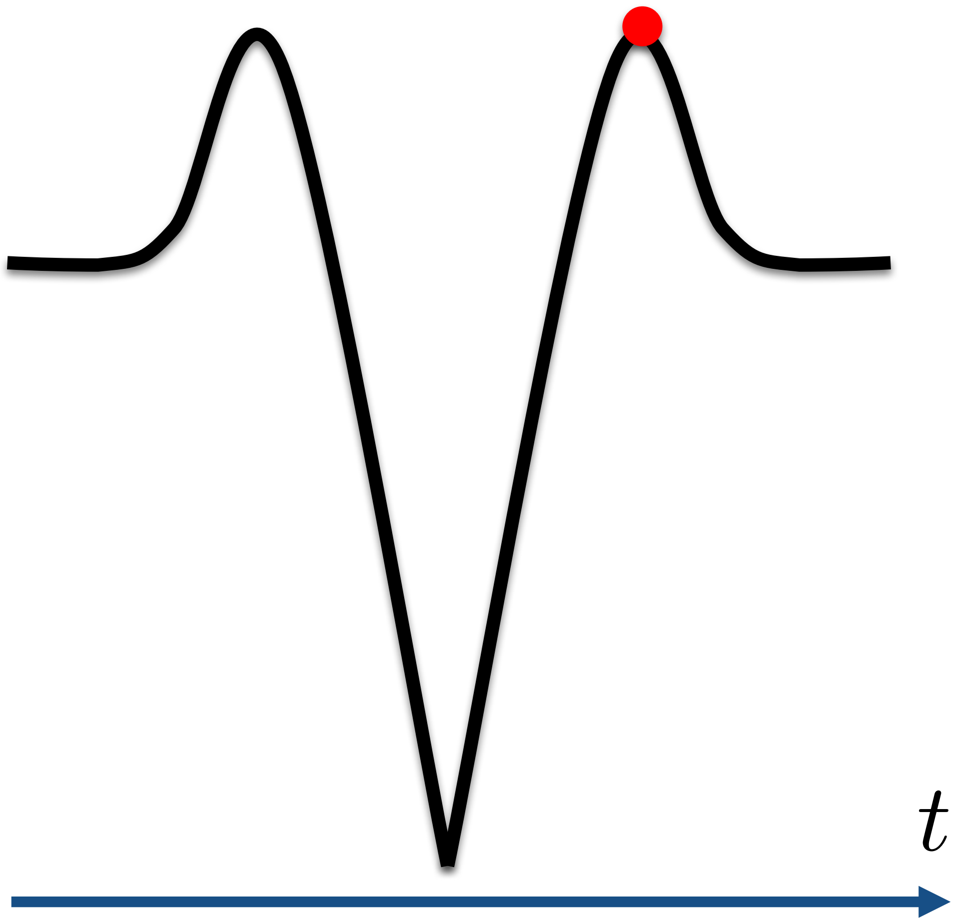

Objective Function

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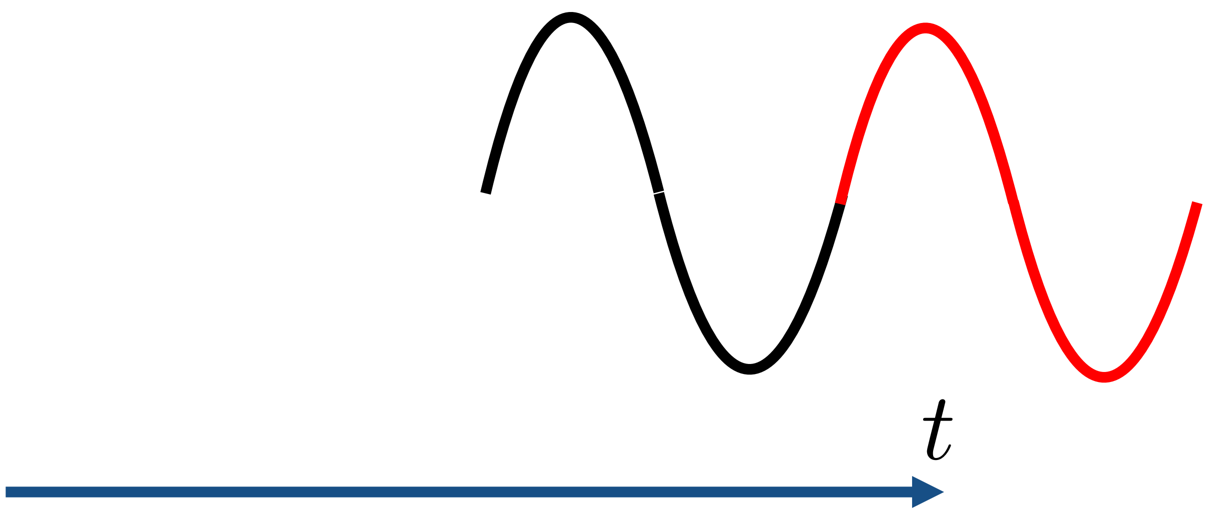
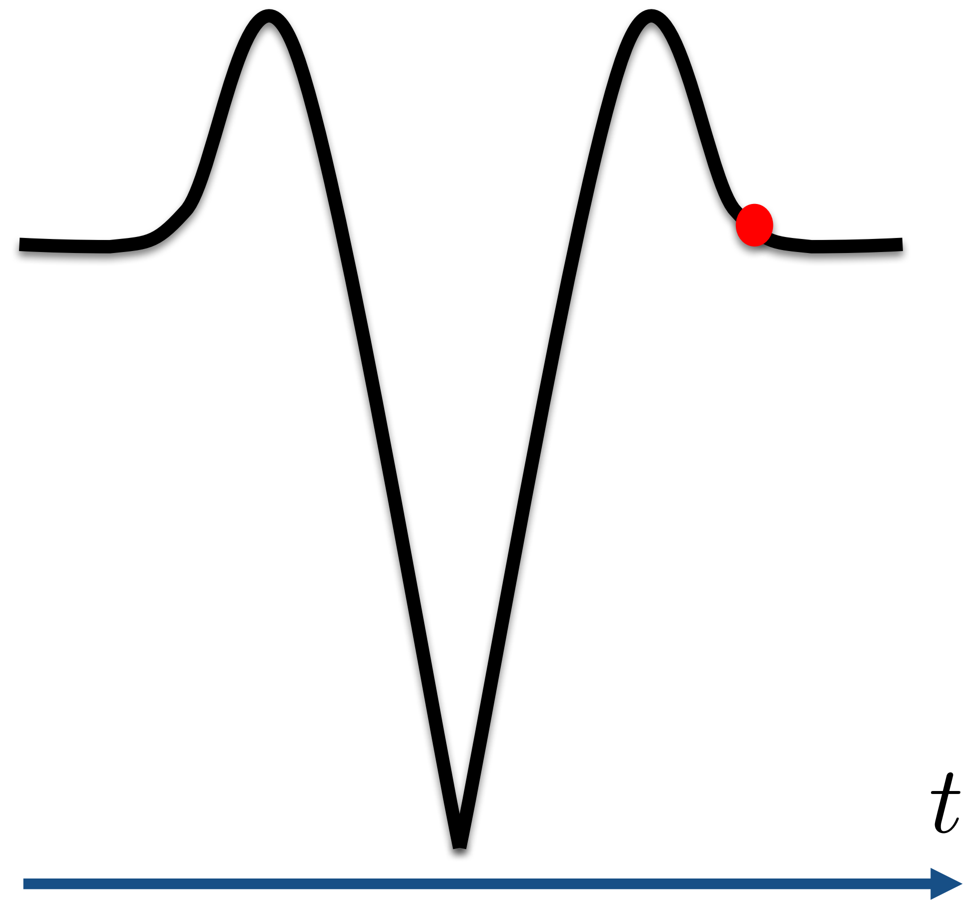


Modeled Data

Observed Data



Objective Function

Behavior of Objective Function

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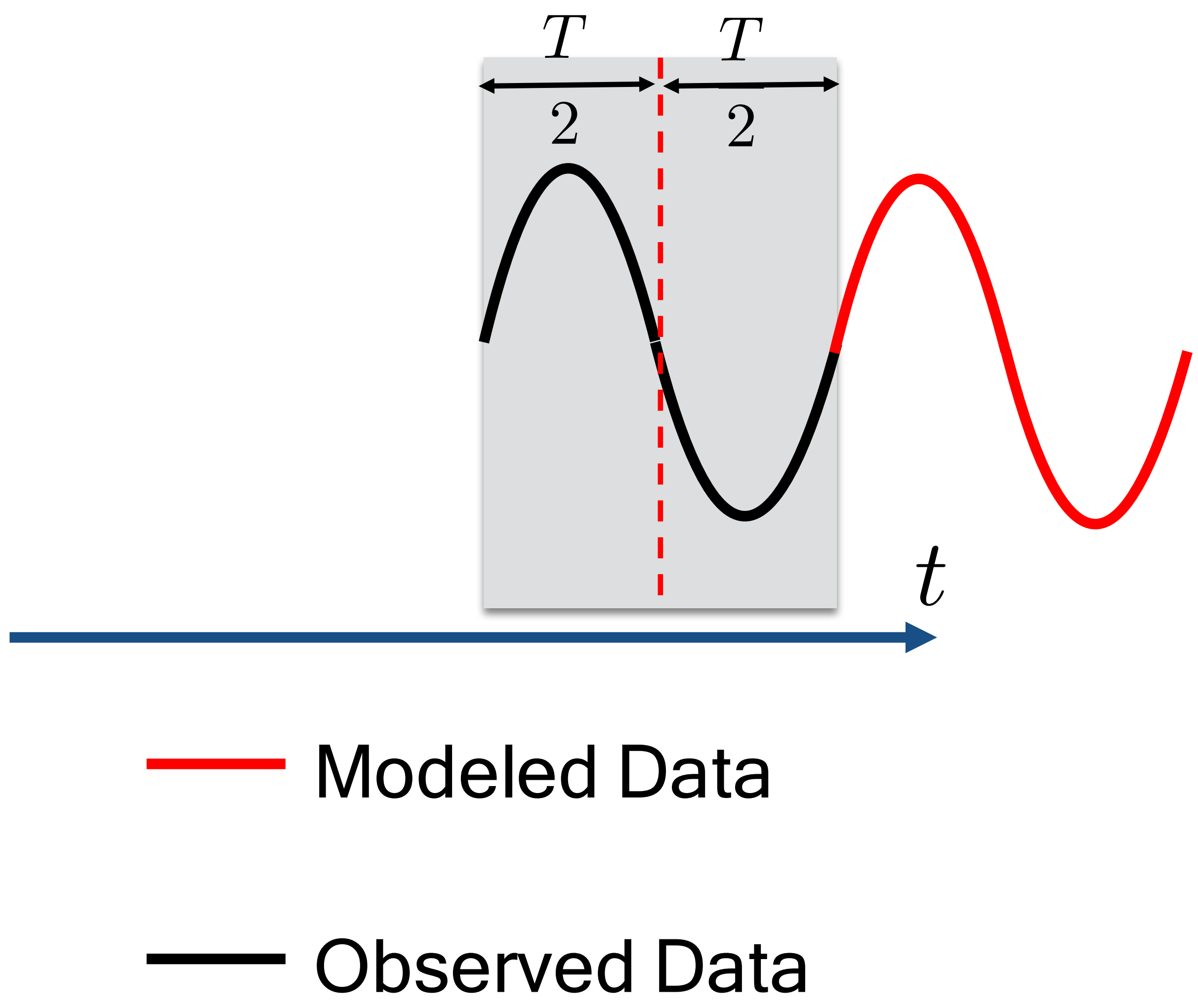
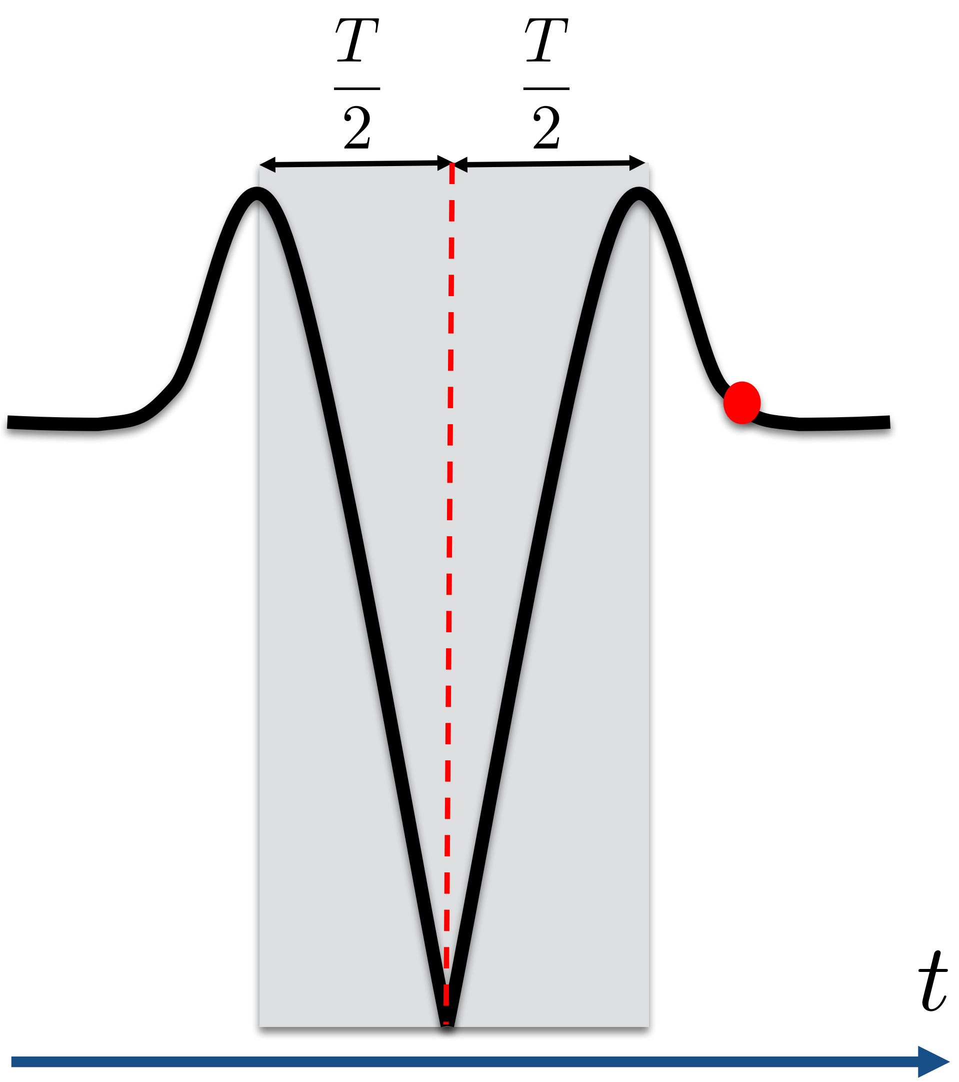




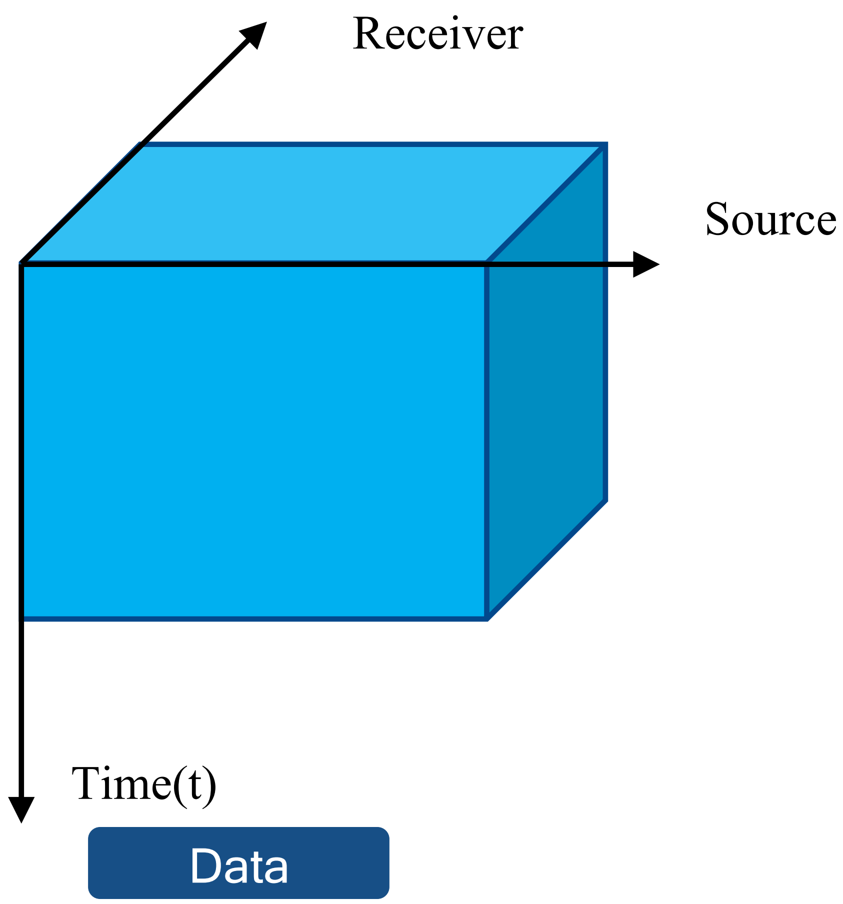
Objective Function

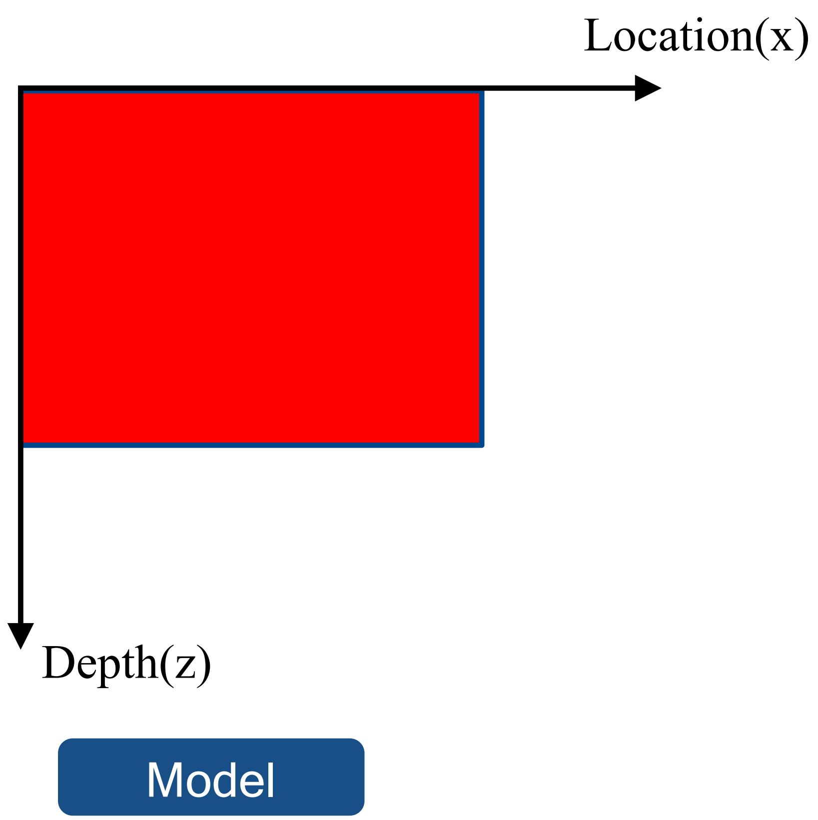
- "Cycle Skipping" problem (local minima)
- start within half wavelength:

  good initial, low frequency, far offset

Model Extension

- Our goal: immune to "cycle skipping"
- Solution: Hug the data!!!





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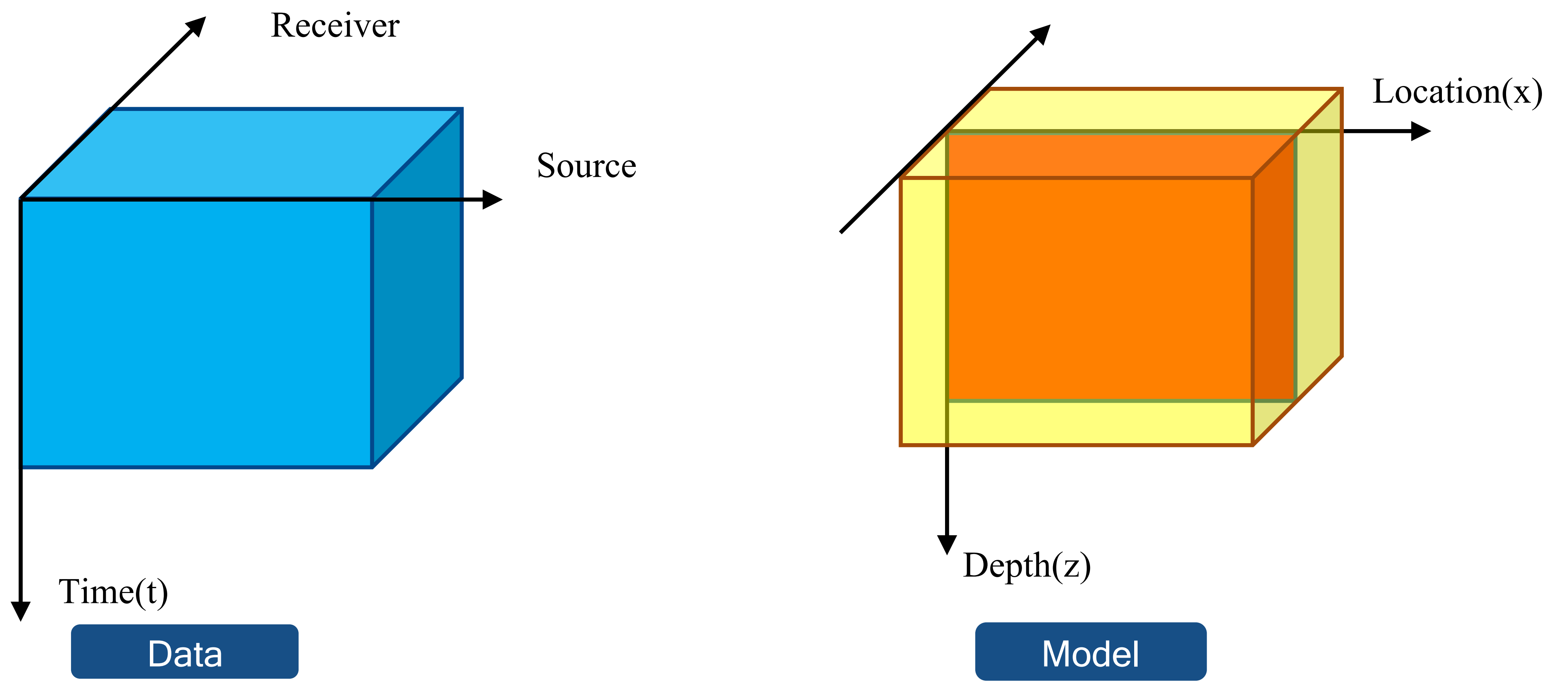

Model Extension

- Extended Model: $\mathcal{M} \to \bar{\mathcal{M}}$
- Extended Modeling

$$\bar{\mathcal{F}} : \bar{\mathcal{M}} \to \mathcal{D} \qquad \mathcal{F}[\mathbf{m}] = \bar{\mathcal{F}}[\bar{\mathbf{m}}]$$

- Extension Parameter:

  Many choices: surface / subsurface offset, source, reflection angle…

- Extended model are not physical

Model Extension + Physical Constraint

Model Extension

- Extended Model: $\mathcal{M} \rightarrow \bar{\mathcal{M}}$
- Extended Modeling

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- Extension Parameter:

Many choices: surface / subsurface offset, source, reflection angle…

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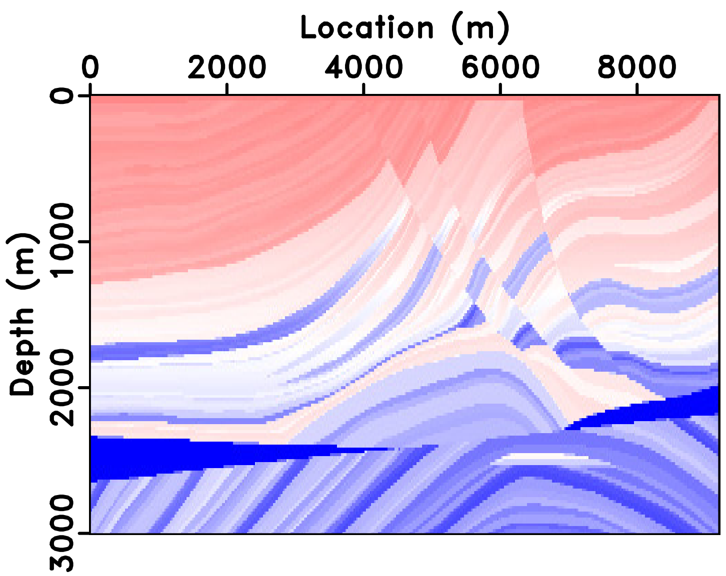
Model Extension + Physical Constraint

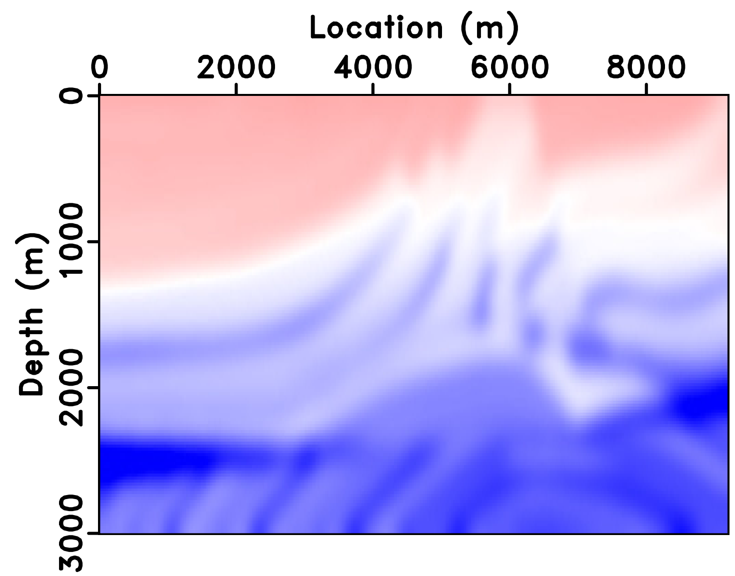
Model Separation

- Separate model into two parts

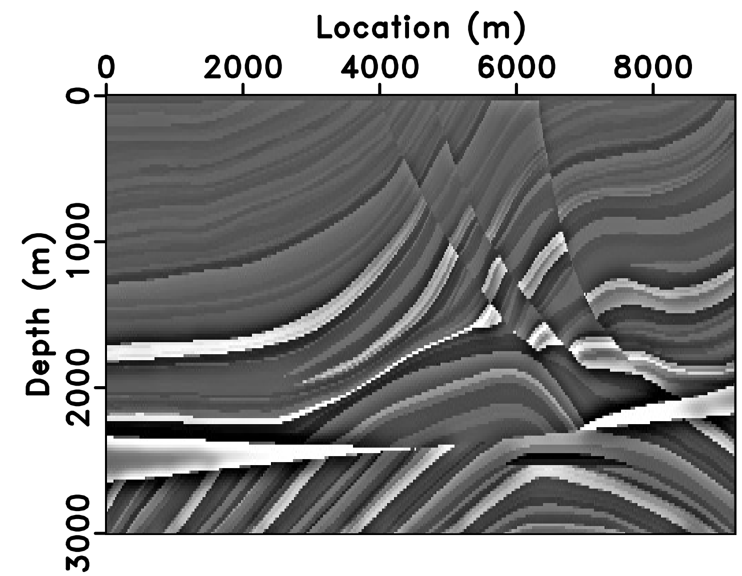
$$\mathbf{m} = \mathbf{m}_0 + \delta\mathbf{m}$$



Velocity Model $\mathbf{m}$



Background Model $\mathbf{m}_0$



Reflectivity Model $\delta\mathbf{m}$

- Two-step Problem

$$\mathcal{F}[\mathbf{m}] \approx \mathcal{F}[\mathbf{m}_0] + F[\mathbf{m}_0]\delta\mathbf{m}$$

Born Modeling Operator

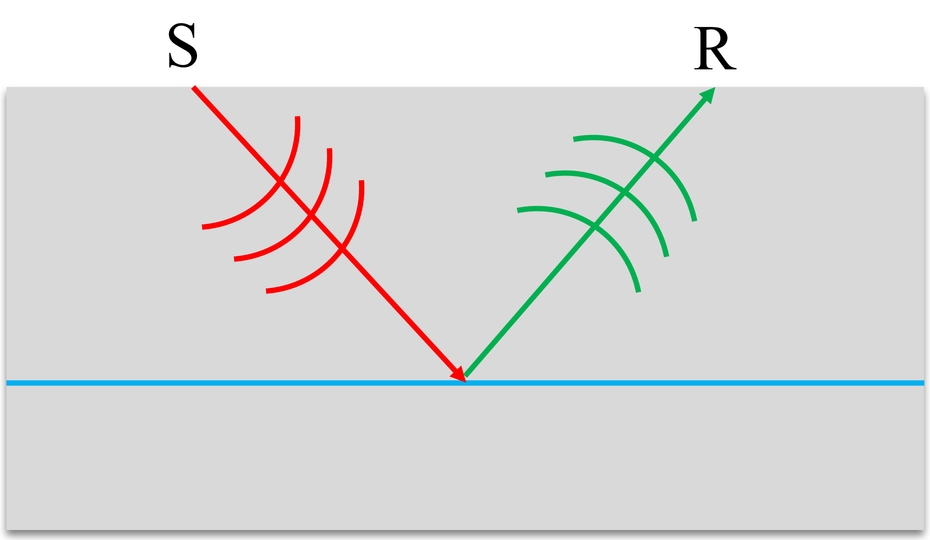
Born modeling and its adjoint

- Born modeling (single scattering) operator

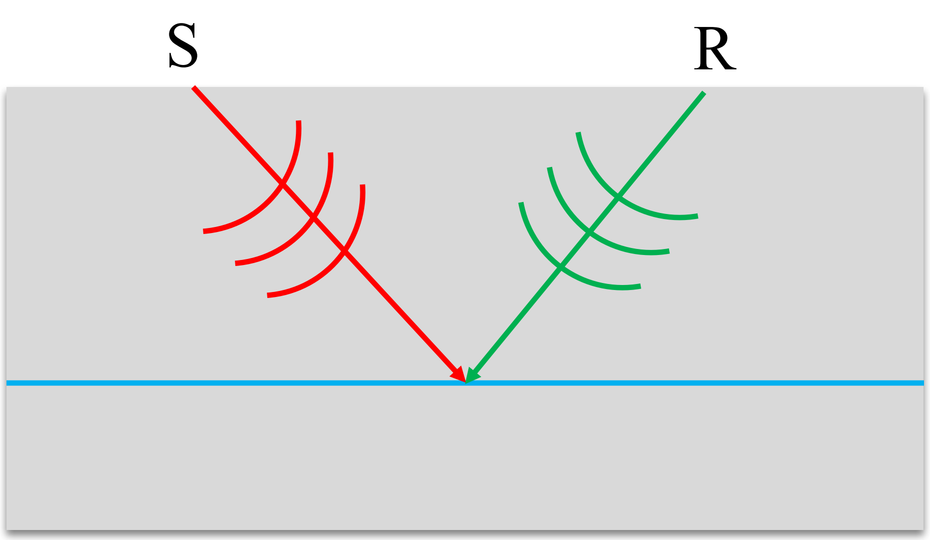
$$(F[v_0]\delta v)(\mathbf{x_s},\mathbf{x_r},t)=\frac{\partial^2}{\partial t^2}\int d\mathbf{x}d\tau G(\mathbf{x_s},\mathbf{x},\tau)\frac{2\delta v(\mathbf{x})}{v_0(\mathbf{x})^3}G(\mathbf{x},\mathbf{x_r},t-\tau)$$

- Adjoint Operator

$$(F^*[v_0]\delta d)(\mathbf{x})=\frac{2}{v_0(\mathbf{x})^3}\int d\mathbf{x_s}d\mathbf{x_r}dtd\tau G(\mathbf{x_s},\mathbf{x},\tau)G(\mathbf{x},\mathbf{x_r},t-\tau)\frac{\partial^2}{\partial t^2}\delta d(\mathbf{x_s},\mathbf{x_r},t)$$



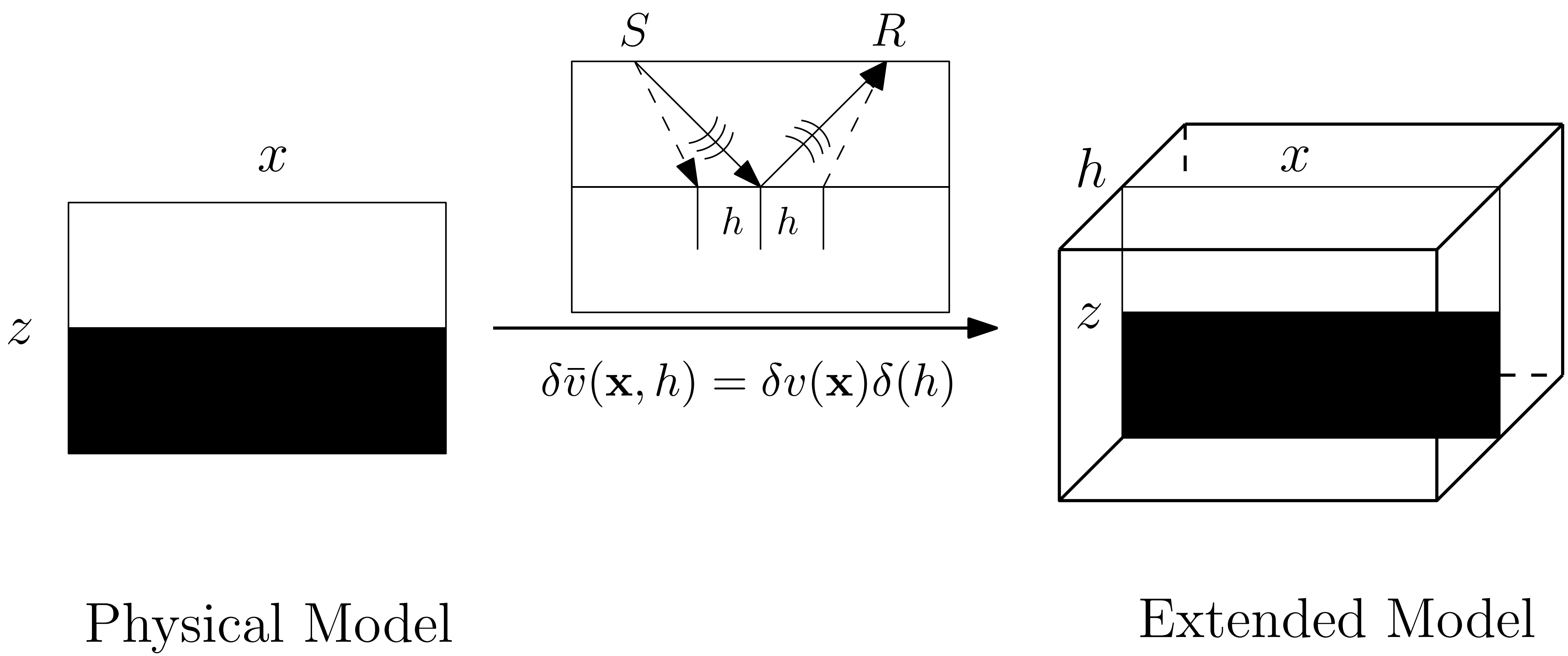
Born Modeling



Adjoint Operator

Model Extension + Model Separation

- Subsurface offset extension



- Physical Meaning

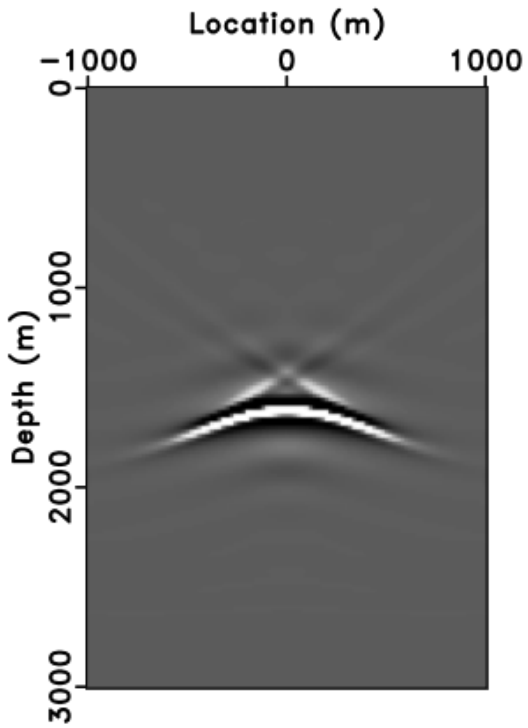
Action at a distance: stress leads to strain at a distance

Extended Born modeling Operator

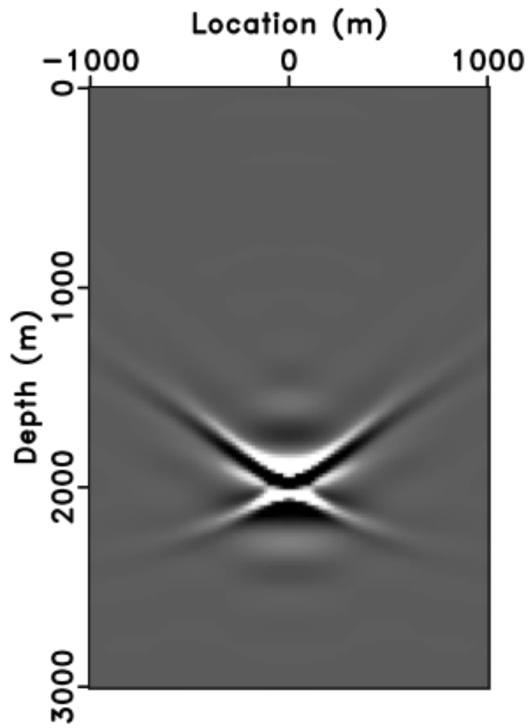
➢ Extended Born modeling operator and its adjoint

$$(\bar{F}[v_0]\delta\bar{v})(\mathbf{x_s},\mathbf{x_r},t)=\frac{\partial^2}{\partial t^2}\int d\mathbf{x}d\mathbf{h}d\tau G(\mathbf{x_s},\mathbf{x}-\mathbf{h},\tau)\frac{2\delta\bar{v}(\mathbf{x},\mathbf{h})}{v_0(\mathbf{x})^3}G(\mathbf{x}+\mathbf{h},\mathbf{x_r},t-\tau)$$

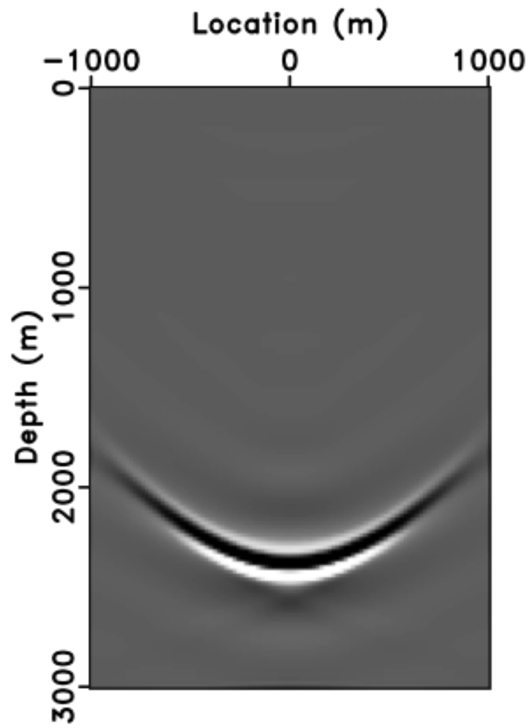
$$(\bar{F}^*[v_0]\delta d)(\mathbf{x},\mathbf{h})=\frac{2}{v_0(\mathbf{x})^3}\int d\mathbf{x_s}d\mathbf{x_r}dtd\tau G(\mathbf{x_s},\mathbf{x}-\mathbf{h},\tau)G(\mathbf{x}+\mathbf{h},\mathbf{x_r},t-\tau)\frac{\partial^2}{\partial t^2}\delta d(\mathbf{x_s},\mathbf{x_r},t)$$



V=2km/s



V=2.5km/s



V=3km/s

Correct Velocity Model

MVA

- Partially Linearized Problem

$$\bar{F}[\mathbf{m}_0]\delta\bar{\mathbf{m}} \simeq \mathbf{d} - \mathcal{F}[\mathbf{m}_0]$$

- Find $\delta\bar{\mathbf{m}}$ to fit the data
- Find $\mathbf{m}_0$ to satisfy the semblance condition

- Migration Velocity Analysis (MVA) (Shen and Symes,2003)

Update velocity based on migrated image volume

MVA via DSO

$$J_{\text{MVA}}[\mathbf{m}_0] = \frac{1}{2}||AI(x,z,h)||^2$$

- A : Annihilator, $A=h$
- $I(x,z,h)$ can be computed via various migration

- J is quadratic in image and data (implicitly), regardless of the frequency components
- Smooth in velocity

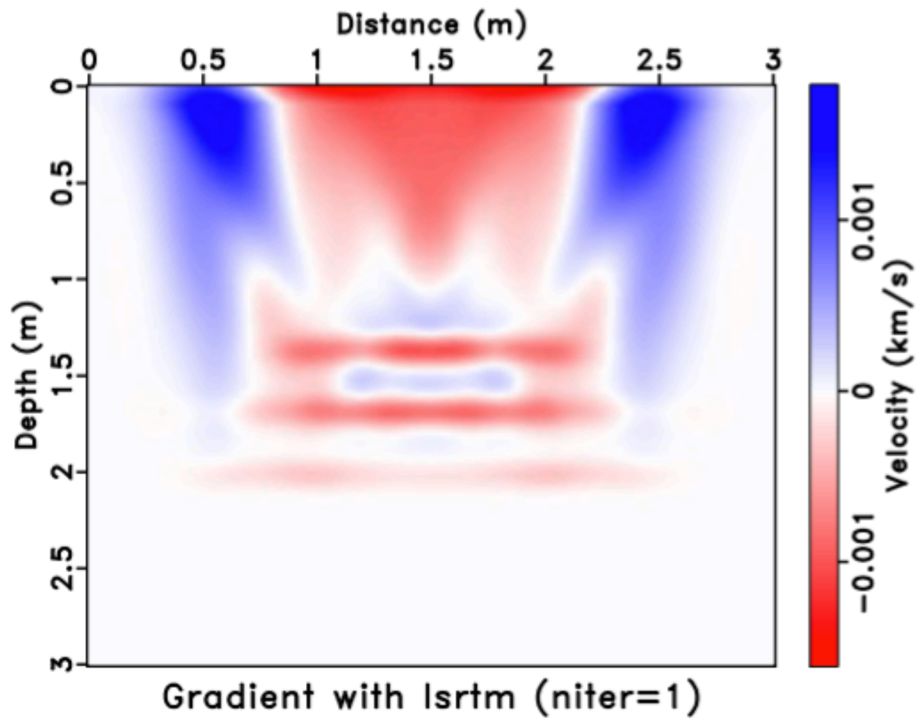
Only choice (Stolk & Symes, 2003)

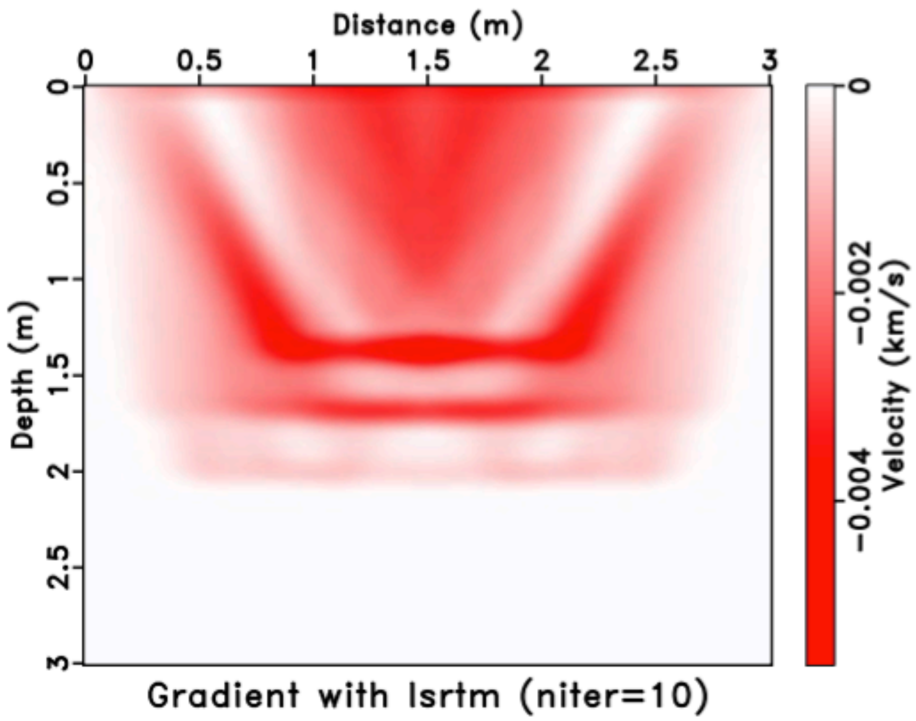
"Gradient Artifacts"

- Gradient artifacts : updating to the wrong direction

(Fei and Williamson, 2010; Vyas and Tang, 2010)

- Root reason: imperfect image volume





(Liu and Symes, 2014)

Approximate Inverse Operator

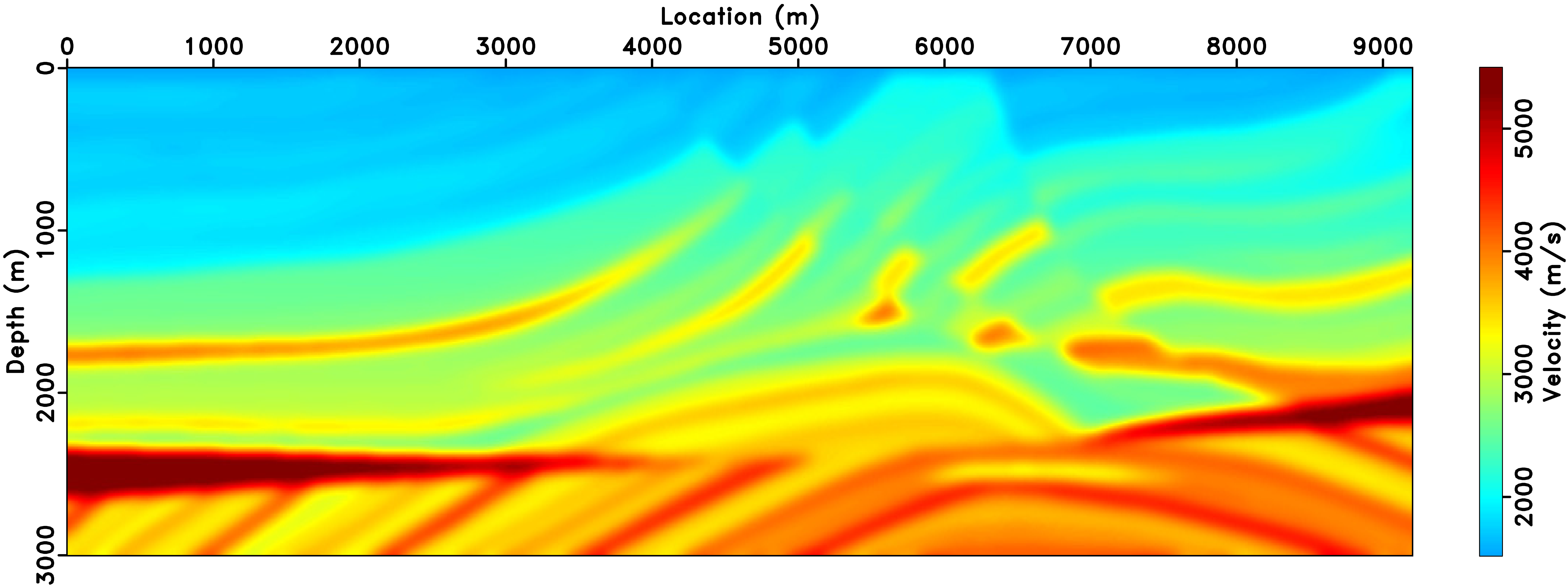
$$\bar{F}^{\dagger} \simeq W_{model}^{-1} \bar{F}^{T} W_{data}$$

(Ten Kroode, 2012; Hou and Symes,2015)

- $W_{model}^{-1} = 4v_0^5 |k_{xz}||k_{hz}| \quad W_{data} = I_t^4 D_{z_S} D_{z_R}$
- Derivation is based on High Frequency Approx.
- Implementation doesn't involve any ray tracing
- Invert the data even when velocity is wrong

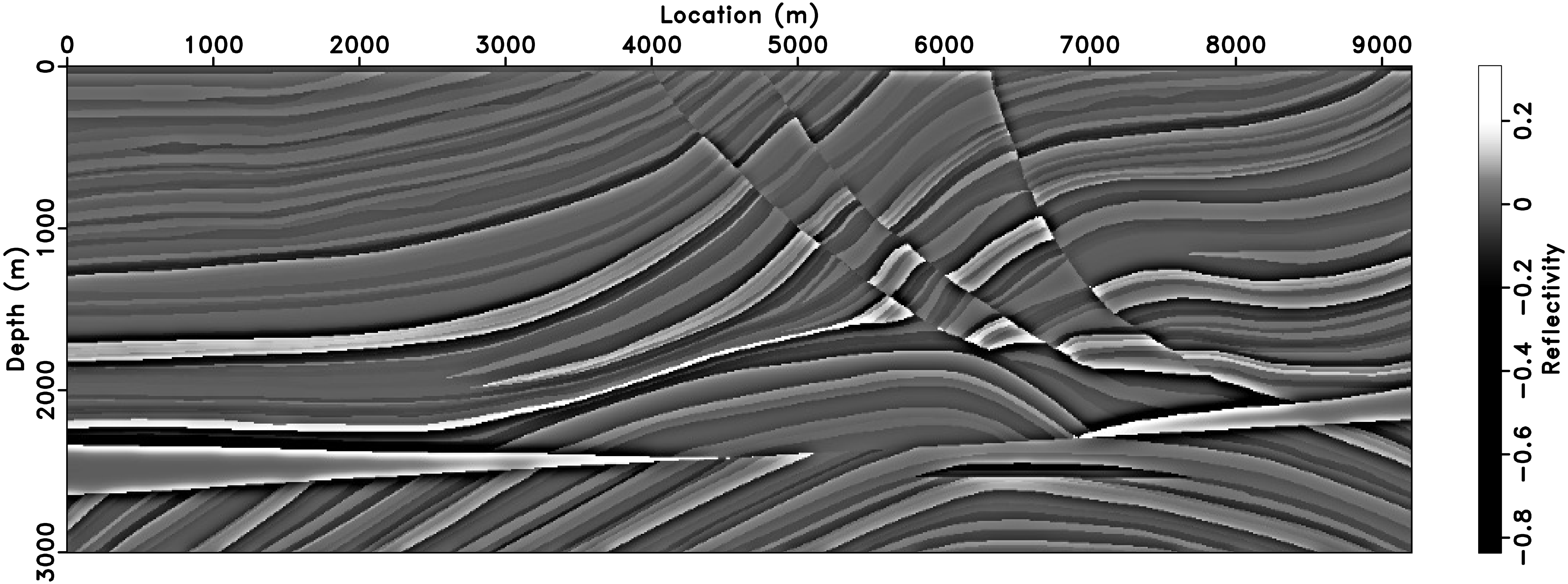
Approximate Inverse Operator

- 2-8 finite difference, 231 shots & 461 receivers
- 2.5-5-30-35Hz Bandpass wavelet
- 1ms time sample, 10m grid interval



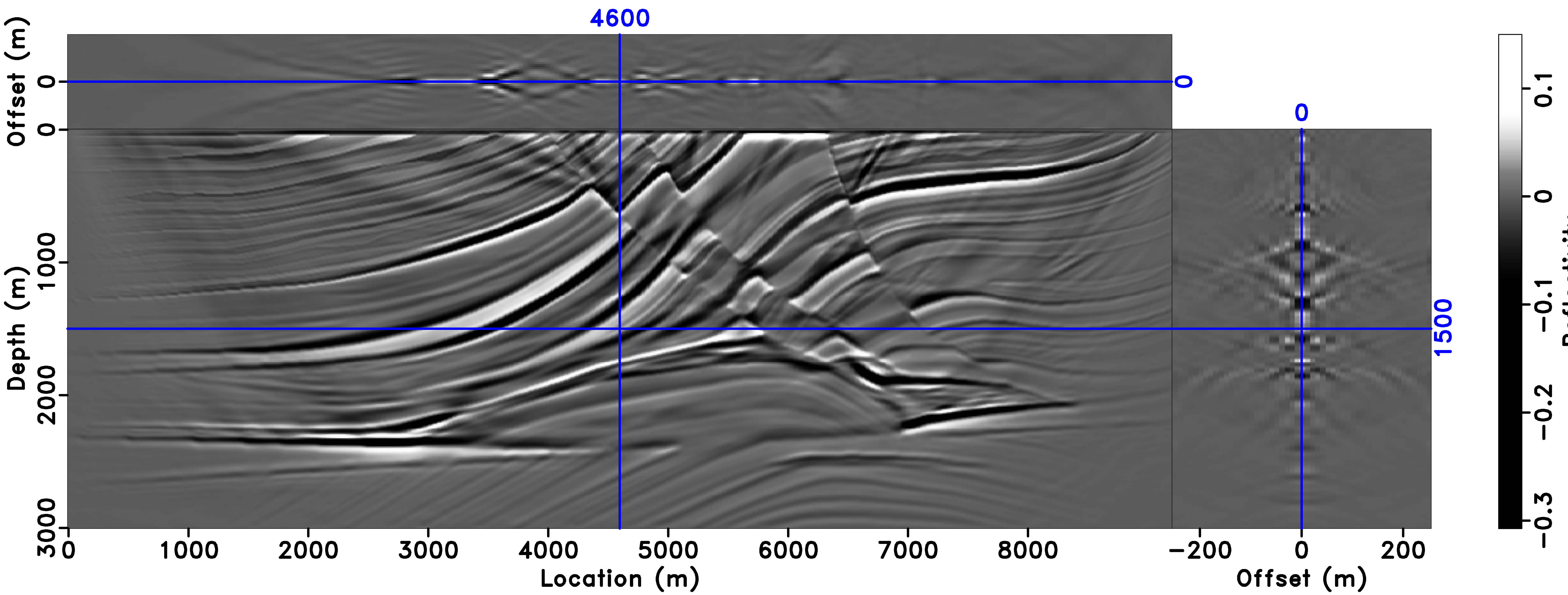
Background Velocity Model

Approximate Inverse Operator



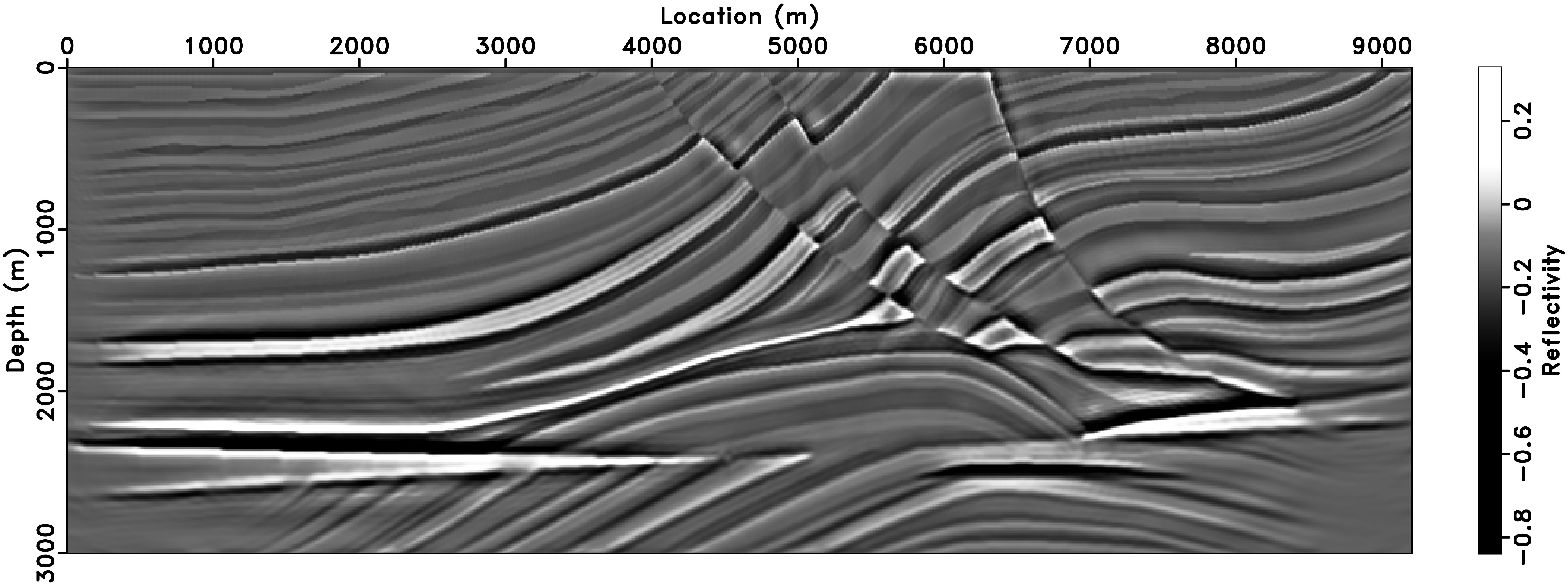
Reflectivity Model

Approximate Inverse Operator



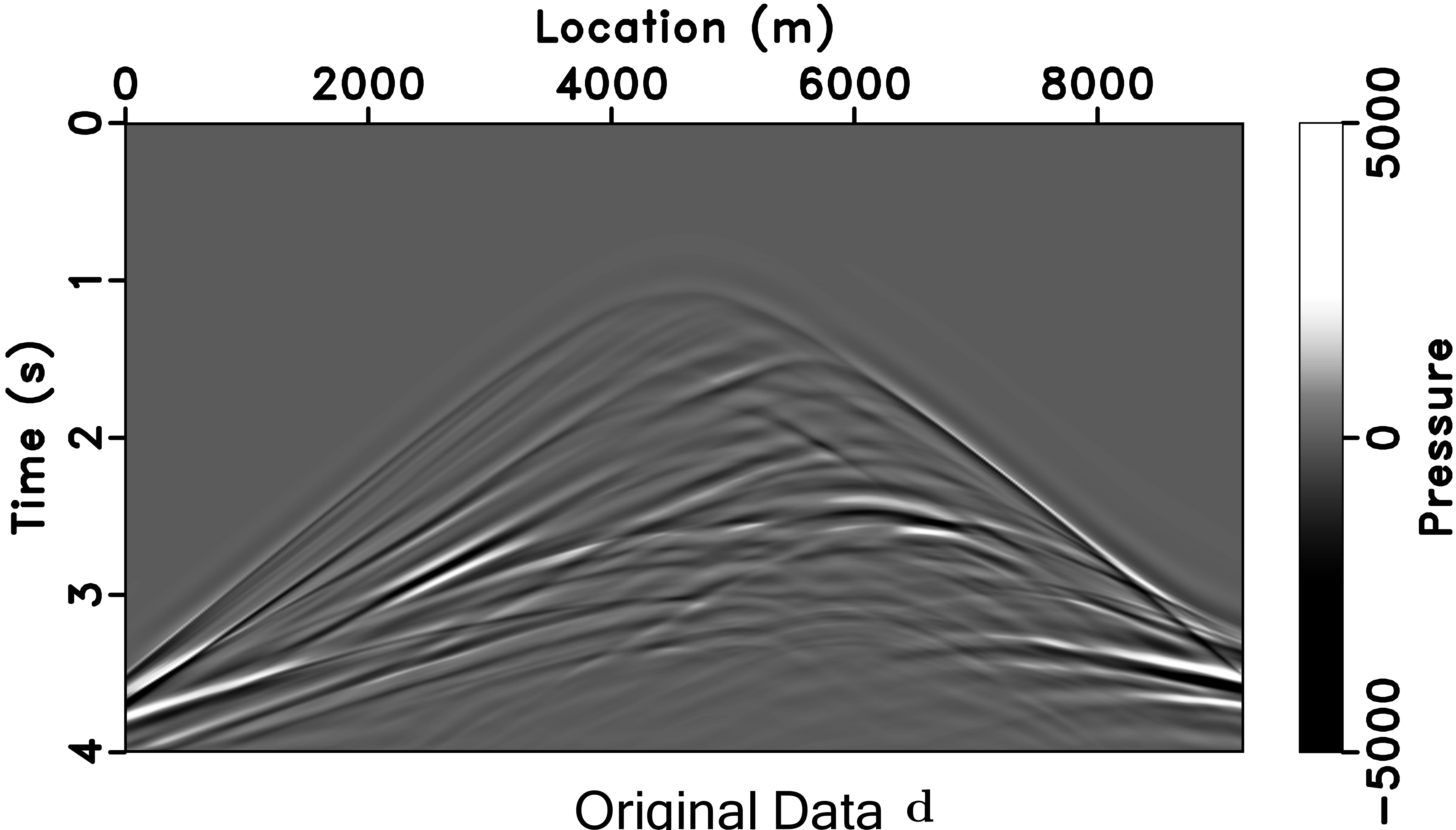
Inverted Model $\bar{F}^{\dagger}\mathbf{d}$

Approximate Inverse Operator



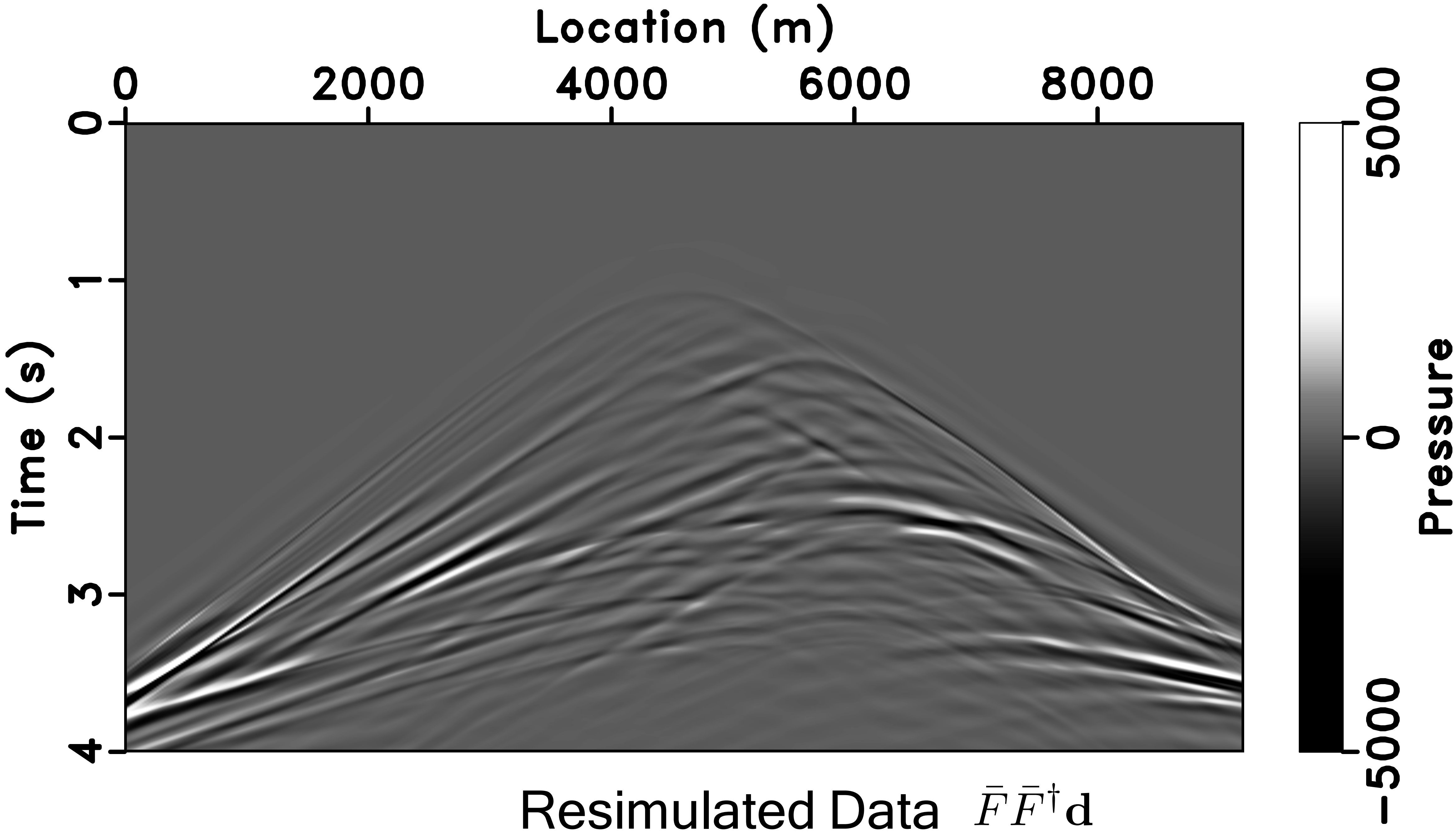
Stacked Model $\sum_h I(\mathbf{x}, h)$

Approximate Inverse Operator



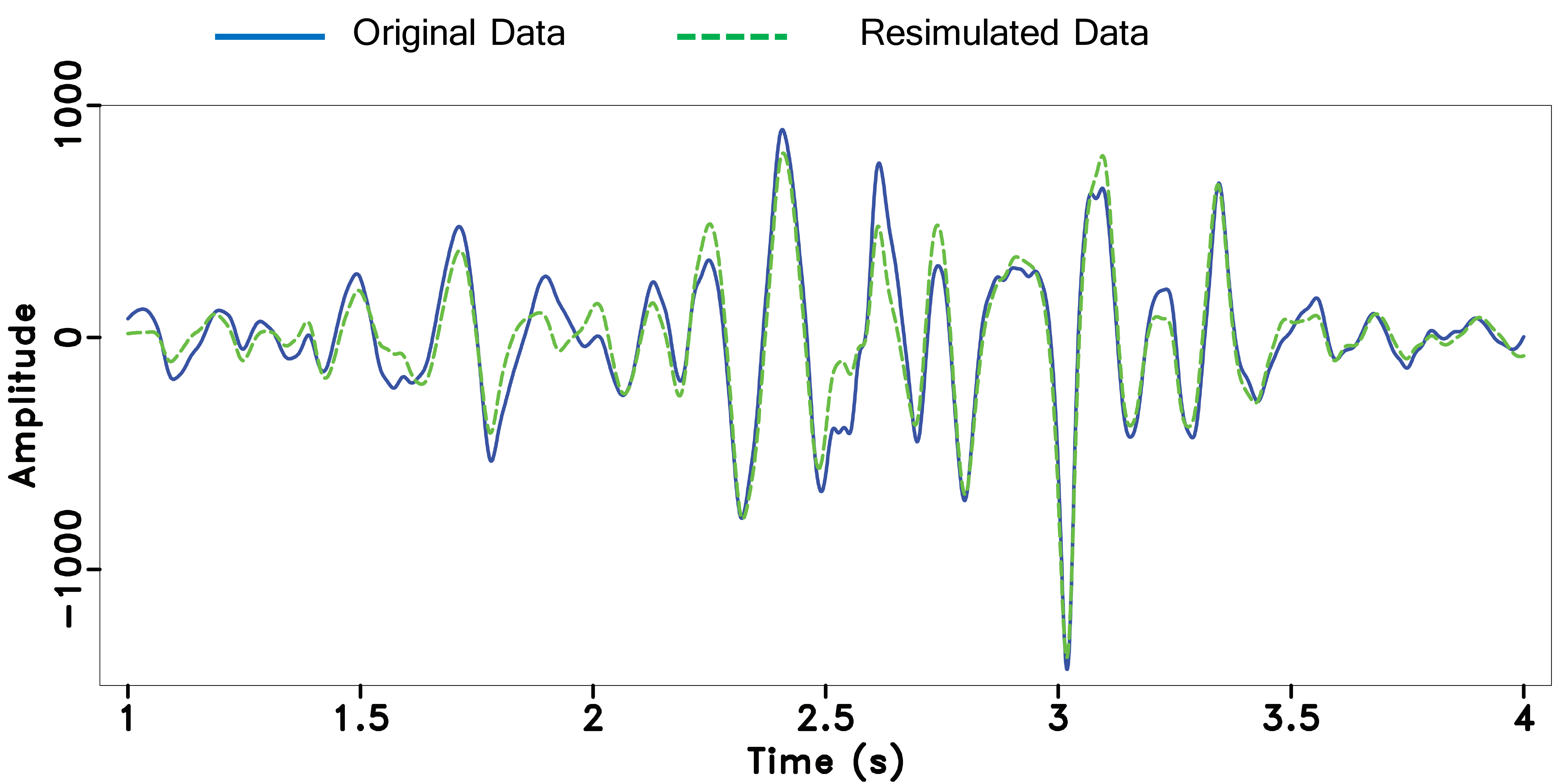
Original Data d

Approximate Inverse Operator



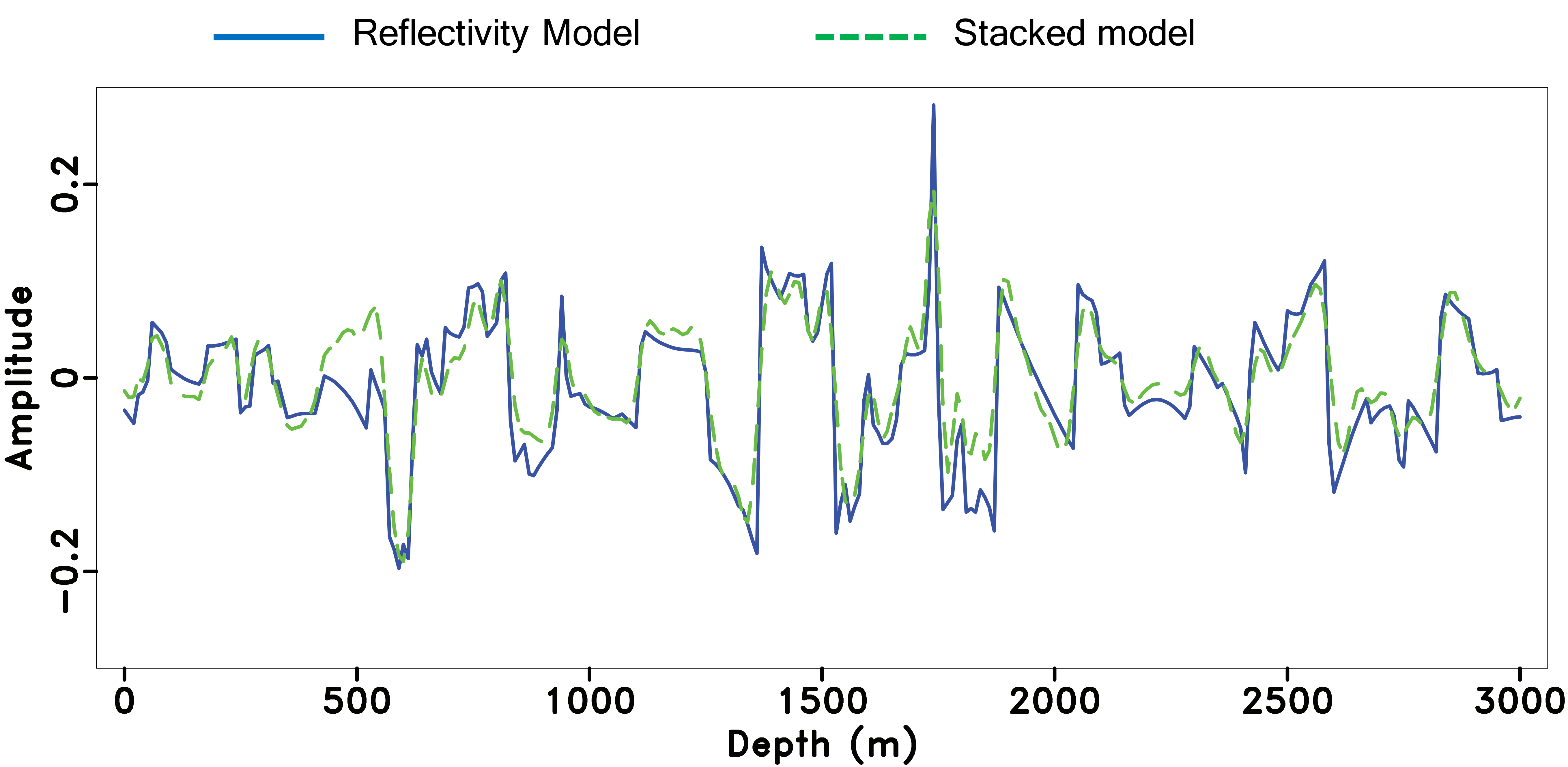
Resimulated Data $\bar{F}\bar{F}^{\dagger}\mathbf{d}$

Approximate Inverse Operator



Single Data Trace Comparison

Approximate Inverse Operator



Single Image Trace Comparison

Imaging Operators

- **Conventional RTM Operator**

$$I(\mathbf{x},h) = \int d\mathbf{x_s} d\mathbf{x_r} dt d\tau G(\mathbf{x_s},\mathbf{x}-h,\tau)G(\mathbf{x}+h,\mathbf{x_r},t-\tau)d(\mathbf{x_s},\mathbf{x_r},t)$$

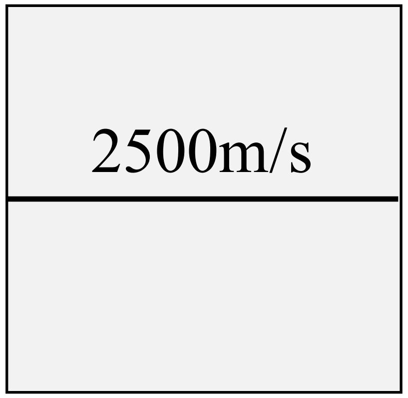
- **Adjoint Operator**

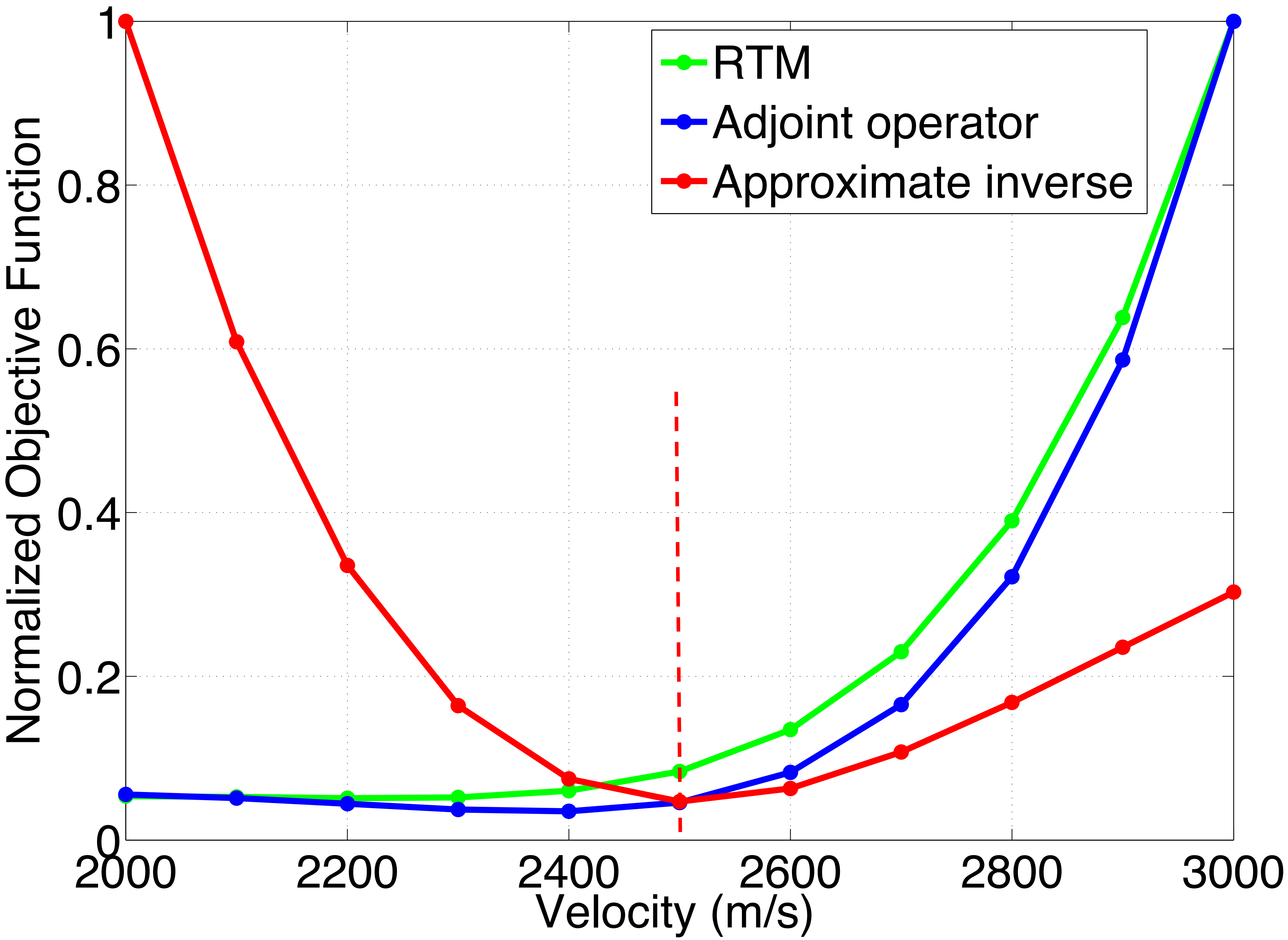
$$I(\mathbf{x},h) = \frac{2}{v_0(\mathbf{x})^3}\int d\mathbf{x_s} d\mathbf{x_r} dt d\tau G(\mathbf{x_s},\mathbf{x}-h,\tau)G(\mathbf{x}+h,\mathbf{x_r},t-\tau)\frac{\partial^2}{\partial t^2}d(\mathbf{x_s},\mathbf{x_r},t)$$

- **Approximate Inverse Operator**

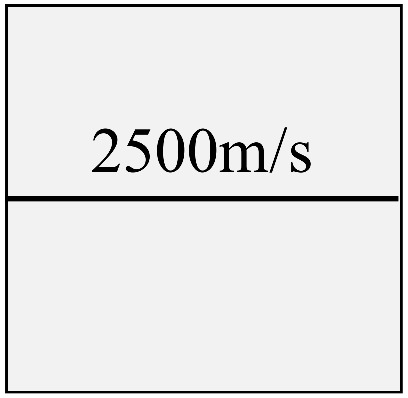
$$\bar{F}^\dagger \simeq W_{model}^{-1}\bar{F}^T W_{data}$$

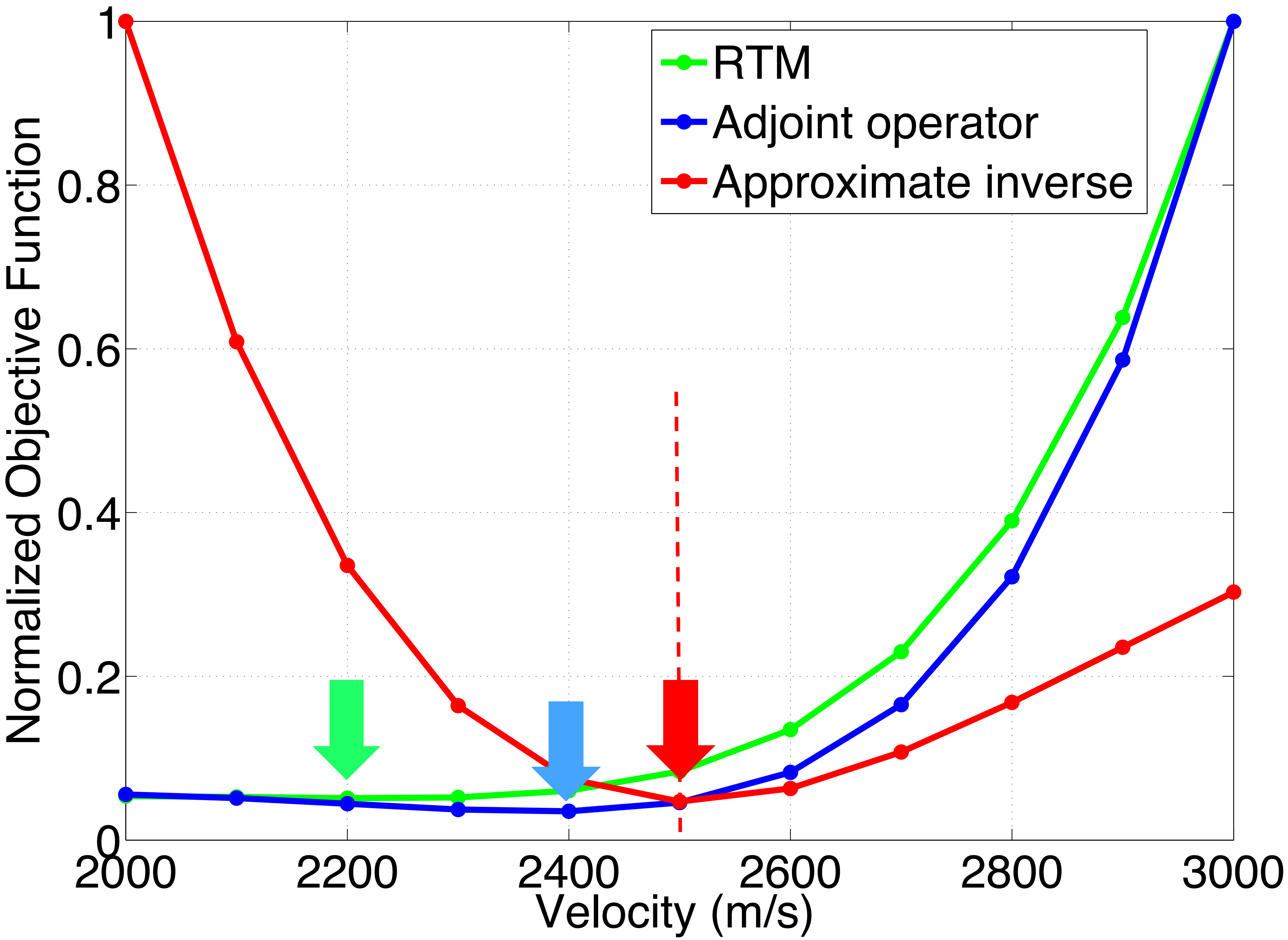
Objective Function Behavior





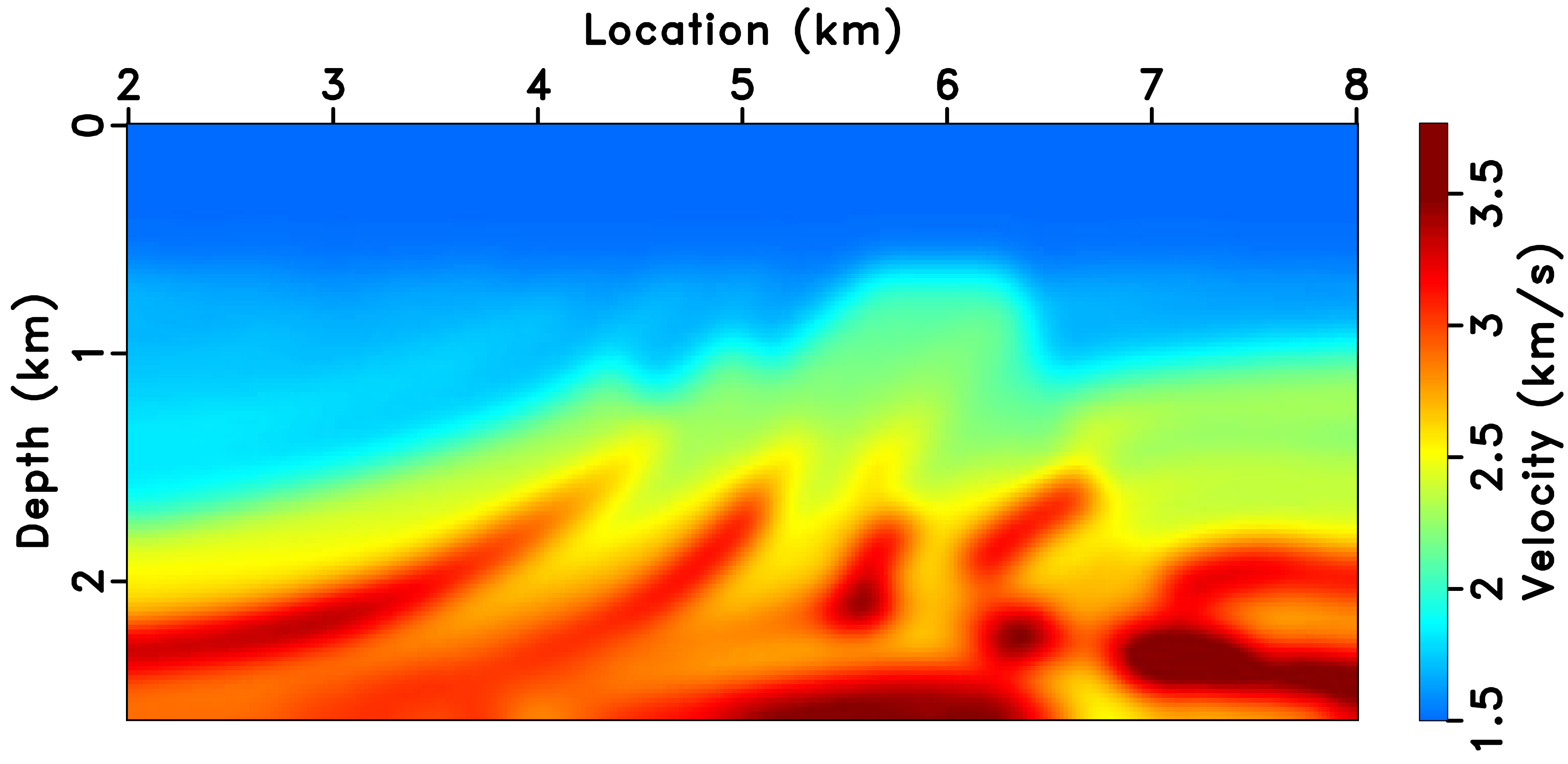
Objective Function Behavior





Numerical Experiment

- 2-8 finite difference, 151 shots & 301 receivers
- 2.5-5-20-25Hz Bandpass wavelet
- 2ms time sample, 20m grid interval



True Model

Numerical Experiment

- 2-8 finite difference, 151 shots & 301 receivers
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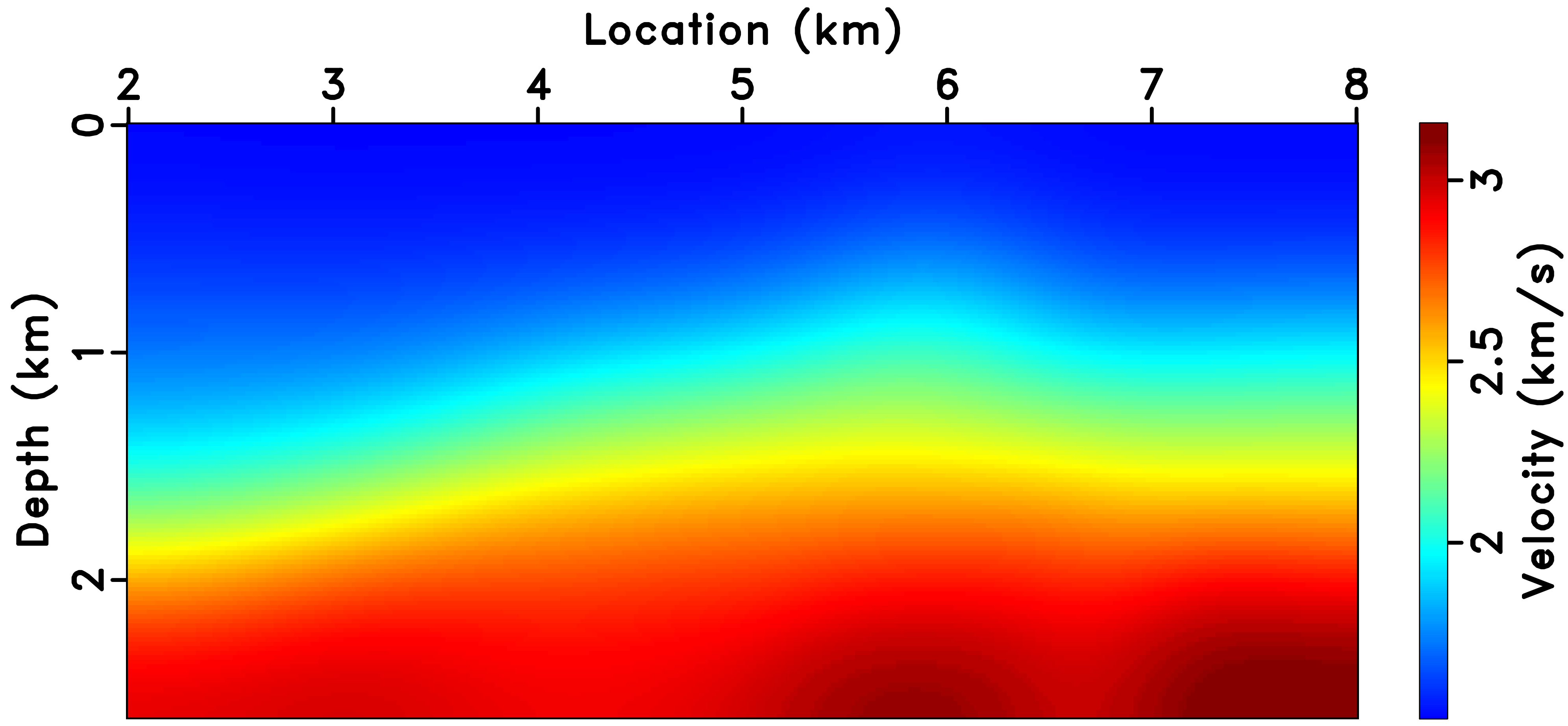
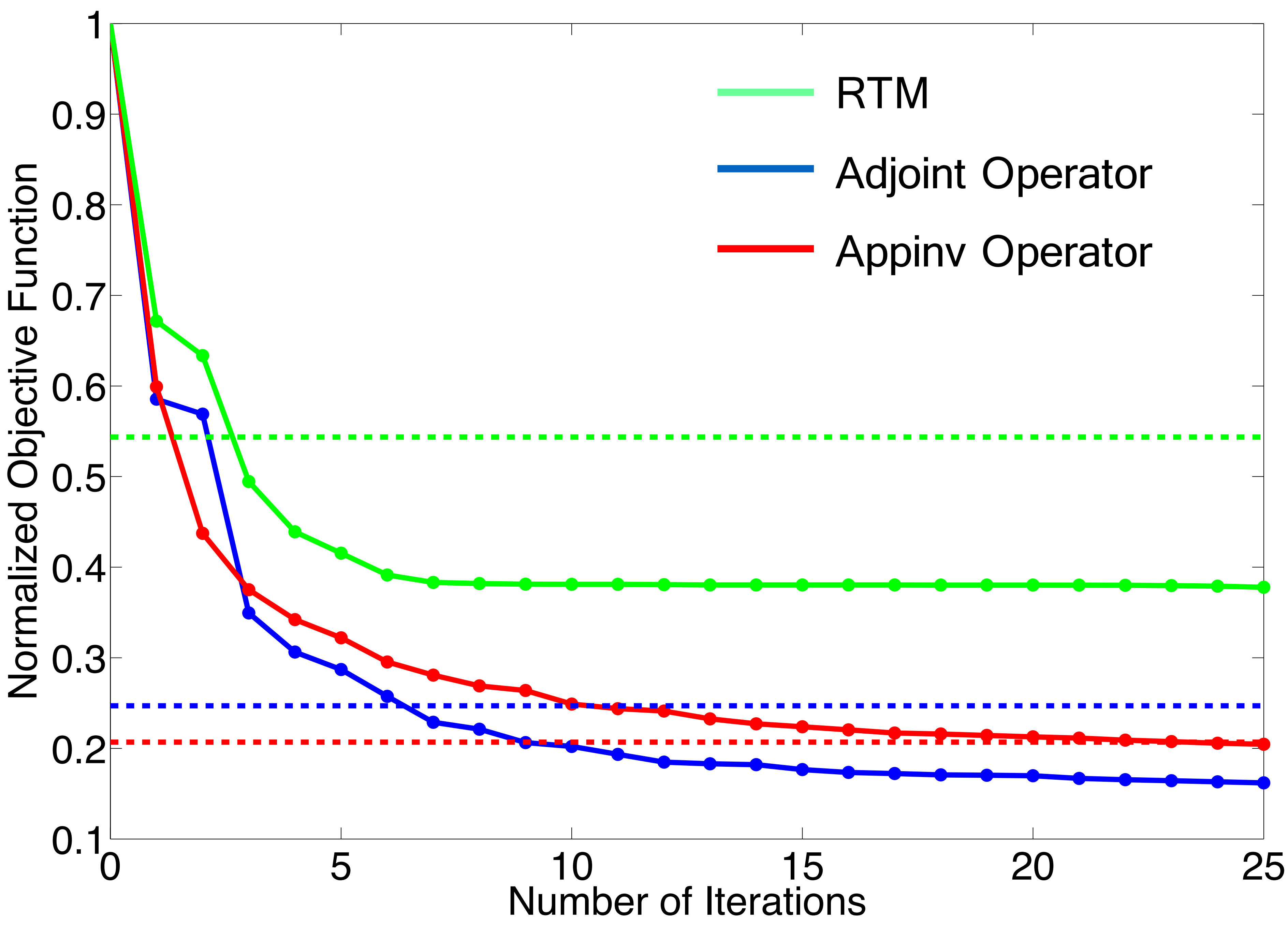
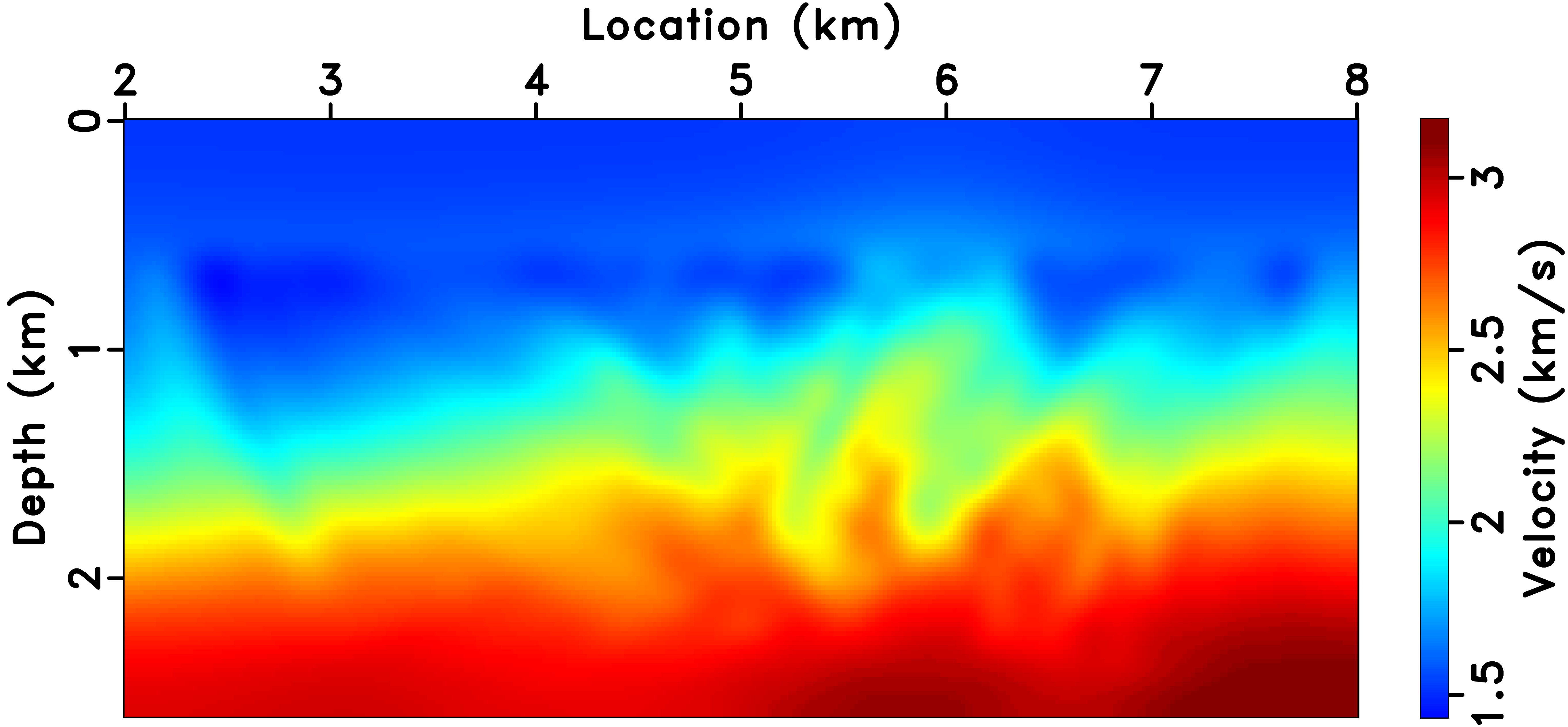


Initial Model

Numerical Experiment

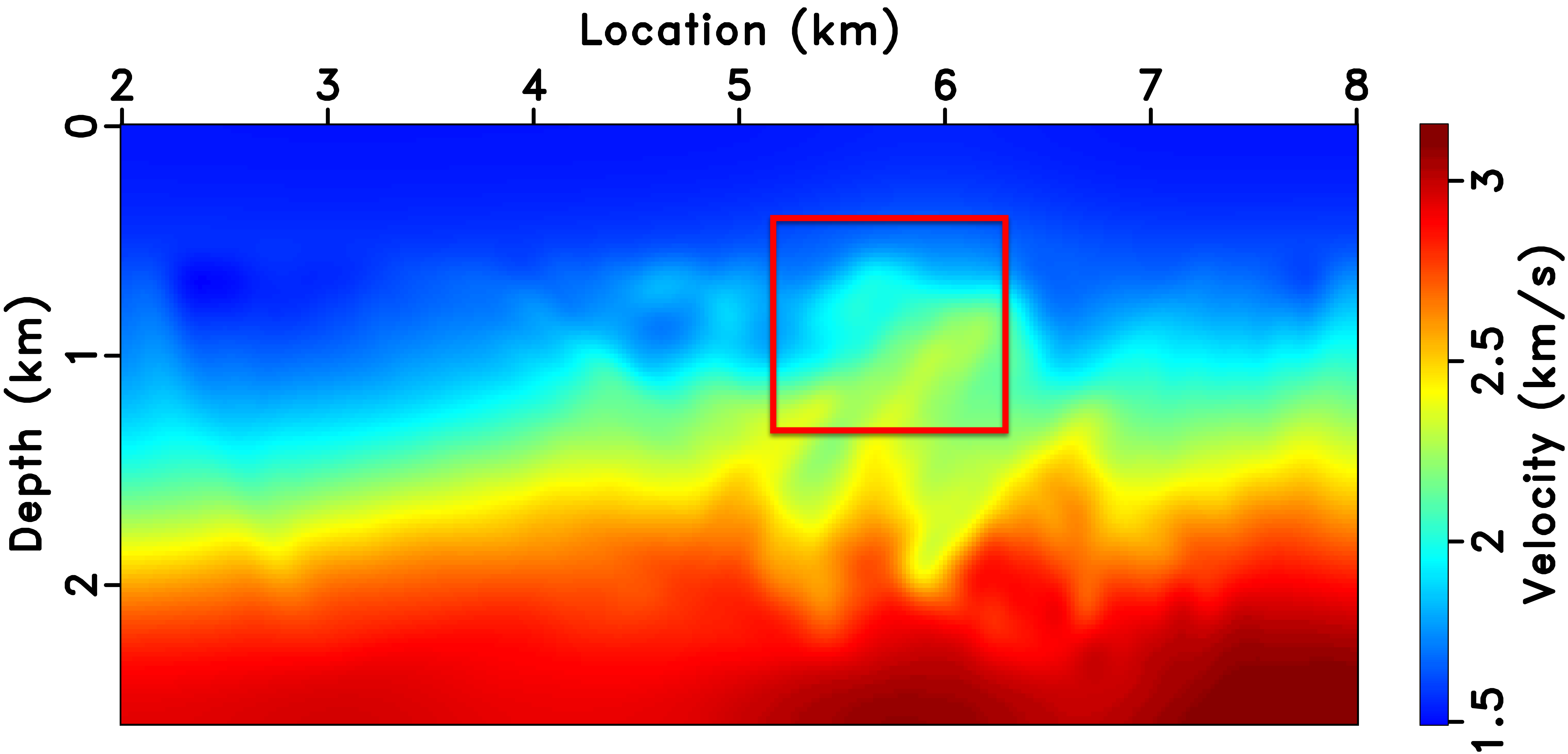


Numerical Experiment



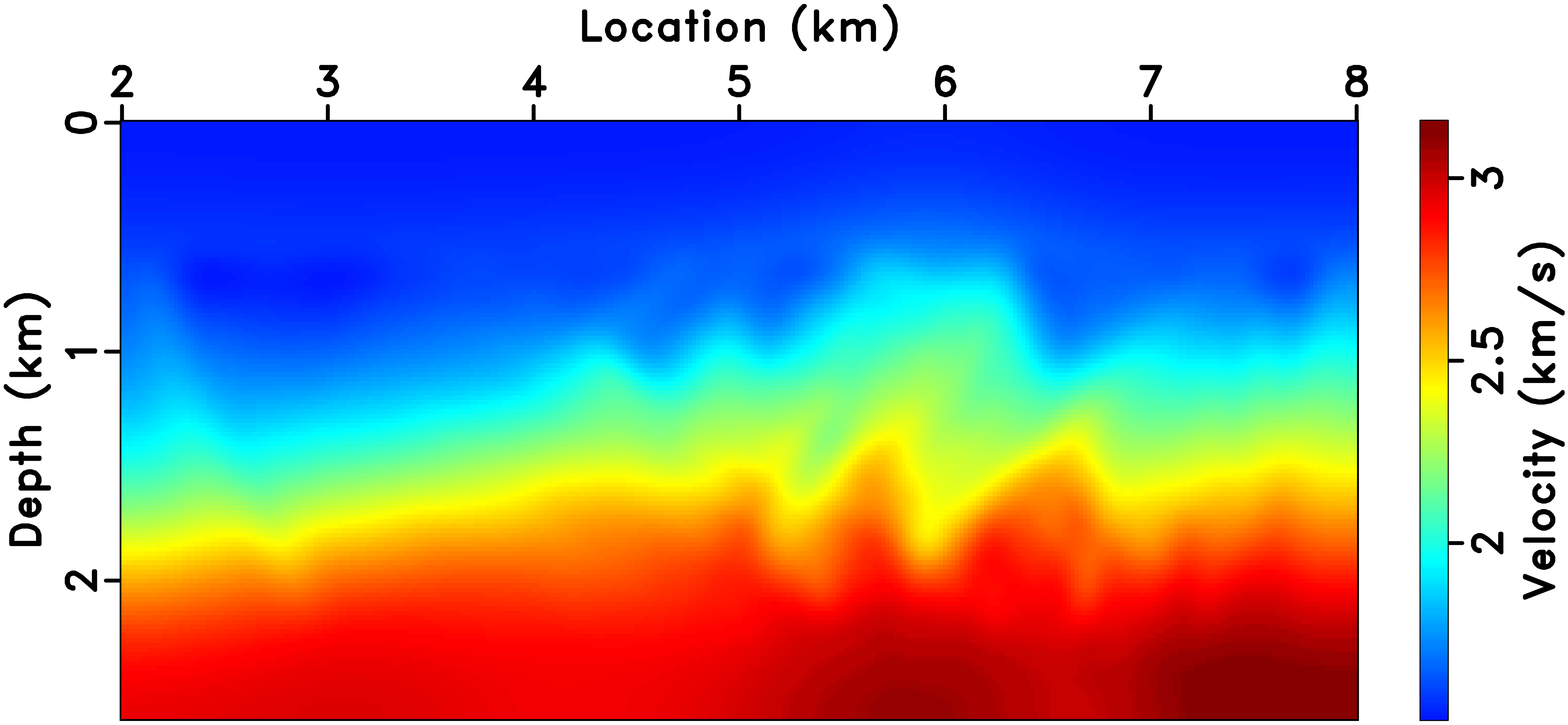
VA with Conventional RTM Operator

Numerical Experiment



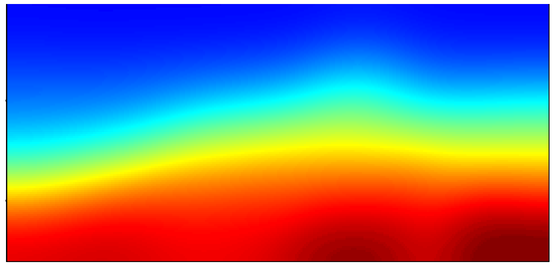
VA with Adjoint Operator

Numerical Experiment



VA with Appinv. Operator

Numerical Experiment



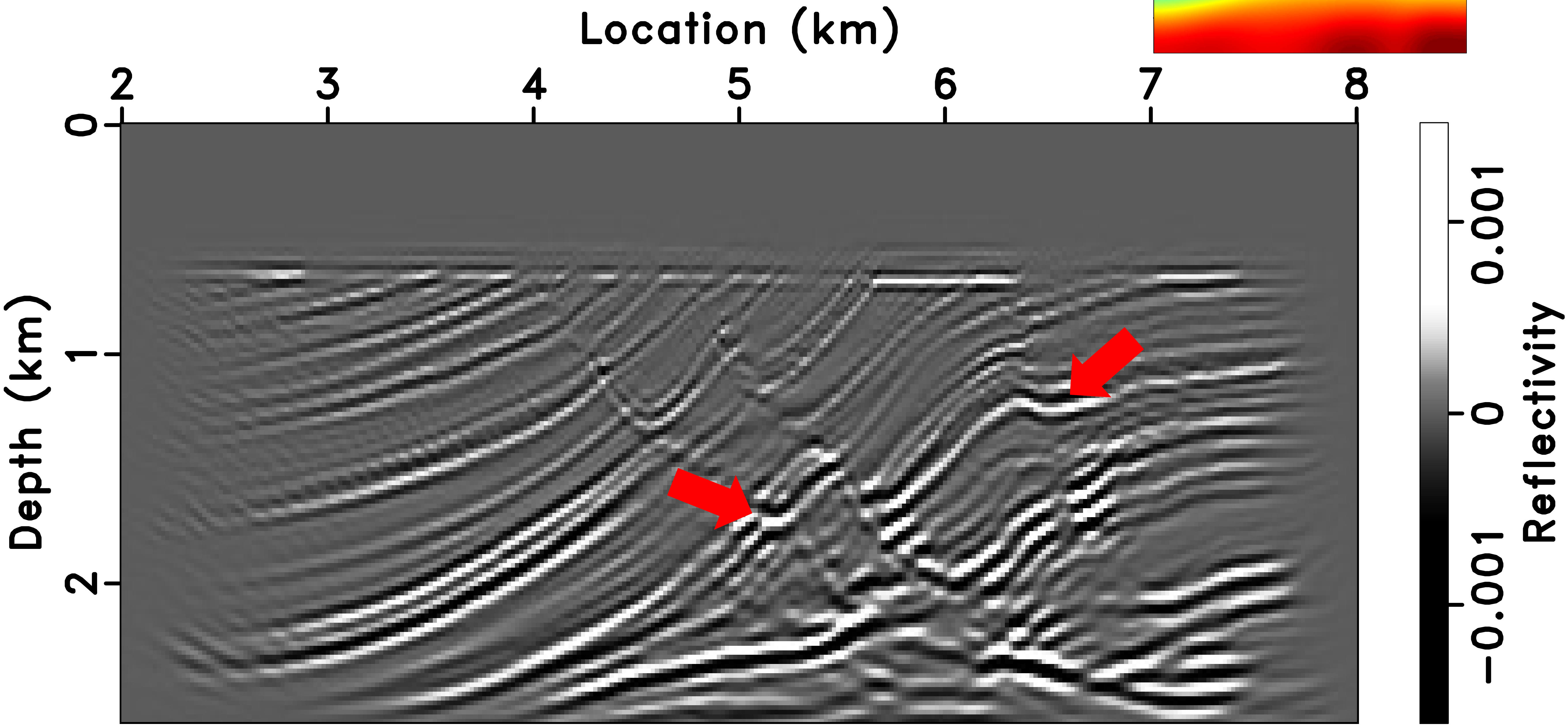
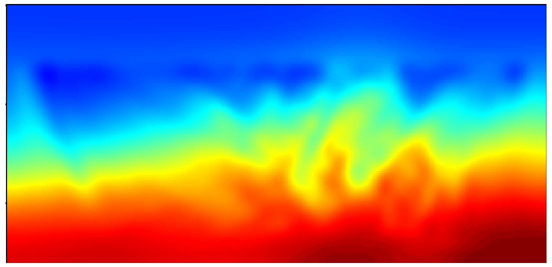


Image with Initial Model

Numerical Experiment



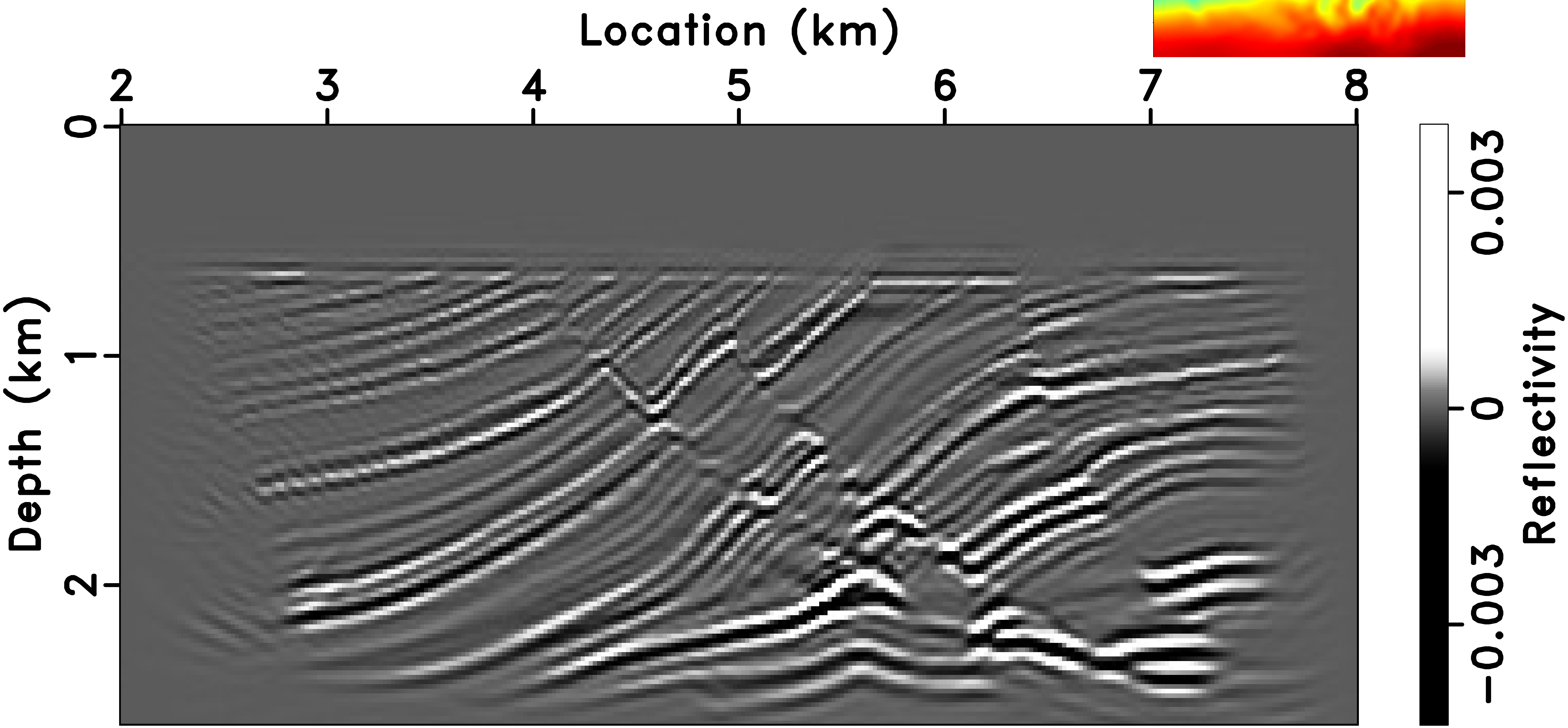
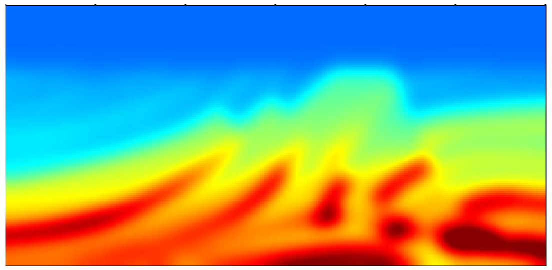


Image with Recovered Model (Conventional RTM)

Numerical Experiment



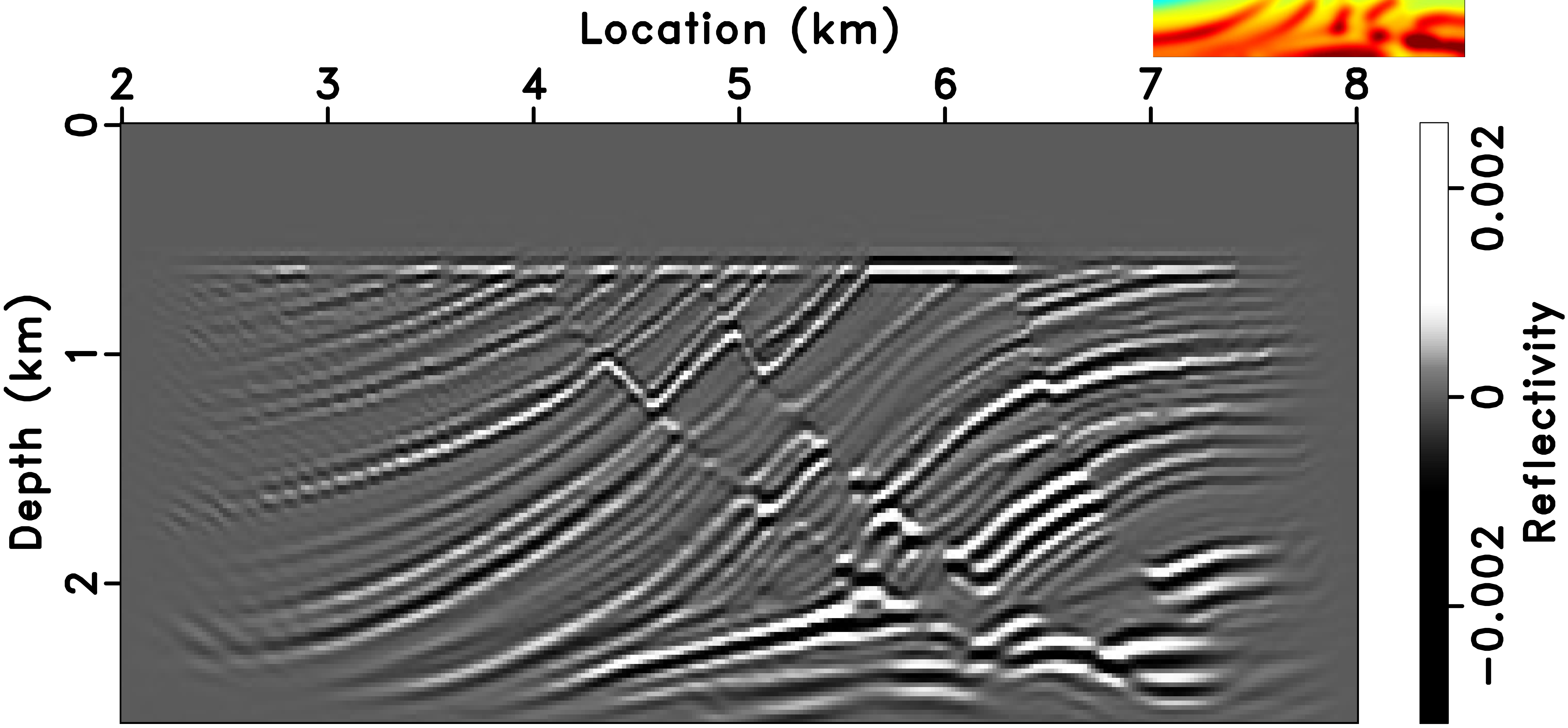


Image with True Model

Numerical Experiment

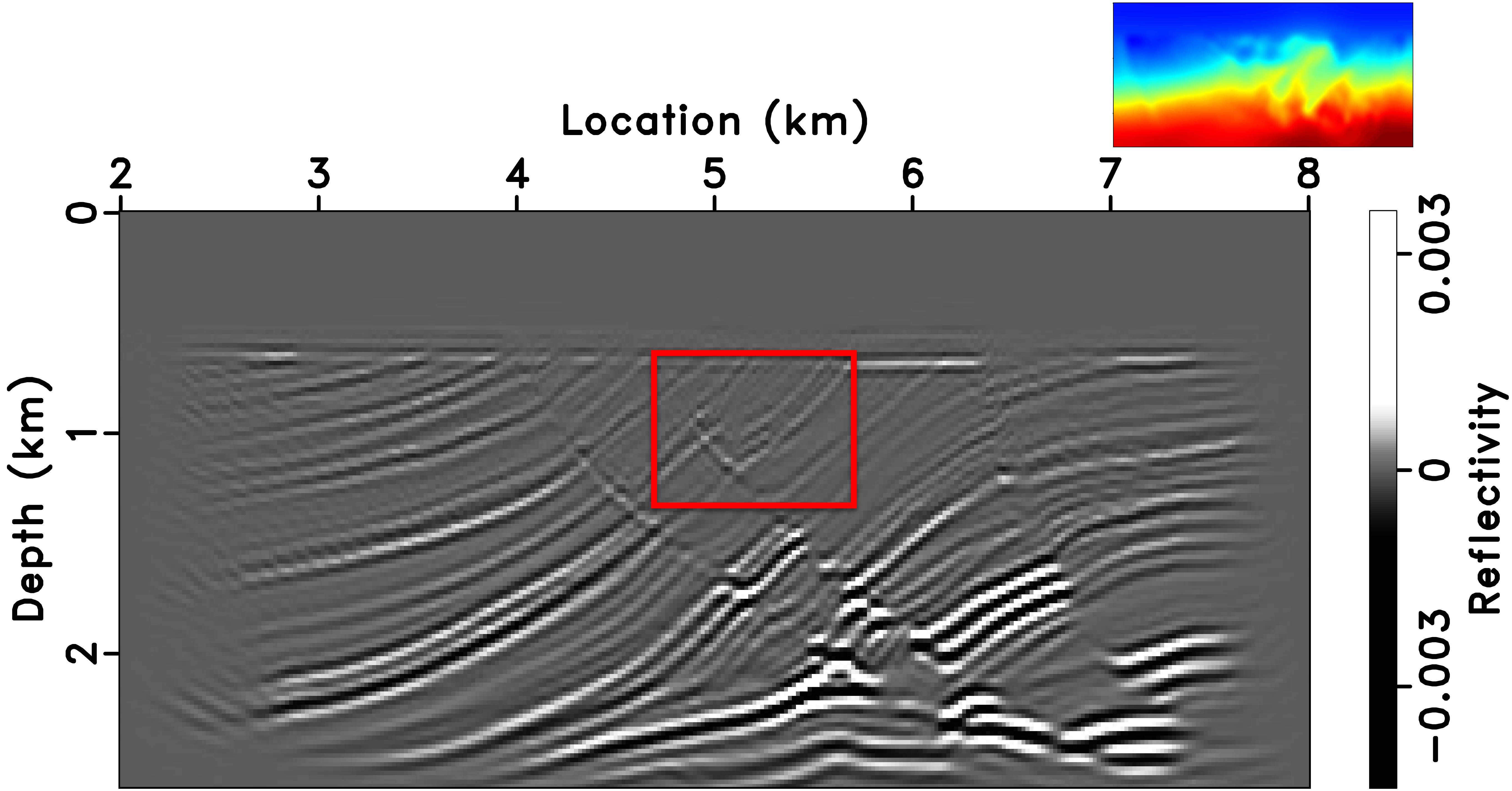
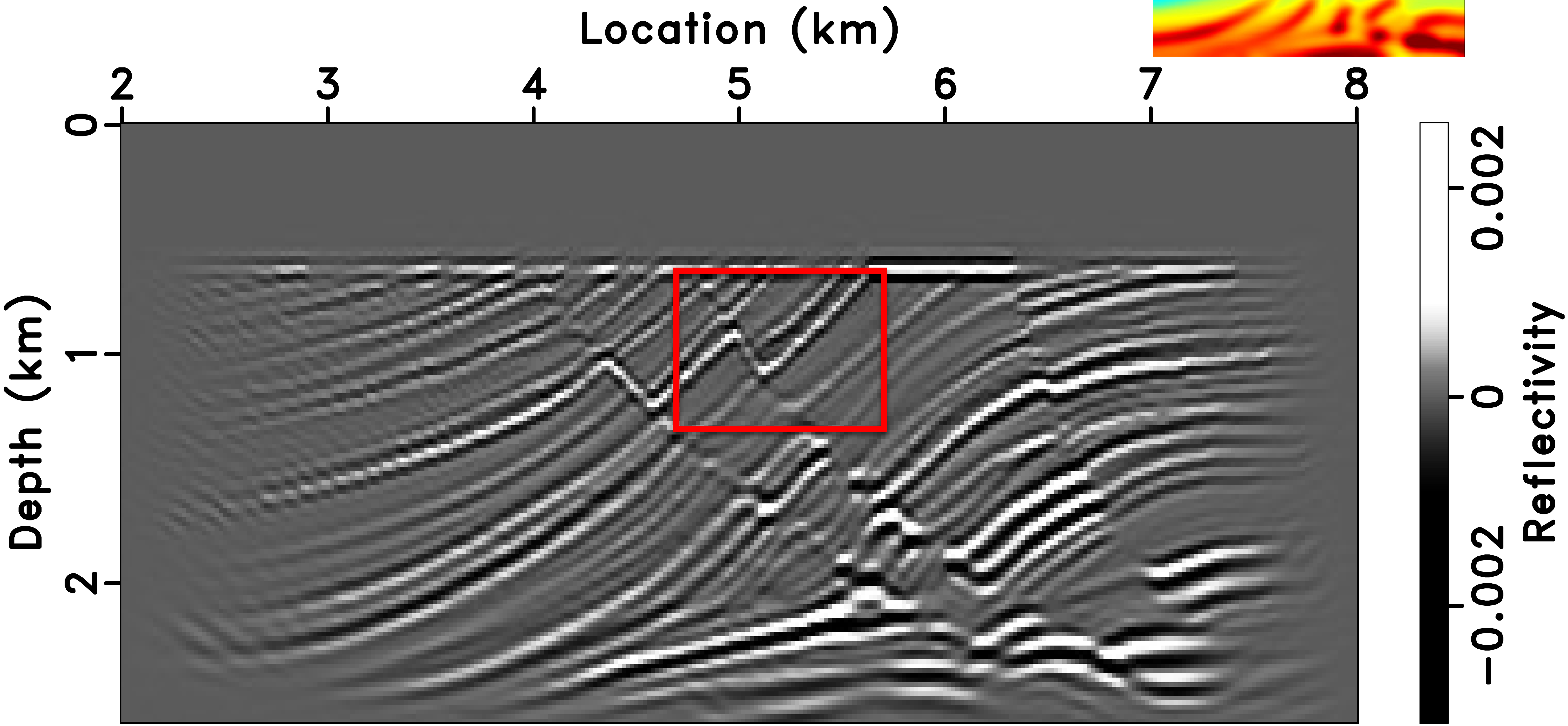


Image with Recovered Model (Adjoint Operator)

Numerical Experiment



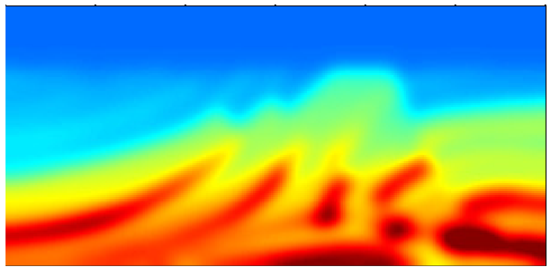
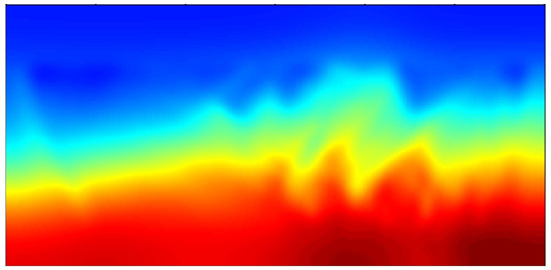


Image with True Model

Numerical Experiment



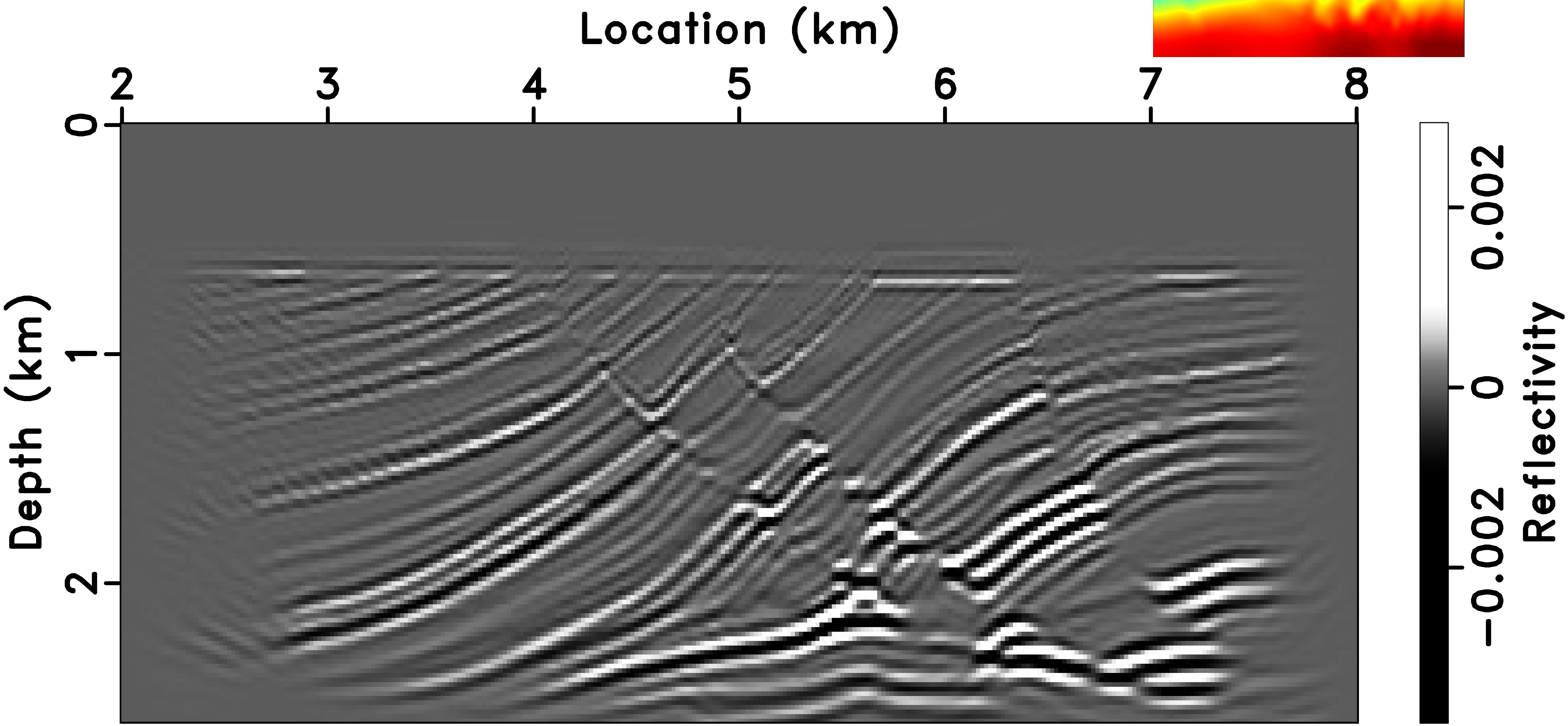
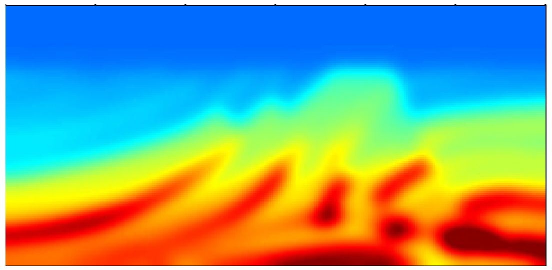


Image with Recovered Model (Appinv)

Numerical Experiment



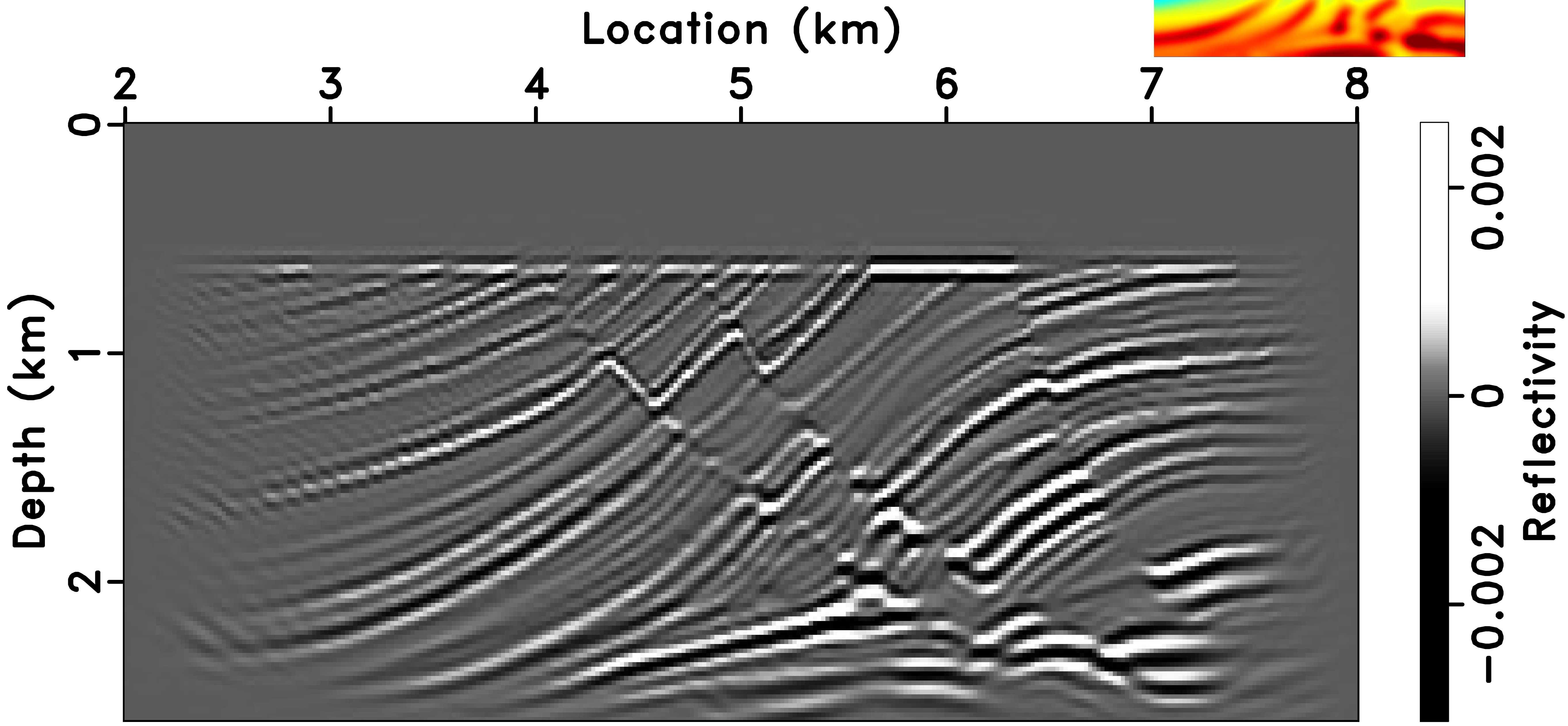
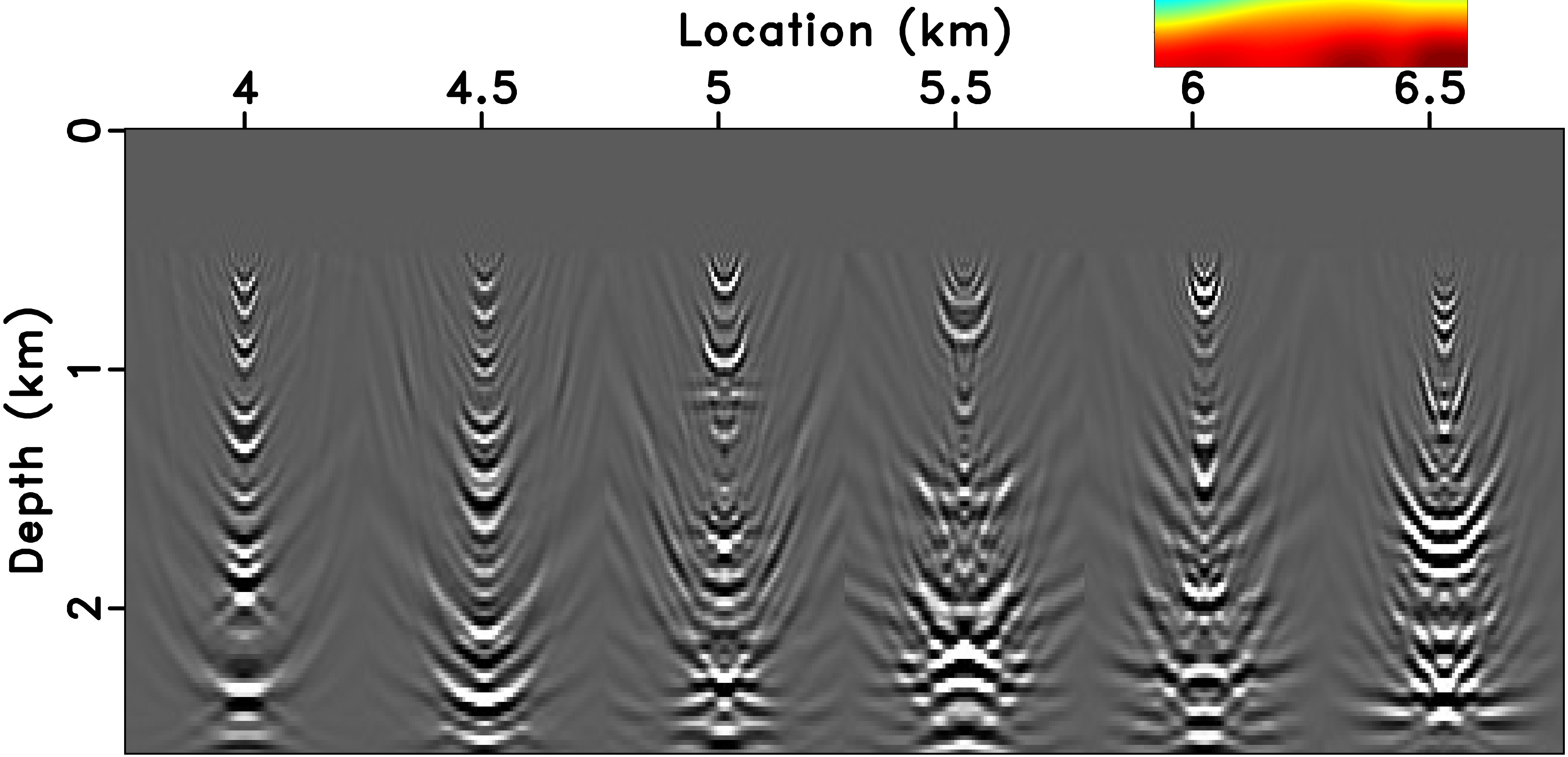
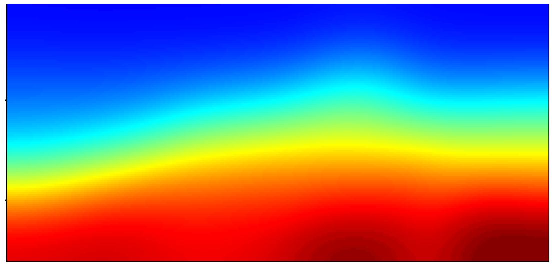


Image with True Model

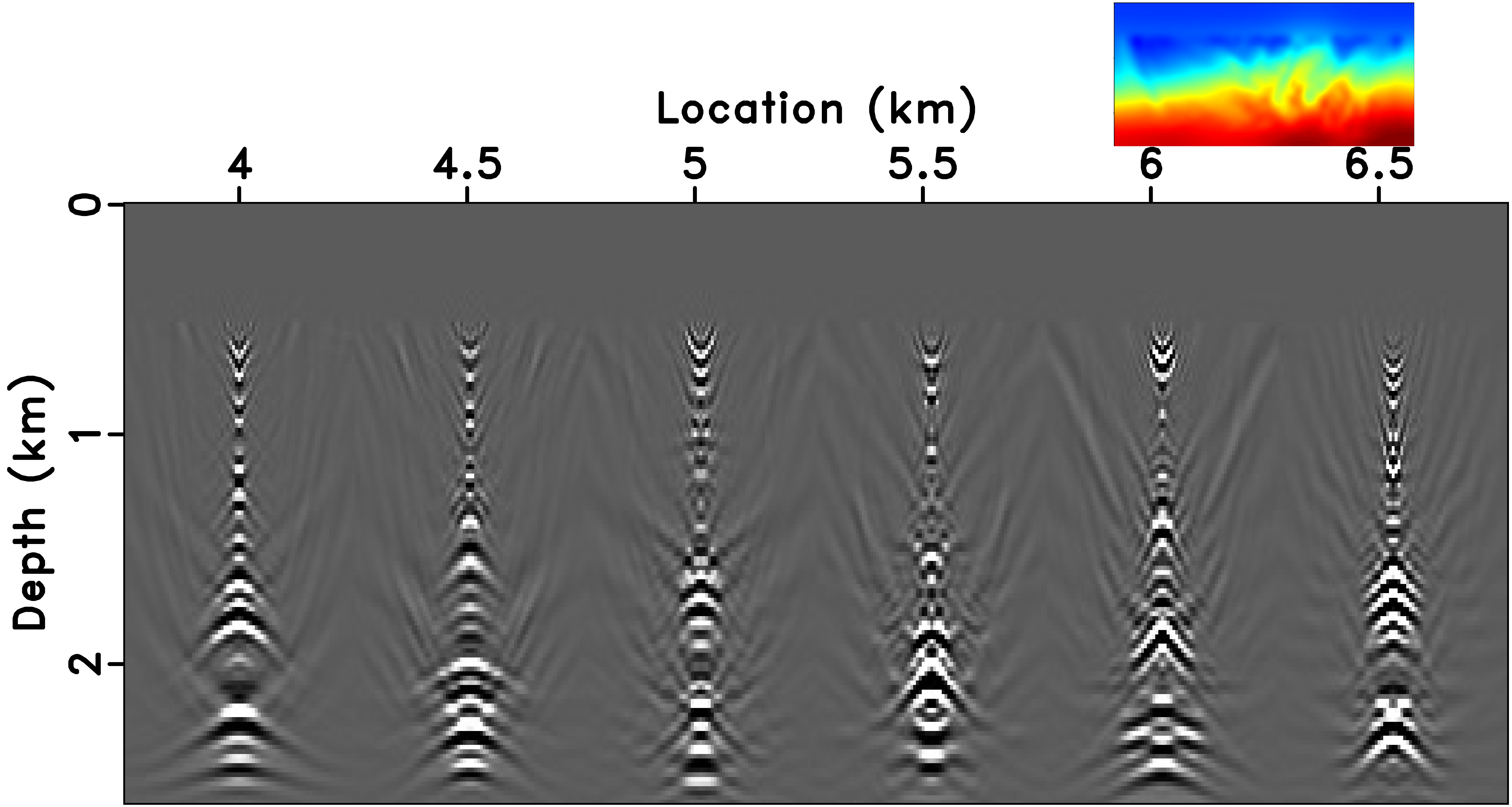
Numerical Experiment





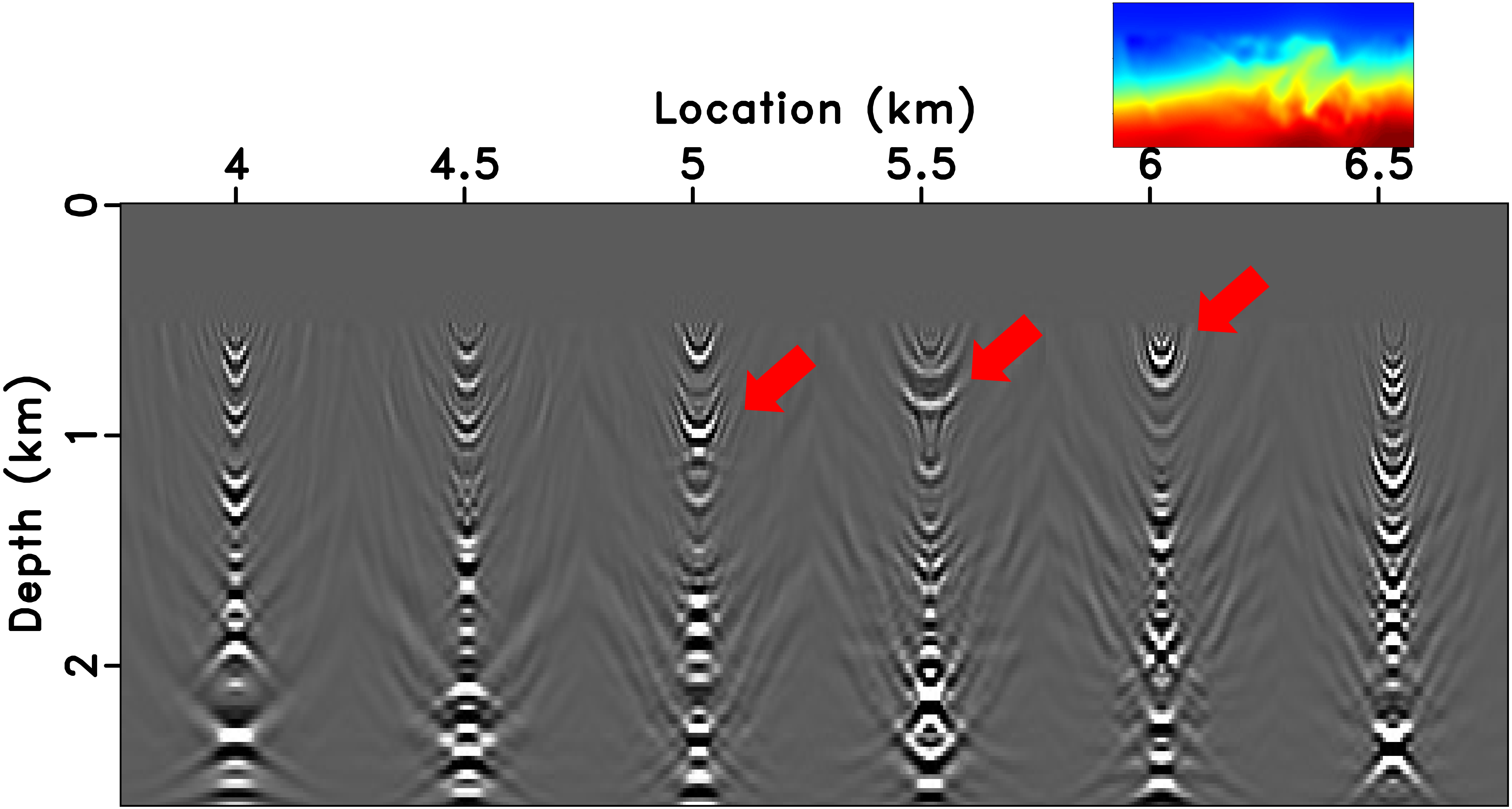
CIG with Initial Model

Numerical Experiment



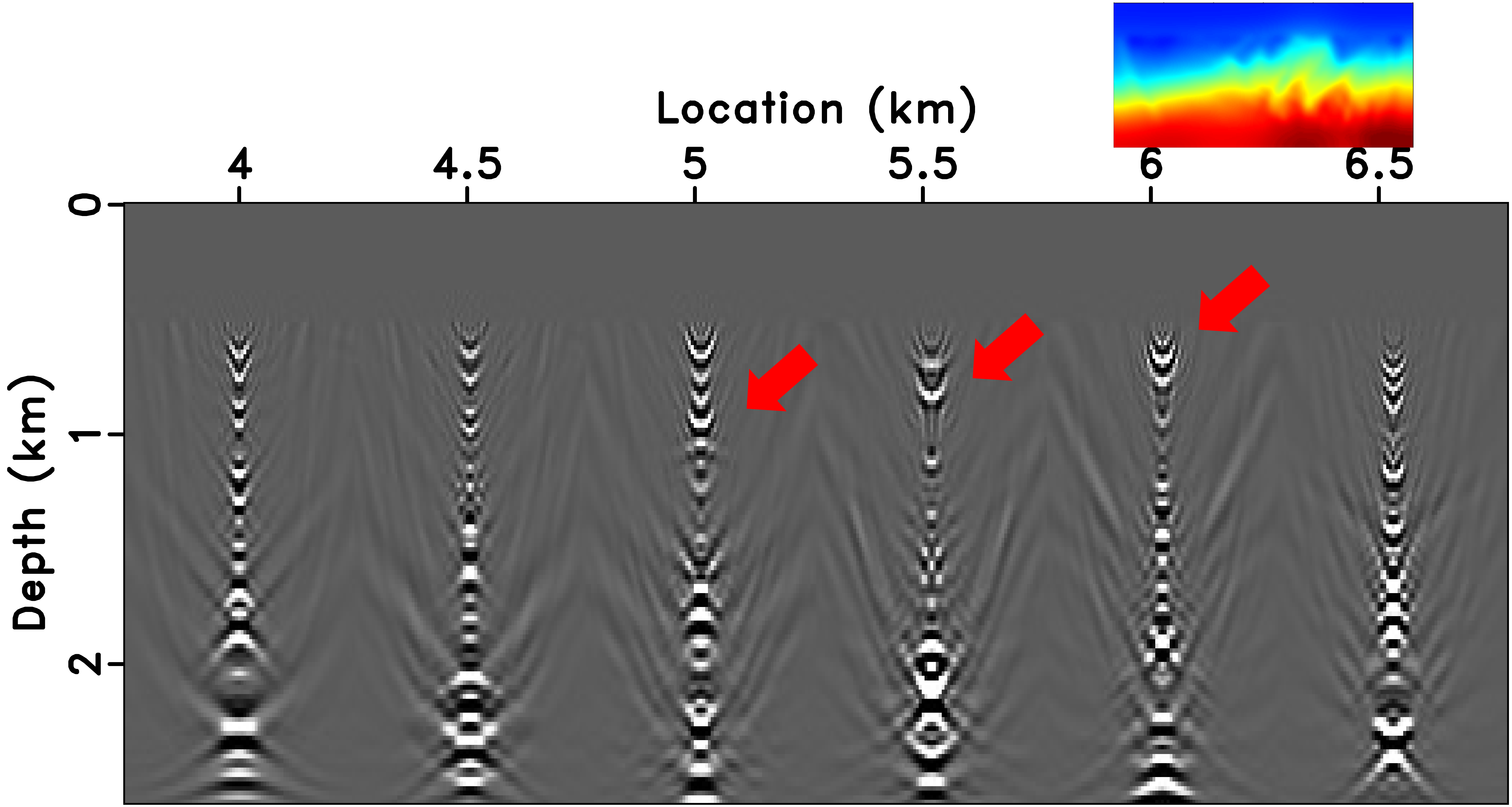
CIG with Recovered Model (RTM)

Numerical Experiment



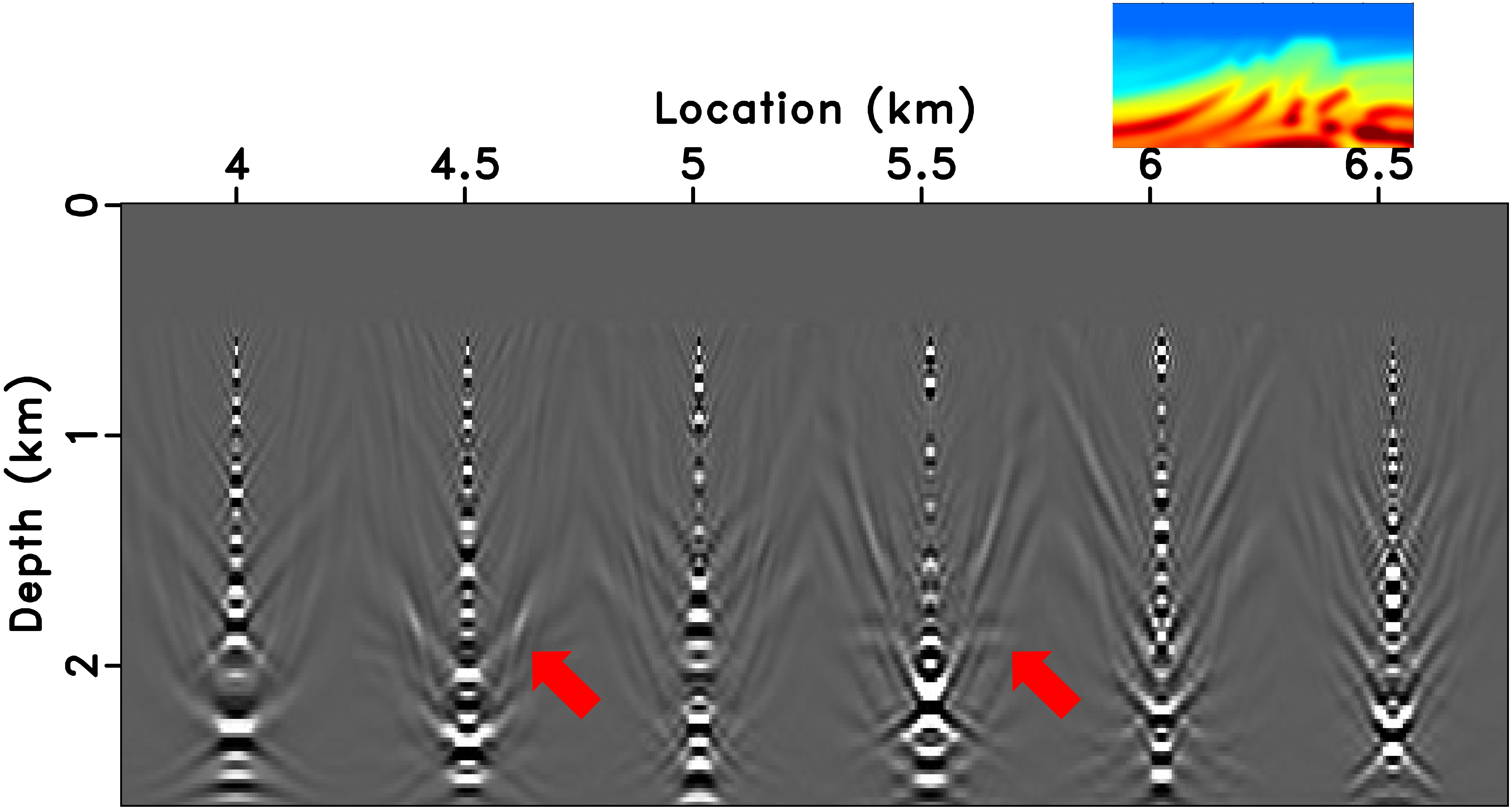
CIG with Recovered Model (Adjoint)

Numerical Experiment



CIG with Recovered Model (Appinv)

Numerical Experiment



CIG with True Model

Conclusion

- MVA complements FWI by extracting long-scale information
- "Gradient Artifacts" are features of the objective function, not gradient
- Approximate inverse operator improve the performance of velocity analysis

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