Yin Huang

Education

- Ph.D. Candidate, Rice University, Houston, TX, USA, 08/2010 - Present
  - Dissertation Topic: Extended waveform inversion in shot coordinate model extension
  - M.A. with Master Thesis: Transparency property of one dimensional acoustic wave equations 12/2012
- M.S, Shanghai Jiao Tong University, Shanghai, China, 09/2006 - 03/2009
  - Dissertation topic: Comparison of numerical methods for saddle point system arising from the mixed finite element method of elliptic problems with nonsmooth coefficients

Research Interests

- Extended Full Waveform Inversion and Born Waveform Inversion
- Migration/Inversion Velocity Analysis, Seismic Imaging
- High Performance Computing

Born Waveform Inversion via Variable Projection and Shot Record Model Extension

Yin Huang

TRIP annual review meeting

April 30, 2015

Introduction

Full waveform inversion (Tarantola, 1984, Virieux & Operto, 2009)

$$J[m] = \frac{1}{2}\|F[m] - d\|^2$$

- d observed data;
- $F[m]$ wave propagation operator;
- has the ability to invert for fine structure of the earth subsurface model by solving a nonlinear model-based least squares data fitting problem;
- initial model needs to be close to true model to avoid local minima problem (Gauthier et al., 1986).

Born modeling and waveform inversion

Scale separation of model $\approx m + \delta m$ (long scale background model plus short scale reflectivity).

Born modeling: $DF[m]\delta m$

Born waveform inversion: given data d, find m and δm that minimizes

$$J_{\text{BWI}}[m, \delta m] = \frac{1}{2}\|DF[m]\delta m - d\|^2.$$

- easy to fit the data;
- suffer from the same local minima problem as FWI.

Variable projection method

Obtain VP objective by minimizing over reflectivity for fixed background model (van Leuwen & Mulder, 2009; Xu et al., 2012)

$$J_{\mathrm{VP}}[m] = \min_{\delta m} J_{\mathrm{BWI}}[m, \delta m] = \frac{1}{2}\|DF[m]\delta m - d\|^2.$$

- less likely to be trapped by a local minimizer;
- may also exhibit cycle skipping in some cases.

VP method assisted by model extension

Introduce model extension to VP objective, to permit better data fit (Kern & Symes 1994)

$$J_{\text{EVP}}[m] = \min_{\delta \bar{m}} J_{\text{EBWI}}[m, \delta \bar{m}] = \frac{1}{2}\|D\bar{F}[m]\delta \bar{m} - d\|^2 + \frac{\alpha^2}{2}\|A\delta \bar{m}\|^2.$$

- $\delta \bar{m}$ extended reflectivity;
- A annihilator, $A = \frac{\partial}{\partial x_s}$ for shot record model extension;
- $\|A\delta \bar{m}\|^2$ differential semblance penalty, the only choice that leads to smooth objective function for shot record (Stolk & Symes, 2003);
- $\alpha \to +\infty$, $J_{\text{EVP}} \to J_{\text{VP}}$.

Value of EVP objective and approximate gradient

Evaluation

$$J_{\mathrm{EVP}}[m] = \min_{\delta\bar{m}} \frac{1}{2}\|D\bar{F}[m]\delta\bar{m} - d\|^2 + \frac{\alpha^2}{2}\|A\delta\bar{m}\|^2.$$

involves solving a least squares migration (LSM)

$$(D\bar{F}[m]^T D\bar{F}[m] + \alpha^2 A^T A)\delta\bar{m} = D\bar{F}[m]^T d.$$

Approximate gradient:

$$\nabla J_{\mathrm{EVP}} = \Lambda^{-1} D^2\bar{F}^T[\delta\bar{m}, D\bar{F}[m]\delta\bar{m} - d].$$

- Λ power of Laplacian operator, Λ^{-1} acts as smoothing operator;
- $D^2\bar{F}^T$ WEMVA or tomographic operator (Biondi & Sava 2004);
- Gradient of $J_{\mathrm{VP}}[m]$ is the same, without model extension.

Main claim

Both extended modeling (EXM) and variable projection (VP) are necessary to enable convergence to a global best fitting model.

	VP	without VP
EXM	✓	✗
without EXM	✗	✗

NOTE: VP objective function without model extension works well to some extent, but suffer from cycle skipping when initial model is too far away from true model.

Extended 2D Constant Density Acoustics

Extended Born modeling: $D\bar{F}[m]\delta\bar{m} = \delta u(\mathbf{x}_r, \mathbf{x}_s, t)$, with $m = c^2$ and $\delta\bar{m} = \delta\bar{c}^2$

$$\begin{aligned}
\left(\frac{\partial^2}{\partial t^2} - c^2(\mathbf{x})\Delta_{\mathbf{x}}\right) u(\mathbf{x}, \mathbf{x}_s, t) &= \delta(\mathbf{x} - \mathbf{x}_s)\omega(t), \\
u(\mathbf{x}, \mathbf{x}_s, t) &= 0, t \ll 0. \\
\left(\frac{\partial^2}{\partial t^2} - c^2(\mathbf{x})\Delta_{\mathbf{x}}\right) \delta u(\mathbf{x}, \mathbf{x}_s, t) &= \delta\bar{c}^2(\mathbf{x}, \mathbf{x}_s)\Delta u(\mathbf{x}, \mathbf{x}_s, t), \\
\delta u(\mathbf{x}, \mathbf{x}_s, t) &= 0, t \ll 0.
\end{aligned}$$

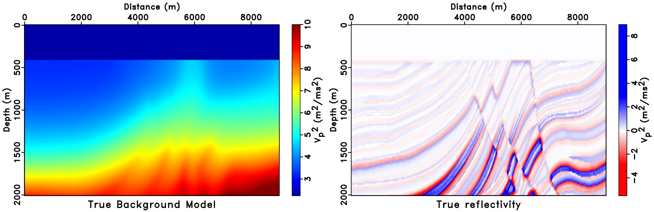
- c velocity of wave propagation,
- u acoustic pressure wave field, $F[c^2] = u(\mathbf{x}_r, \mathbf{x}_s, t)$,
- δu perturbed wave field due to the extended model perturbation $\delta\bar{c}^2$.

Extended 2D Constant Density Acoustics

Numerical discretization:

- finite difference method: 2-nd order in time, 4-th order in space, reflection boundary condition;
- implement the time step function of $\bar{F}[c^2]$;
- automatic differentiation tool TAPENADE (Hascöet and Pascual, 2004) to generate the time step function of $D\bar{F}[c^2]$, $D^2\bar{F}[c^2]$ and their adjoints; $D\bar{F}[c^2]^T$, $D^2\bar{F}[c^2]^T$;
- IWAVE framework: provides i/o, job control, and parallelization;
- RVL optimization software
  https://svn.code.sf.net/p/rsf/code/trunk/trip/

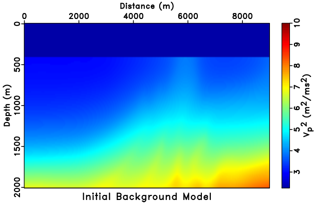
Example 1: truncated marmousi model

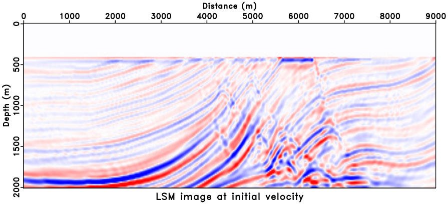


- acquisition geometry:
  110 shots starting from 2km with spacing 64m;
  481 symmetric receivers for each shot with spacing 16m;
- ricker1 wavelet with fpeak=6Hz;
- acquire data until 2.6s;
- 50 steps of conjugate gradient method is used for the LSM;
- steepest descent method with line search for background model updates.

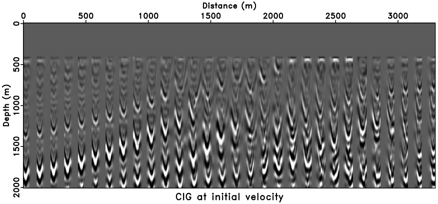
Initial model

Initial background model and reflectivities



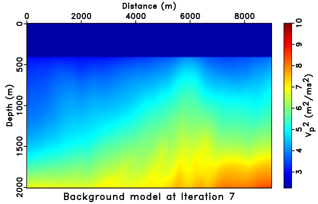


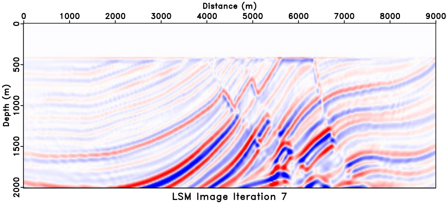
Common image gathers



7 steps of EVP

Background model after 7 steps of EVP and reflectivities





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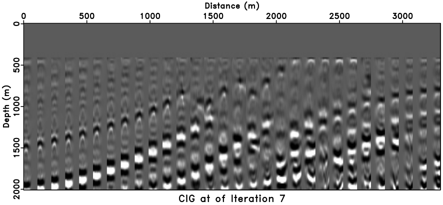
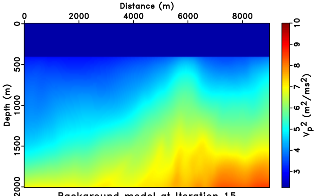
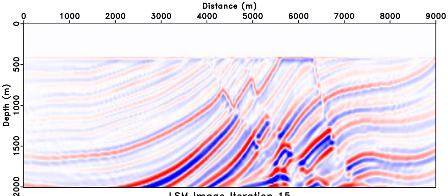


15 steps of EVP

Background model after 15 steps of EVP and reflectivities

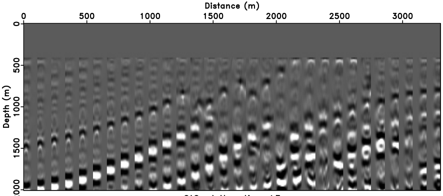


Background model at Iteration 15



LSM Image Iteration 15

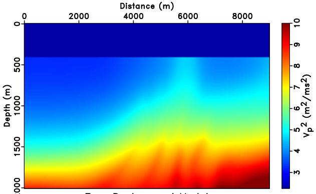
Common image gathers



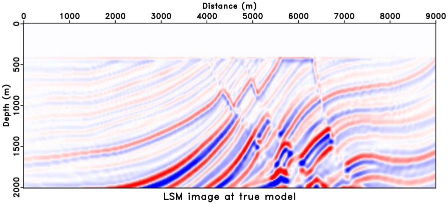
CIG at Iteration 15

At true background model

True background model and reflectivities

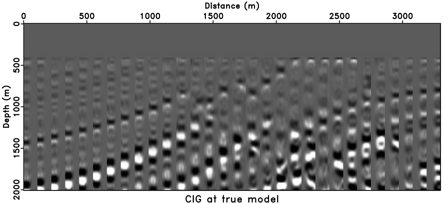


True Background Model



LSM image at true model

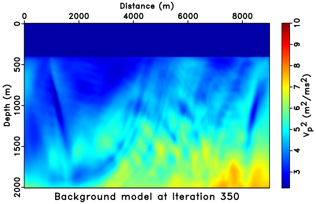
Common image gathers



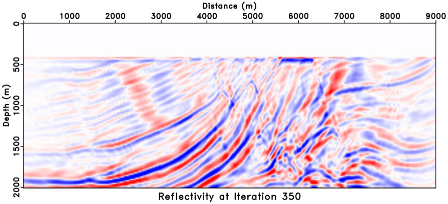
CIG at true model

350 steps of EBWI

Background model after 350 steps of EBWI and reflectivities

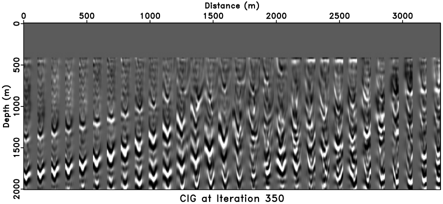


Background model at Iteration 350



Reflectivity at Iteration 350

Common image gathers



CIG at Iteration 350

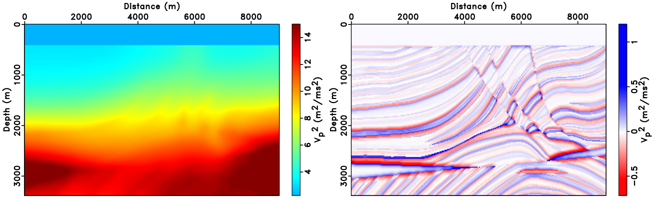
Summary of this example

Figure: Reflectivity model of EVP method

Conclusion from this example: use variable projection method when updating more than one parameters.

NOTE: 350 steps of EBWI is roughly equivalent to 7 steps of EVP in terms of computational cost.

Example 2: marmousi model



True Background Model

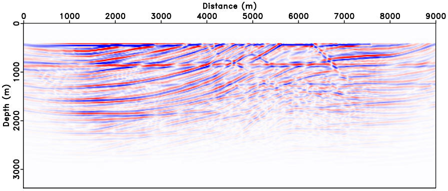
True reflectivity

- acquisition geometry:
  110 shots starting from 2km with spacing 64m;
  481 symmetric receivers for each shot with spacing 16m;
- ricker1 wavelet with fpeak=6Hz;
- acquire data until 4s;
- start with small number of conjugate gradient and increase with background model update;
- steepest descent method with line search for background model updates.

Initial model

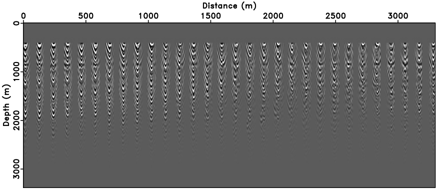
Initial background model and reflectivities

Initial Background Model



LSM image at initial background model

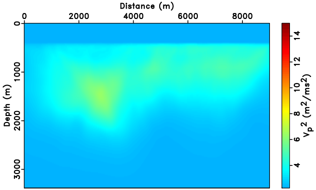
Common image gathers at the initial model



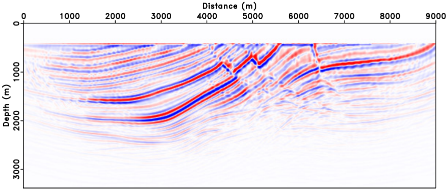
CIG at initial background model

10 steps of EVP

Background model after 10 steps of EVP and reflectivities

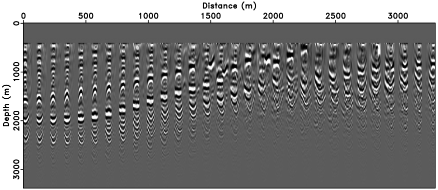


Background model at Iteration 10



LSM Image Iteration 10

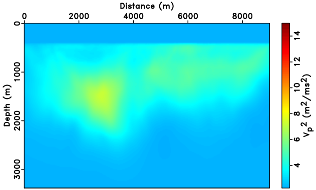
Common image gathers



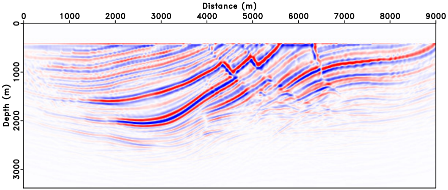
CIG at background model of Iteration 10

18 steps of EVP

Background model after 10 steps of EVP and reflectivities

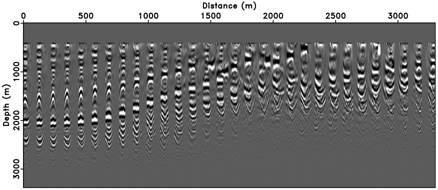


Background model at Iteration 18



LSM Image Iteration 18

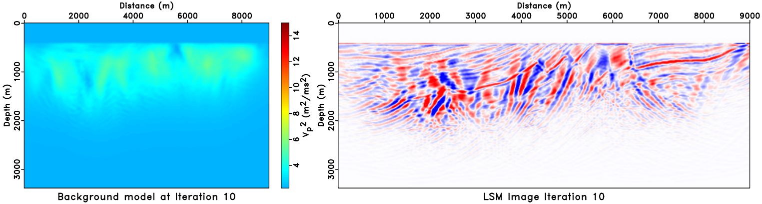
Common image gathers



CIG at background model of Iteration 18

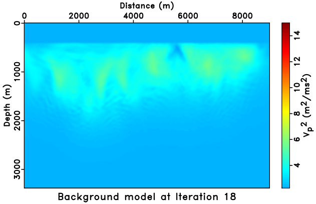
10 steps of VP

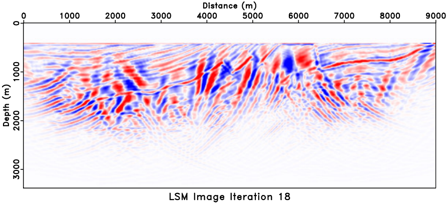
Background model after 10 steps of VP and reflectivities



18 steps of VP

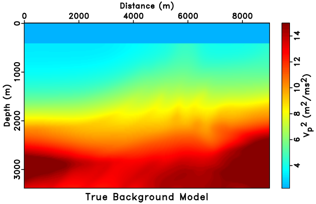
Background model after 18 steps of VP and reflectivities

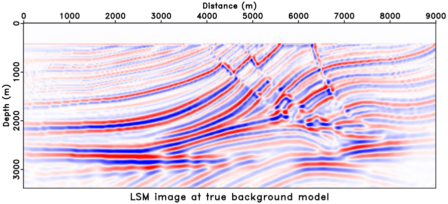




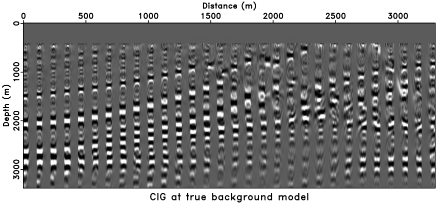
True model

True model and reflectivities at the true model





Common image gathers at the true model



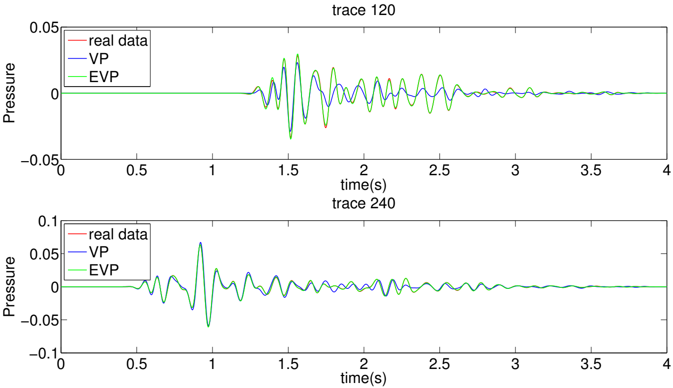
Reflectivity of EVP method

Figure: Reflectivity of EVP method

Reflectivity of VP method

Figure: Reflectivity model of VP method

Summary of this example



Model extension is necessary to a stable inversion.
NOTE: 1 step of VP is roughly equivalent to 1 step of EVP in terms of computational cost.

Discussion

- Compared Born waveform inversion with/without variable projection and with/without model extension;
- Both model extension and variable projection are necessary for a stable Born waveform inversion;
- Hundreds of modeling/migration were involved in the inversion $\Rightarrow$ future work.

Future works

- (in progress) apply preconditioning to accelerate the convergence of the minimization over reflectivity (Tang, 2009; Stolk et al., 2009; Nammour & Symes, 2009)
- compare with a similar method that uses full waveform operator as a prediction operator;
- (in progress) inversion velocity analysis for shot record model extension

$$\begin{array}{ll} \min\limits_{m} & \|A\delta\bar{m}\|^2 \\ with & \delta\bar{m} = \arg\min \|D\bar{F}[m]\delta\bar{m} - d\|^2 \end{array}$$

Acknowledgements

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- Sponsors of The Rice Inversion Project;
- Special thanks to Anatoly Baumstein and Yaxun Tang for helps and inspiring discussions during my internship at Exxonmobil.