Accelerating

Extended Least Squares Migration

Jie Hou

TRIP 2014 Review Meeting

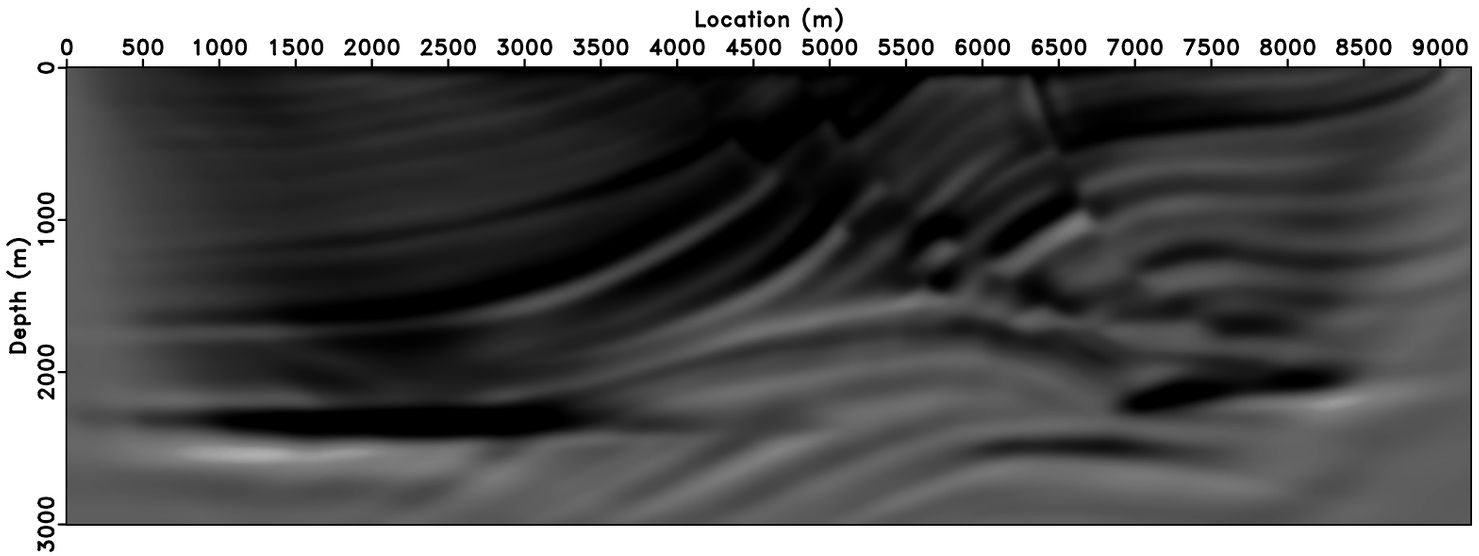
May 1st, 2015

Linearized Inverse Problem

$$Fm = d$$

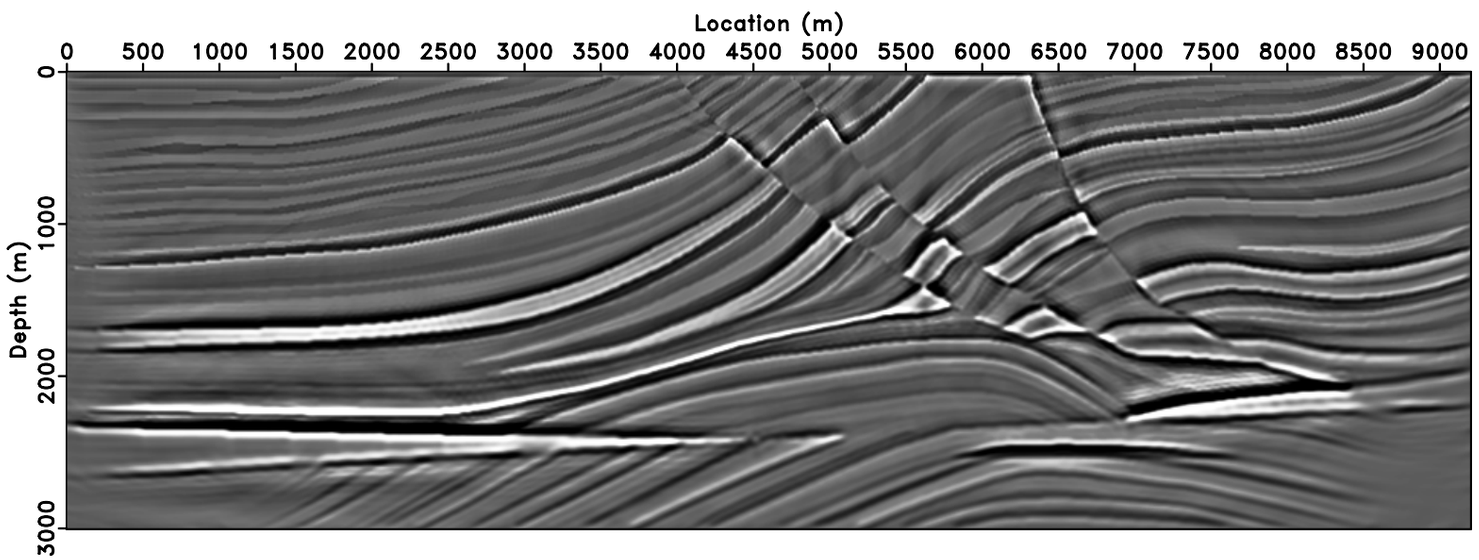
Adjoint Operator

$$m = F^T d$$



Approximate Inverse

$$m = F^{\dagger} d$$



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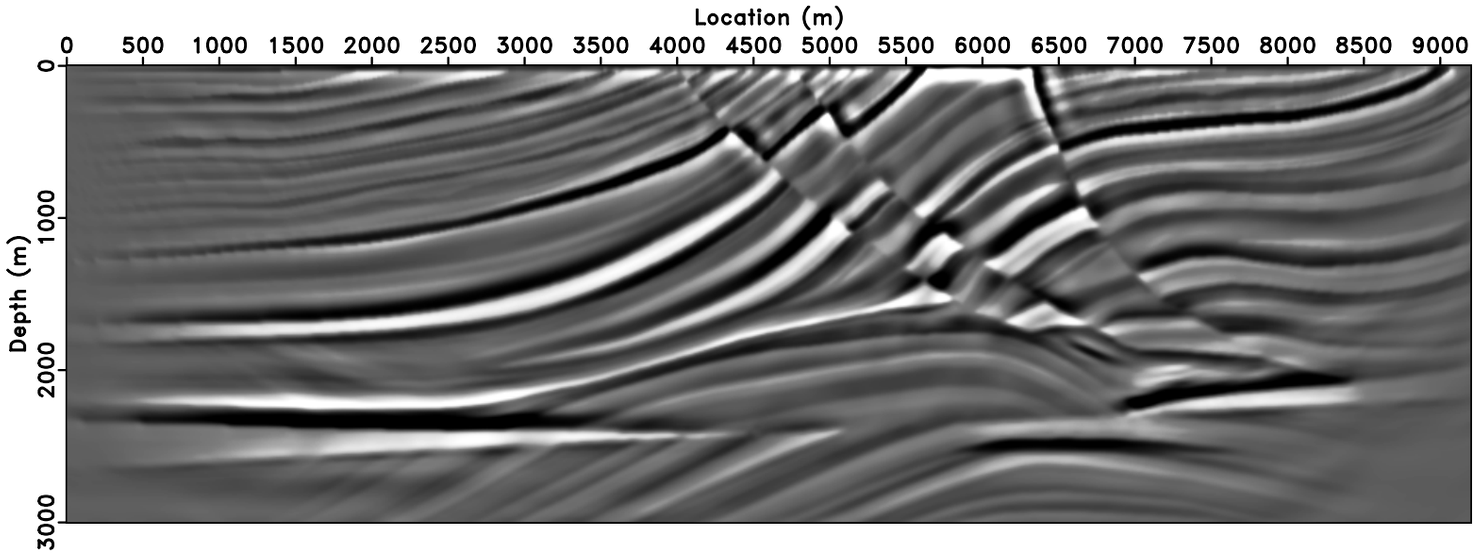
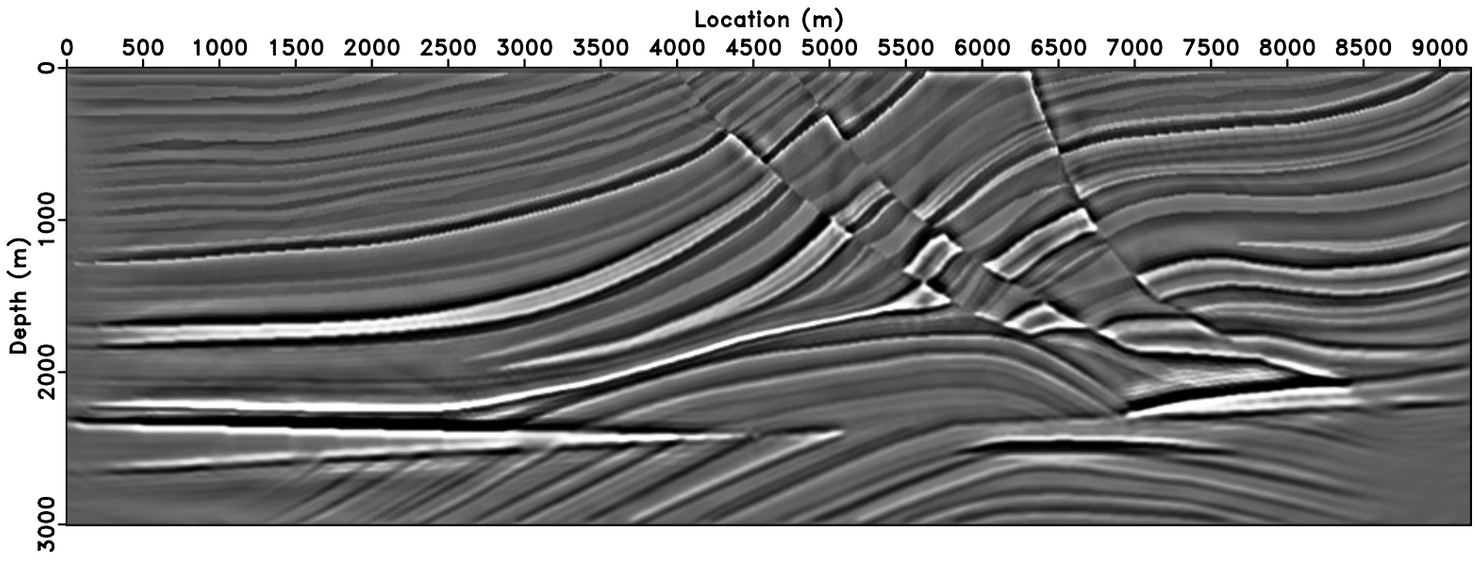
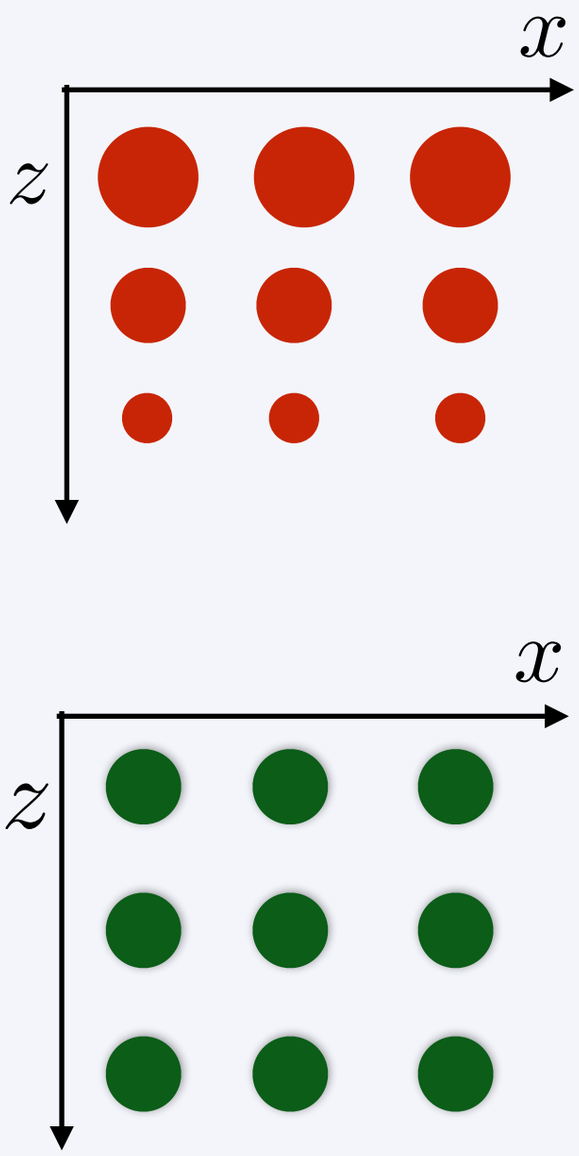


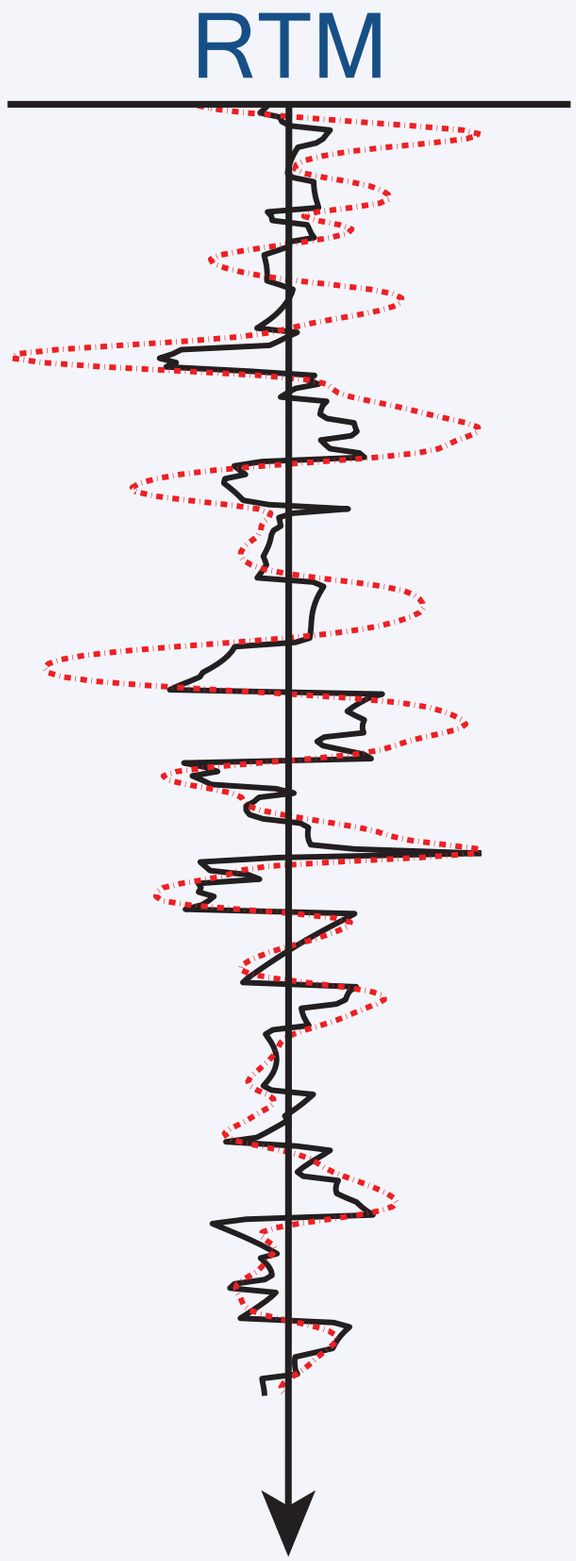
Approximate Inverse

$$m = F^\dagger d$$

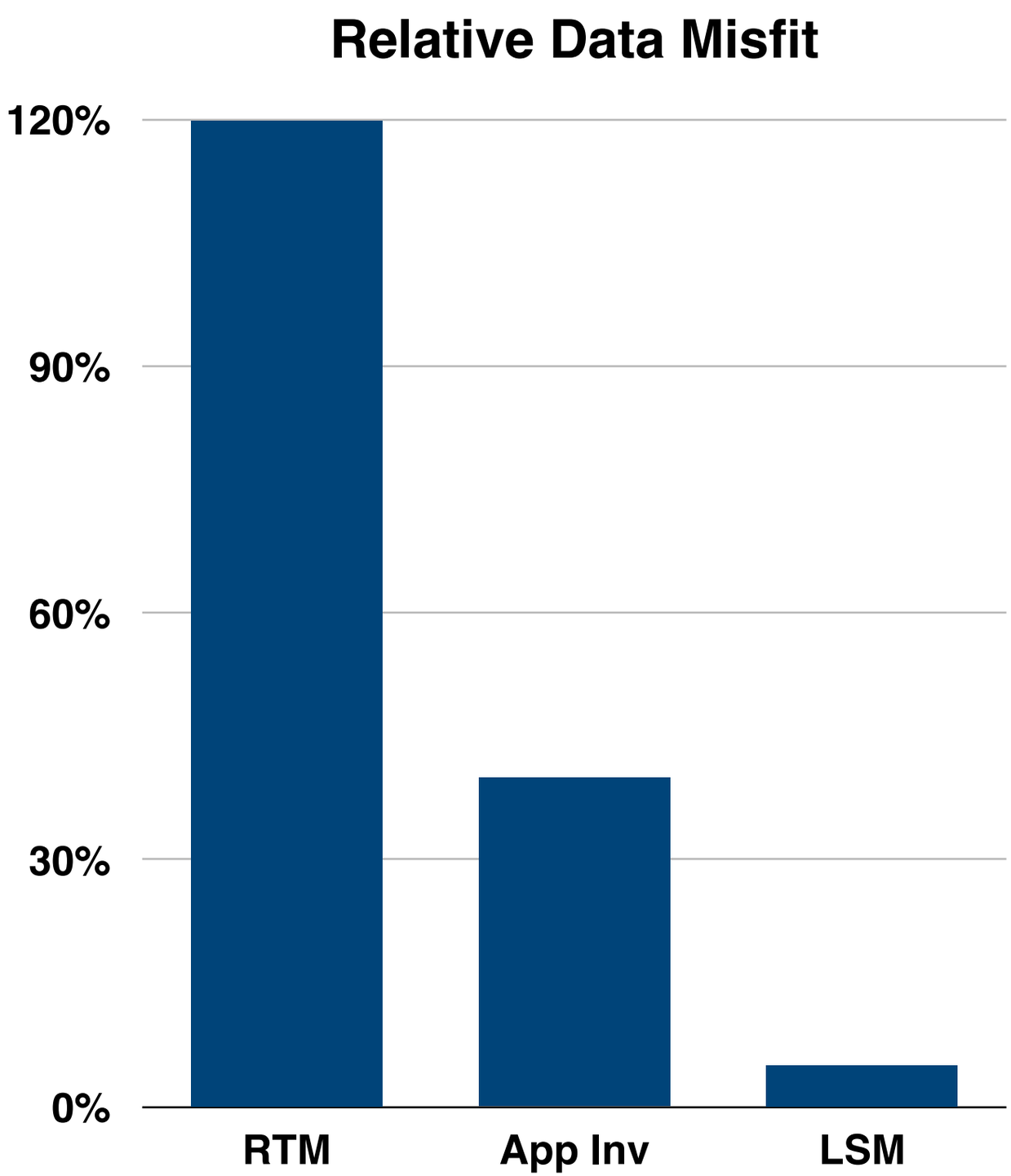


Linearized Inverse Problem



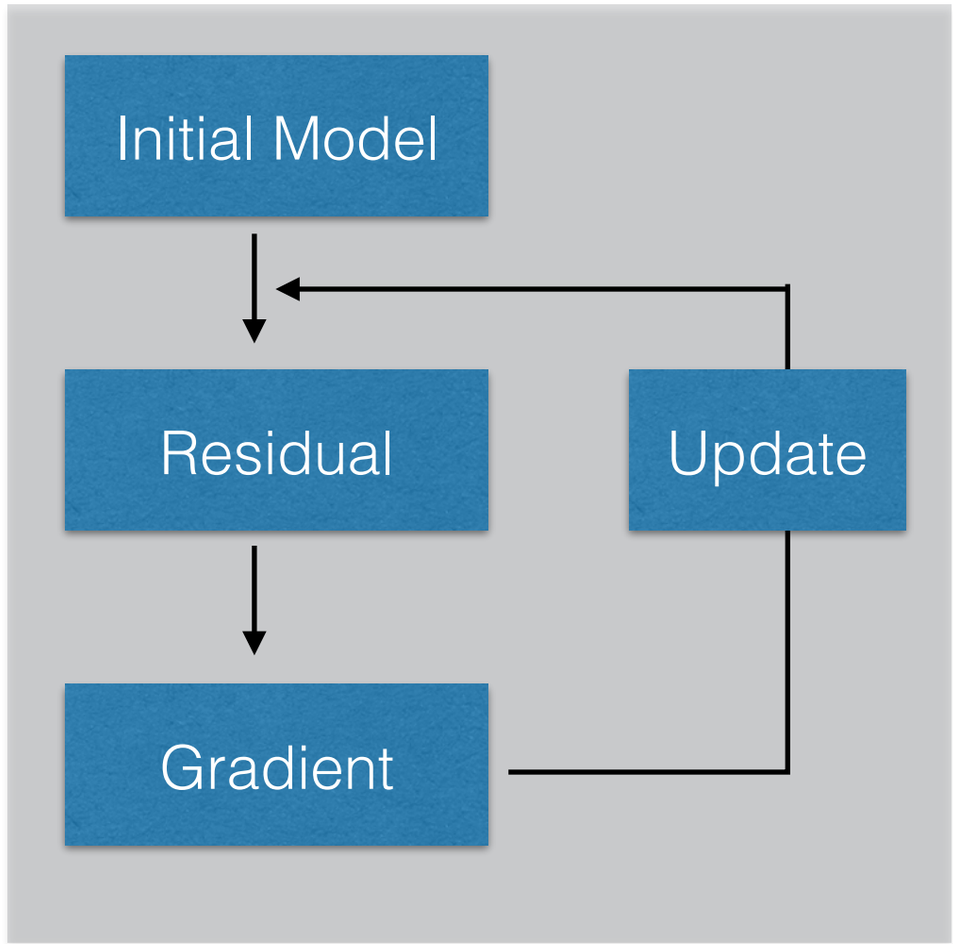


Least Squares Migration



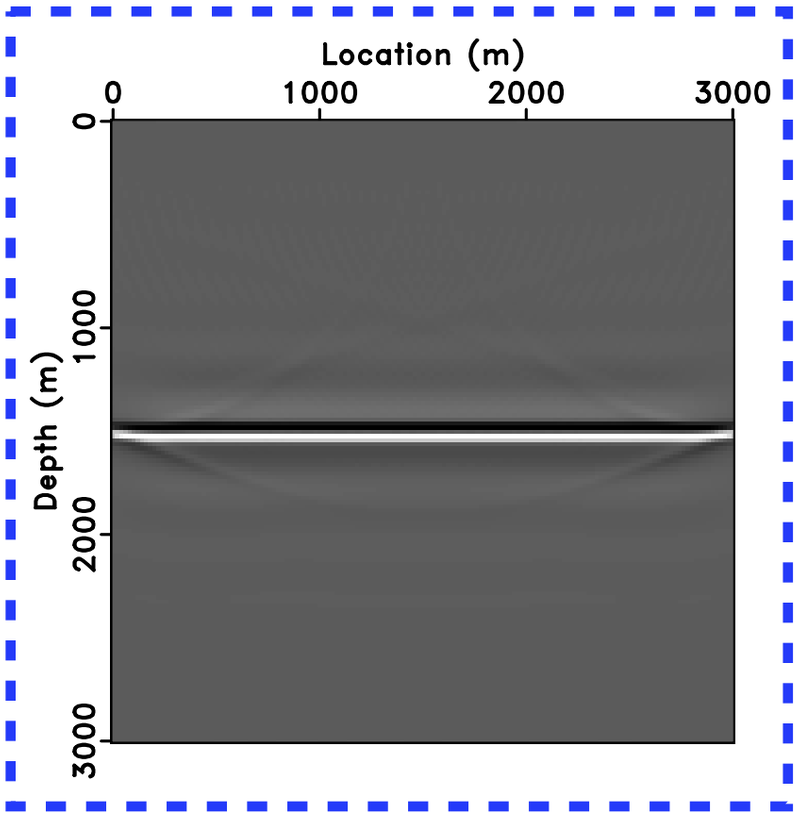
$$J_{LS} = \frac{1}{2} \| Fm - d \|^2$$

$$F^T F m = F^T d$$

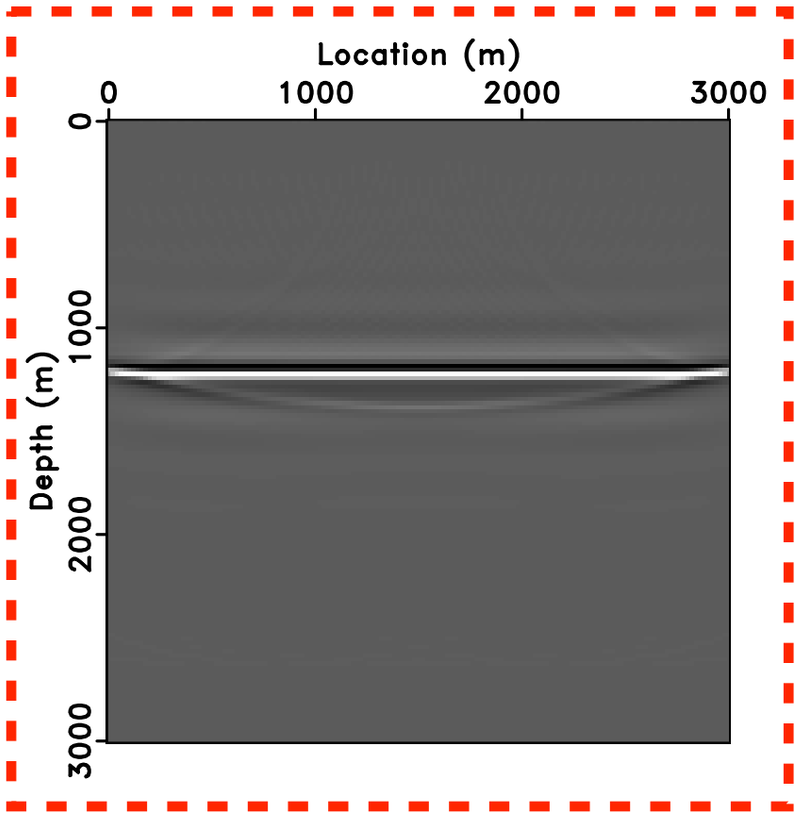


Two Problems of LSM

- Sensitive to Velocity Error
- Expensive Computational Cost



Correct



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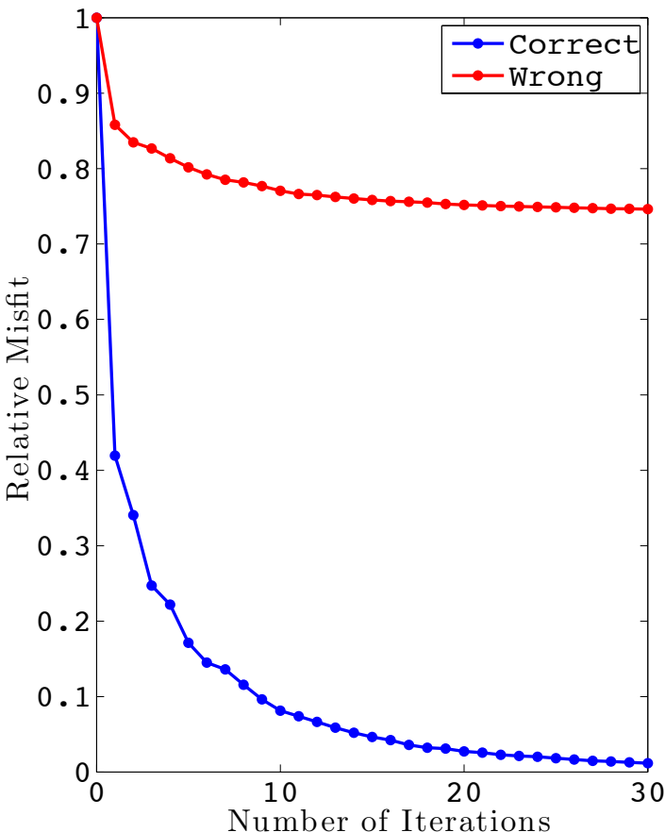
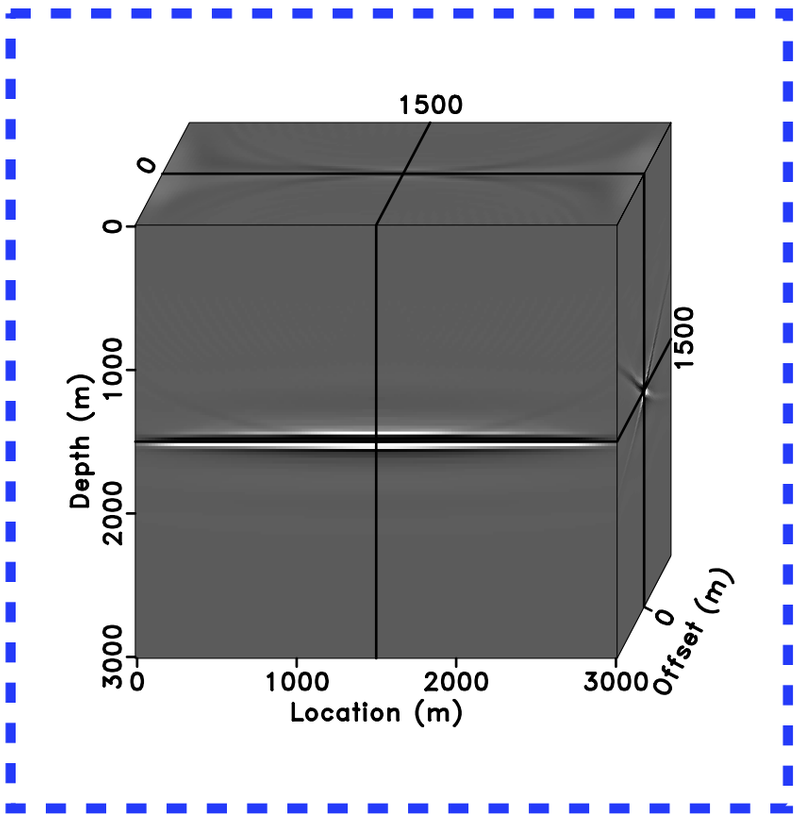
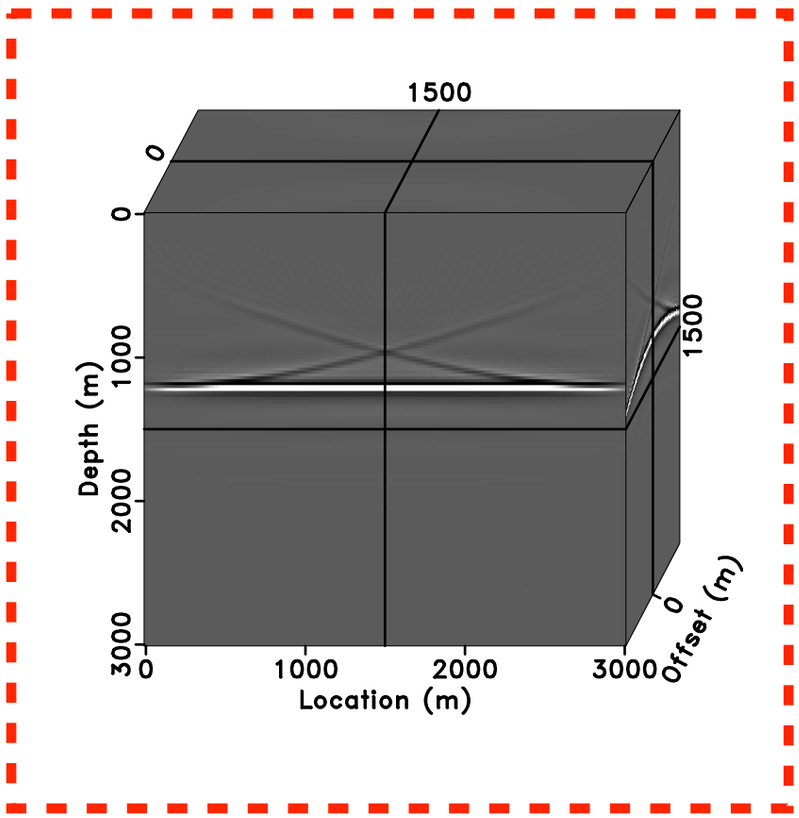


Extended LSM

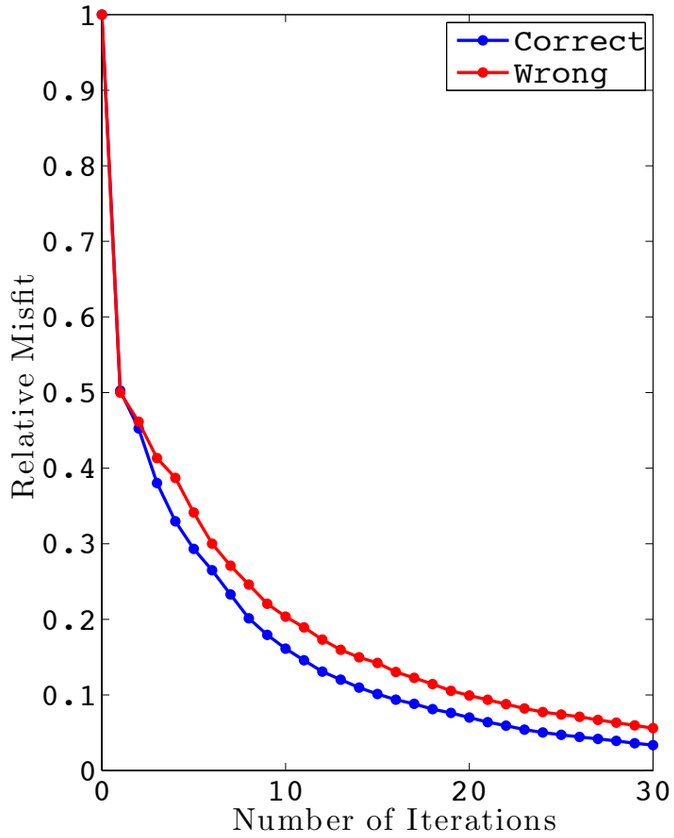
- Sensitive to Velocity Error
- Expensive Computational Cost



Correct



Incorrect



Computational Cost

- Sensitive to Velocity Error
- Expensive Computational Cost

F

$$\int \quad G_s(\mathbf{x},\tau) \qquad m(\mathbf{x}) \qquad G_R(\mathbf{x},t-\tau) \qquad d\mathbf{x}d\tau$$

$\bar{F}$

$$\int G_s(\mathbf{x}-h,\tau) \;\; m(\mathbf{x},h)\; G_R(\mathbf{x}+h,t-\tau)\; dh d\mathbf{x} d\tau$$

F^T

$$\int \quad G_S(\mathbf{x},\tau) \qquad d \qquad G_R(\mathbf{x},t-\tau) \quad d\tau dt dx_S dx_R$$

$\bar{F}^T$

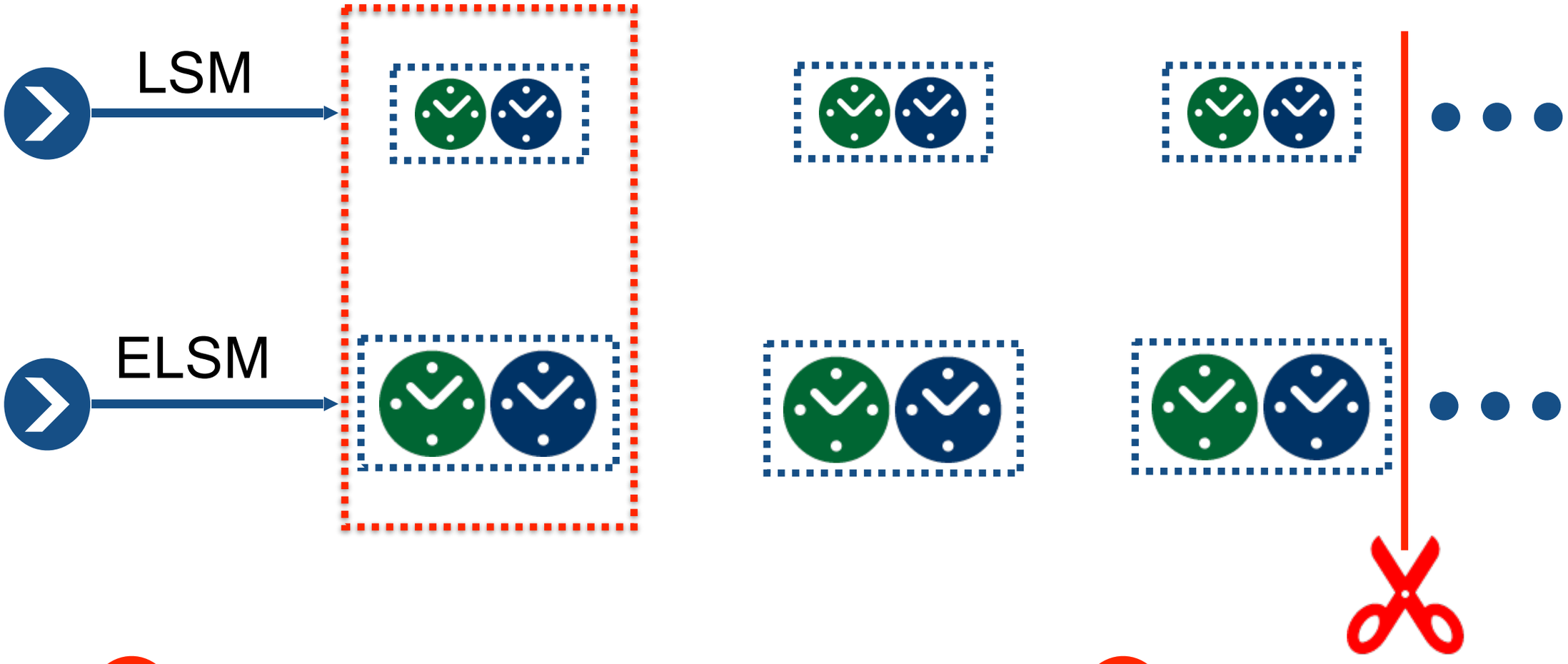
$$\int \quad G_S(\mathbf{x}-h,\tau) \;\; d \;\; G_R(\mathbf{x}+h,t-\tau) d\tau dt dx_S dx_R$$

Acceleration

- Sensitive to Velocity Error
- Expensive Computational Cost

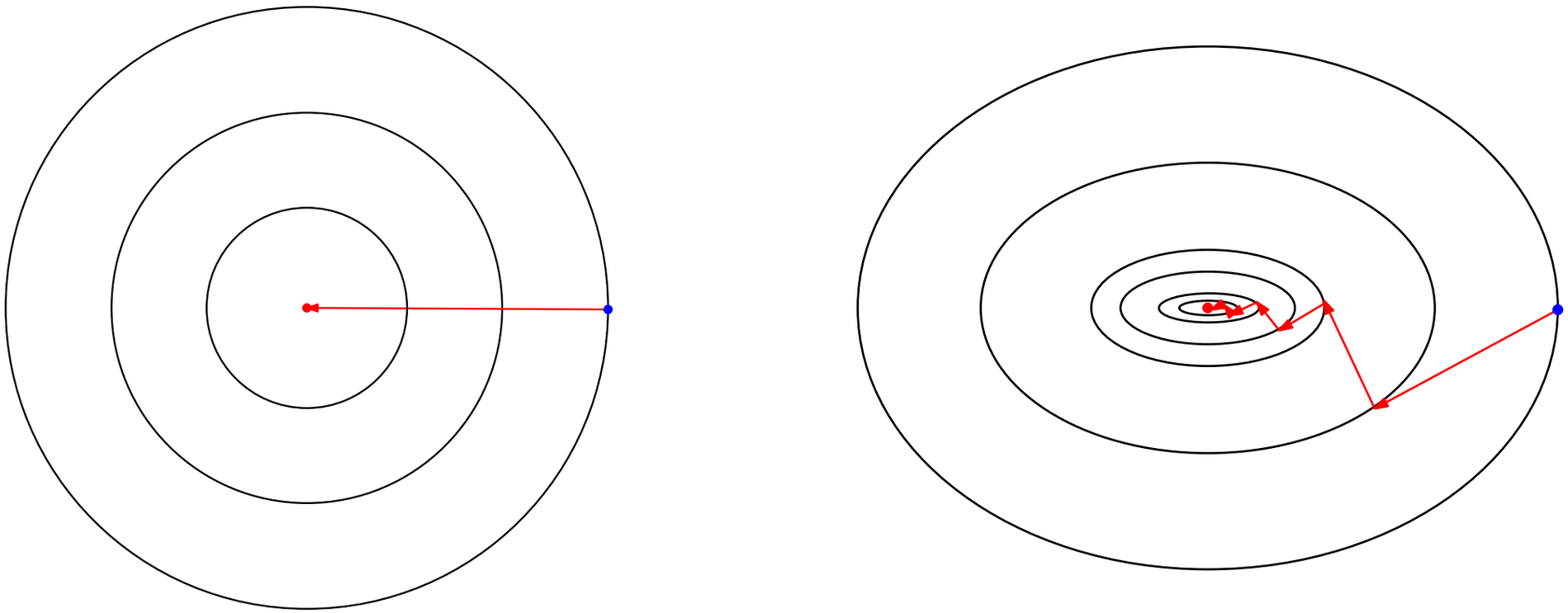
Acceleration

- Sensitive to Velocity Error
- Expensive Computational Cost



Optimization Algorithm

$$J_{LS} = \frac{1}{2} \| Fm - d \|^2 \quad \Leftrightarrow \quad F^T F m = F^T d$$



Steepest Descent Algorithm

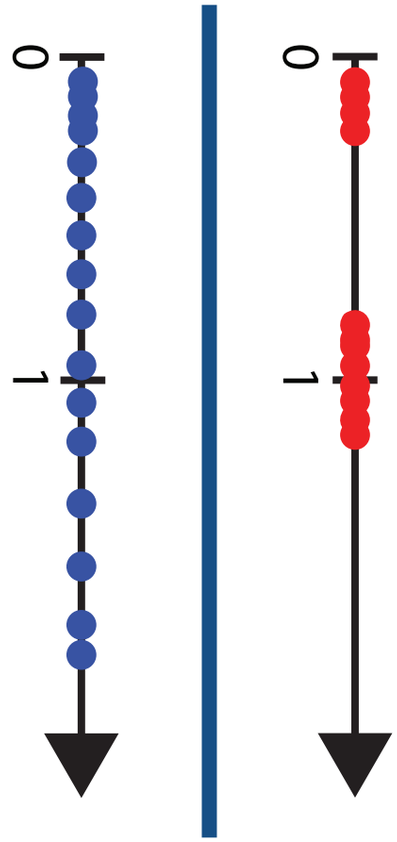
Optimization Algorithm

$$F^T F m = F^T d \quad \Longleftrightarrow \quad F^\dagger F m = F^\dagger d$$

$$< m, m >= m^T m \qquad < m, m >_{model} = m^T W_{model} m$$

$$< d, d >= d^T d \qquad < d, d >_{data} = d^T W_{data} d$$

$$< m, F^T d >=< d, Fm > \qquad < m, F^\dagger d >_{model} =< d, Fm >_{data}$$



Eigenvalue

Weighted Conjugate Gradient

Inner Product

$$< m, F^{\dagger} d >_{model} = < d, Fm >_{data}$$

$$m^T W_{model} F^{\dagger} d = m^T F^T W_{data} d$$

Adjoint

$$F^{\dagger} = W_{model}^{-1} F^T W_{data}$$

Weight Operator

$$W_{model}^{-1} = 4 v_0^5 LP$$

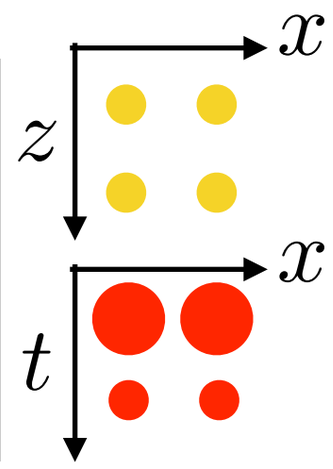
$$W_{data} = I_t^4 D_{z_s} D_{z_r}$$

Weighted Conjugate Gradient

CG

$$< m, m >= m^T m$$

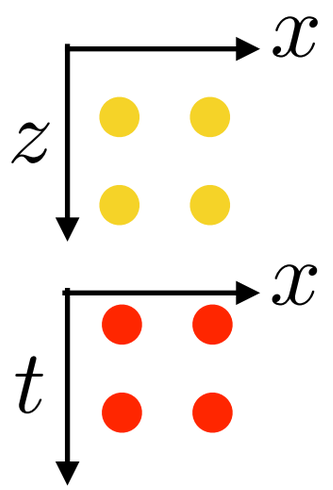
$$< Fm, Fm >= m^T F^T F m$$



WCG

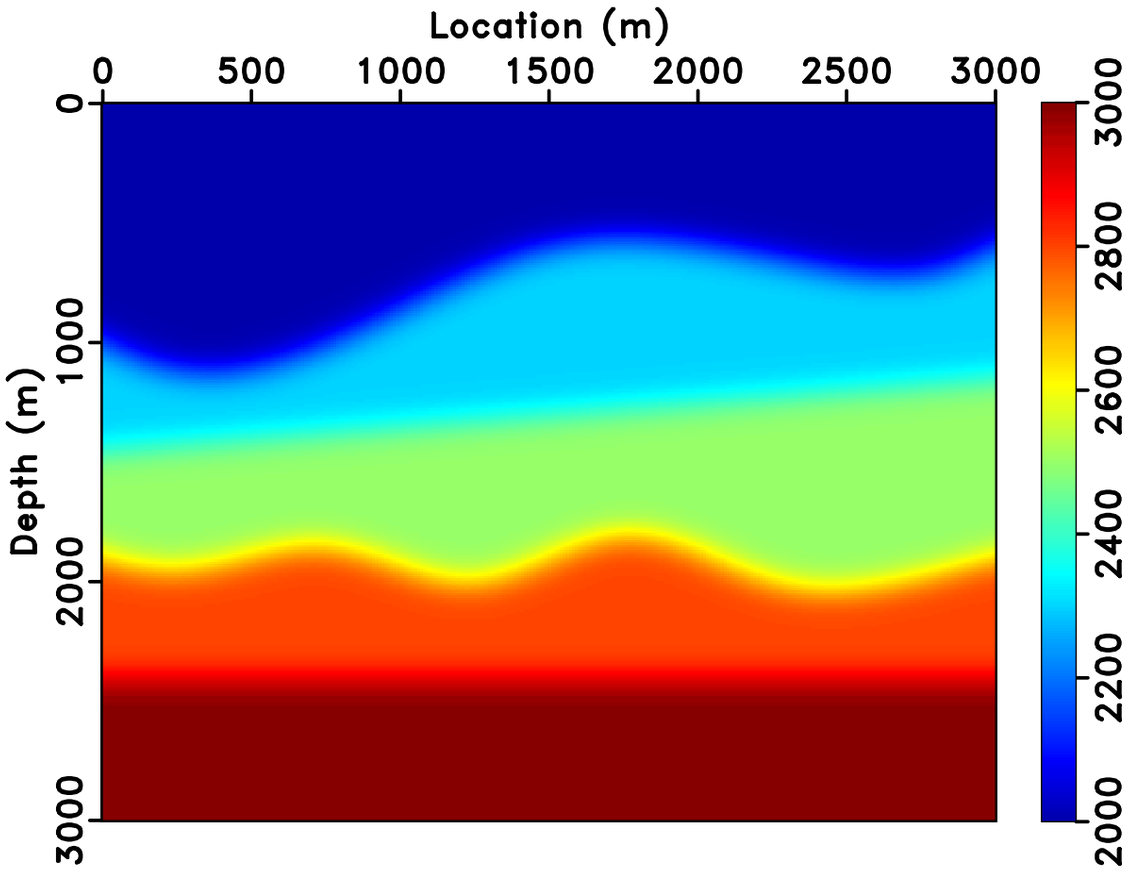
$$< m, m >_{model} = m^T W_{model} m$$

$$< Fm, Fm >_{data} = m^T W_{model} F^\dagger F m$$

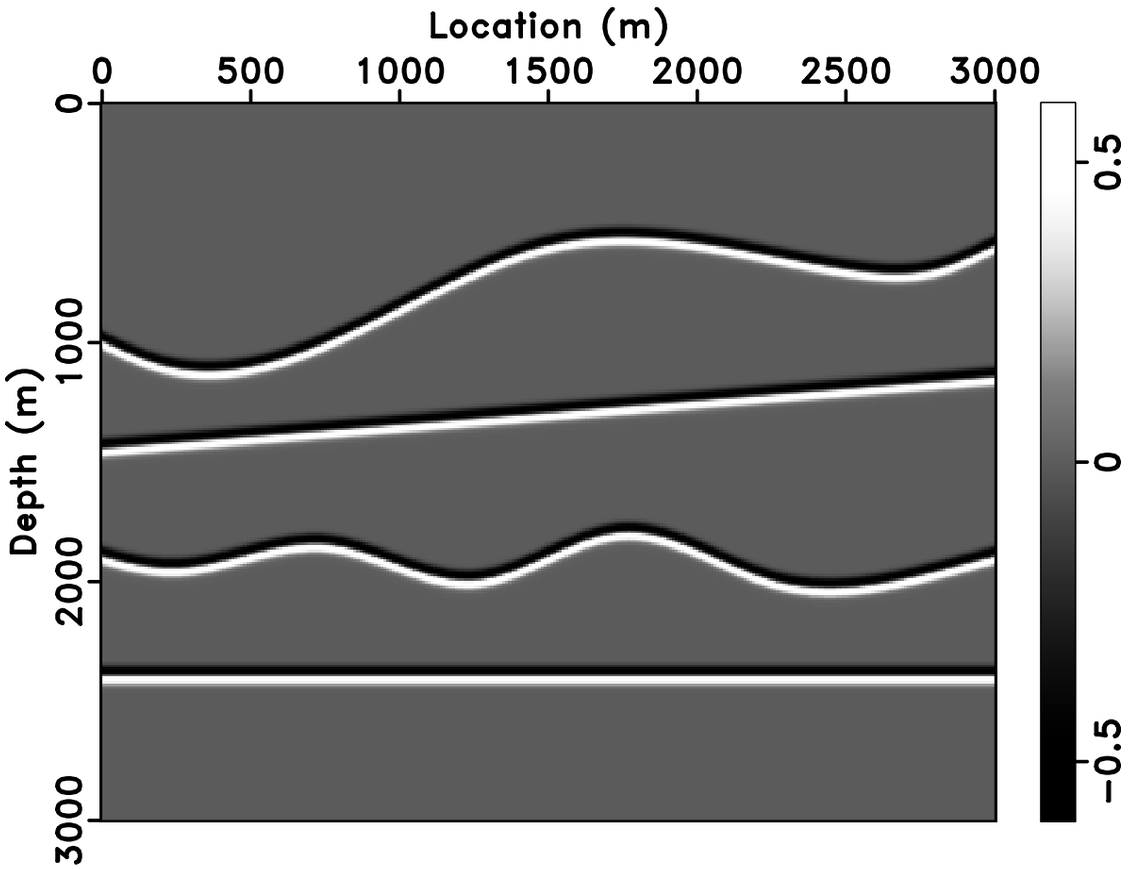


Numerical Example - Simple

- $dx=dz=dh=10m\ dt=1ms\ dx_s=40m\ dx_r=10m$
- 2.5-5-30-35 bandpass wavelet

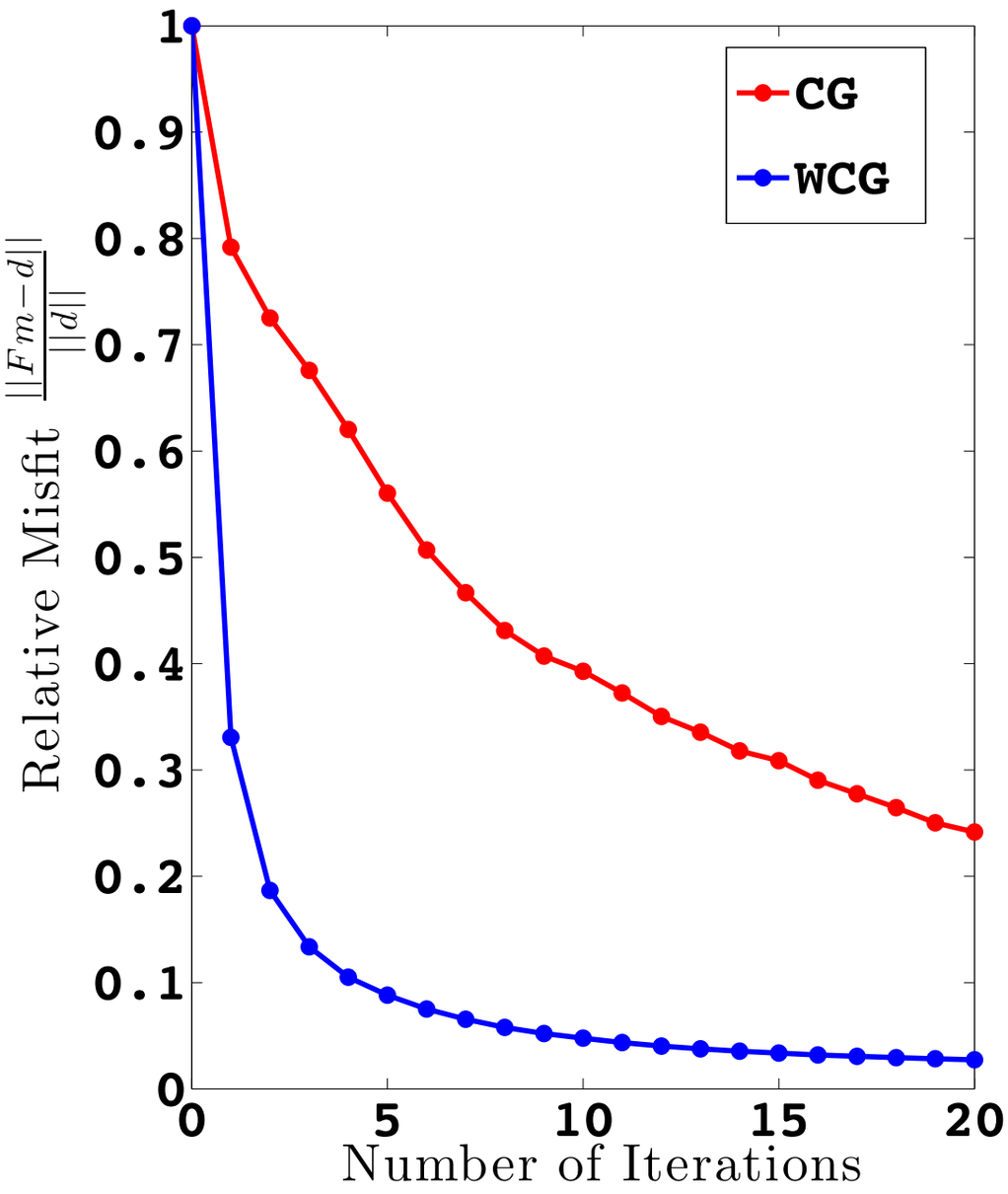


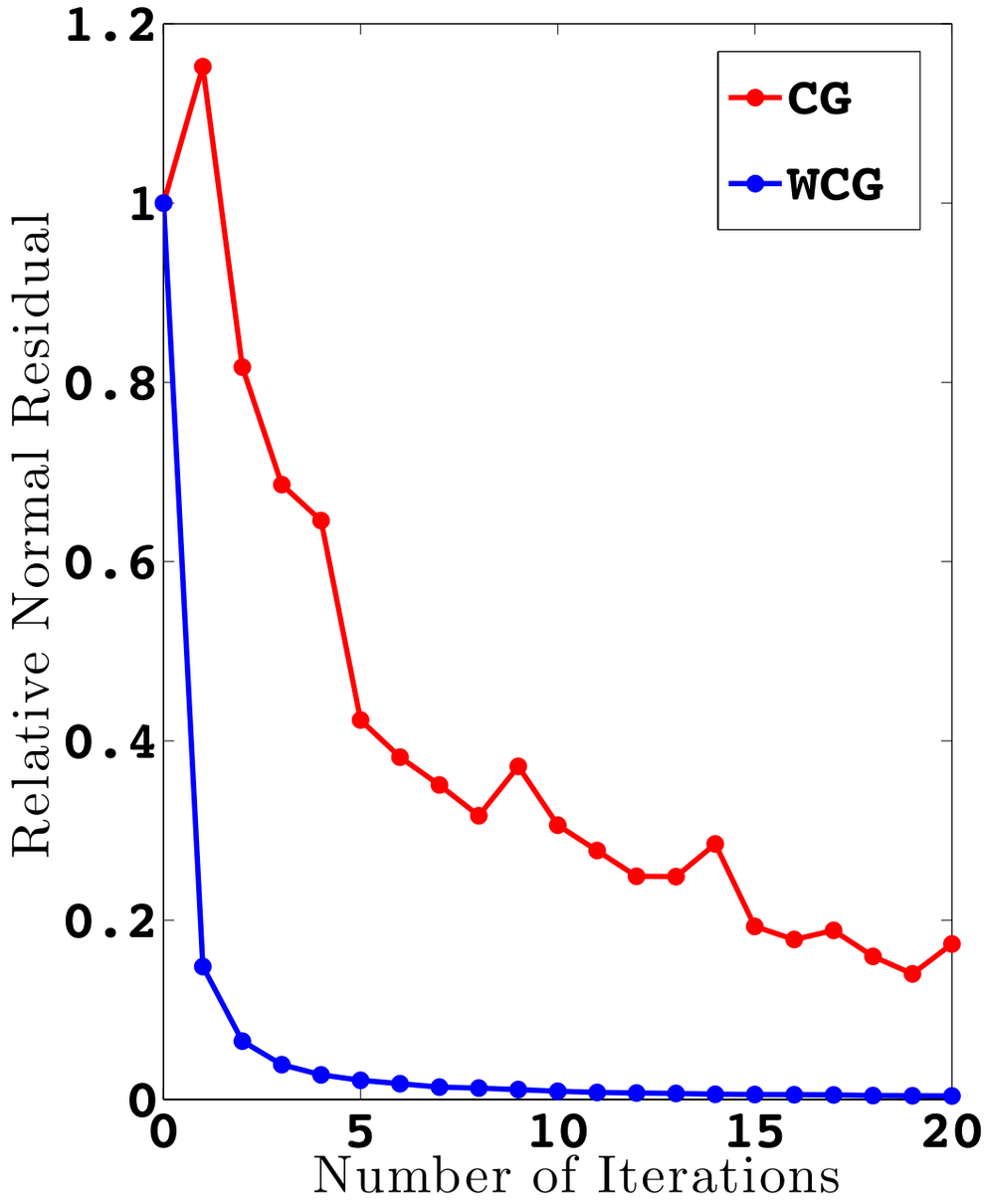
Background Model



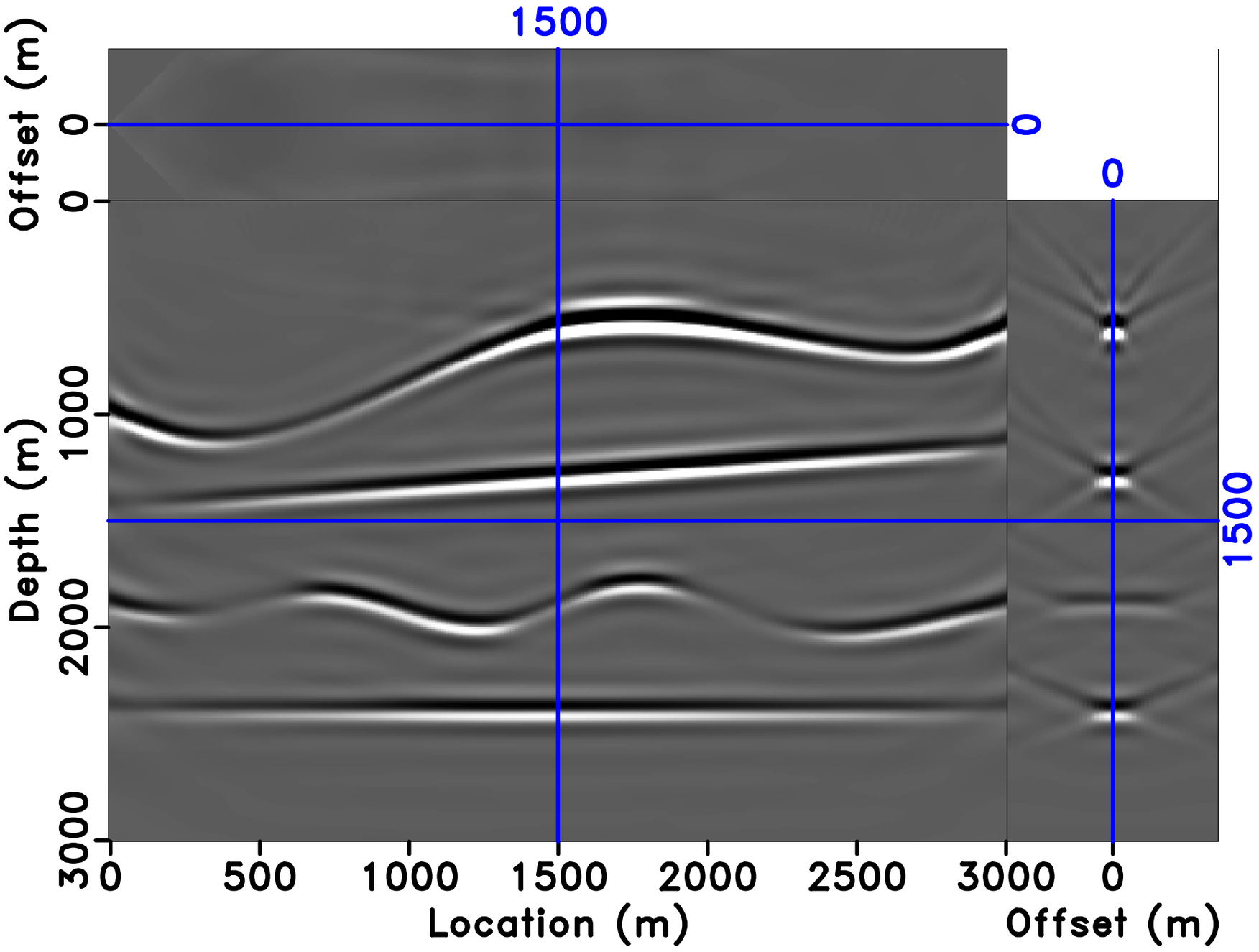
Reflectivity Model

Numerical Example - Simple



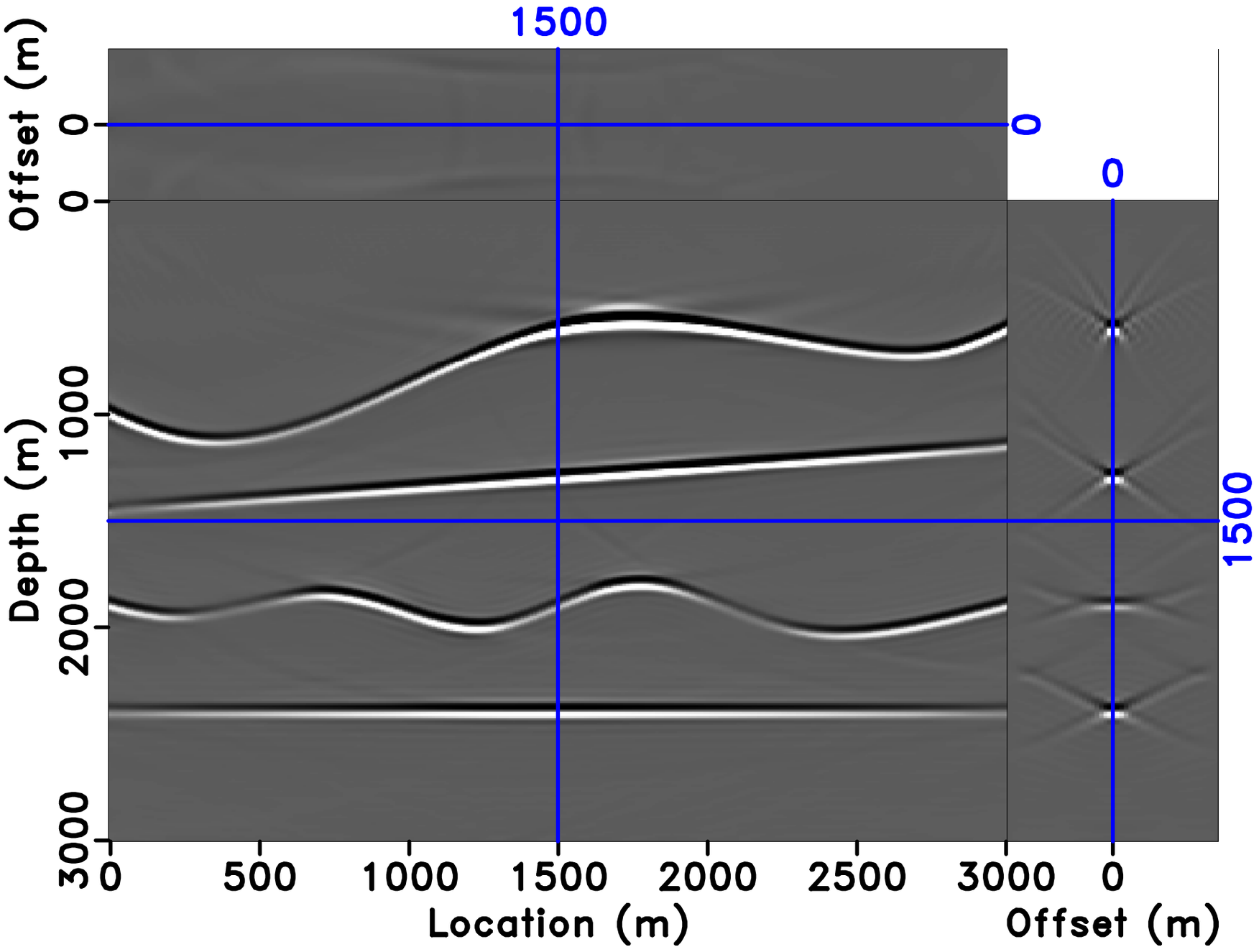


Numerical Example - Simple



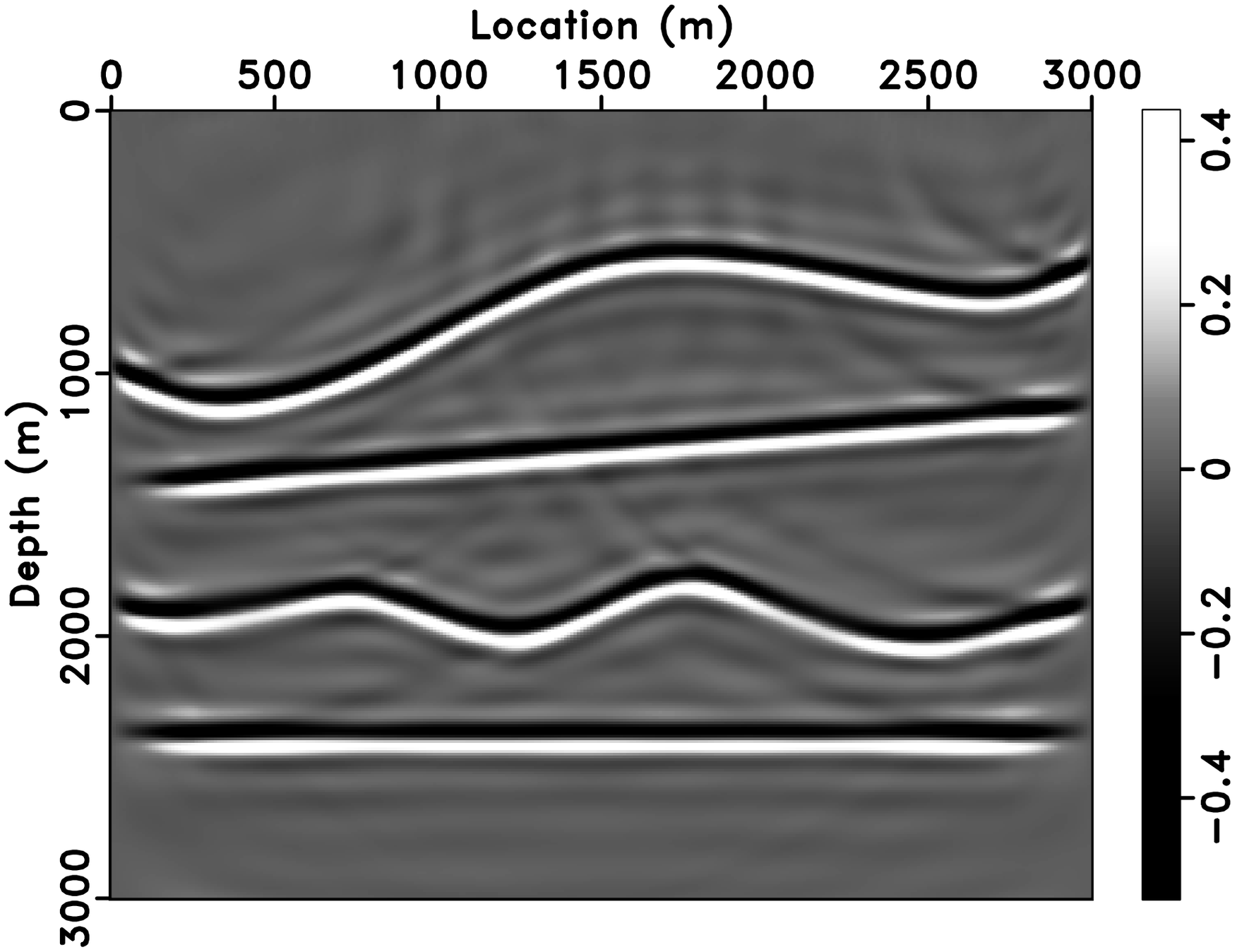
ELSM with 20 iteration CG

Numerical Example - Simple



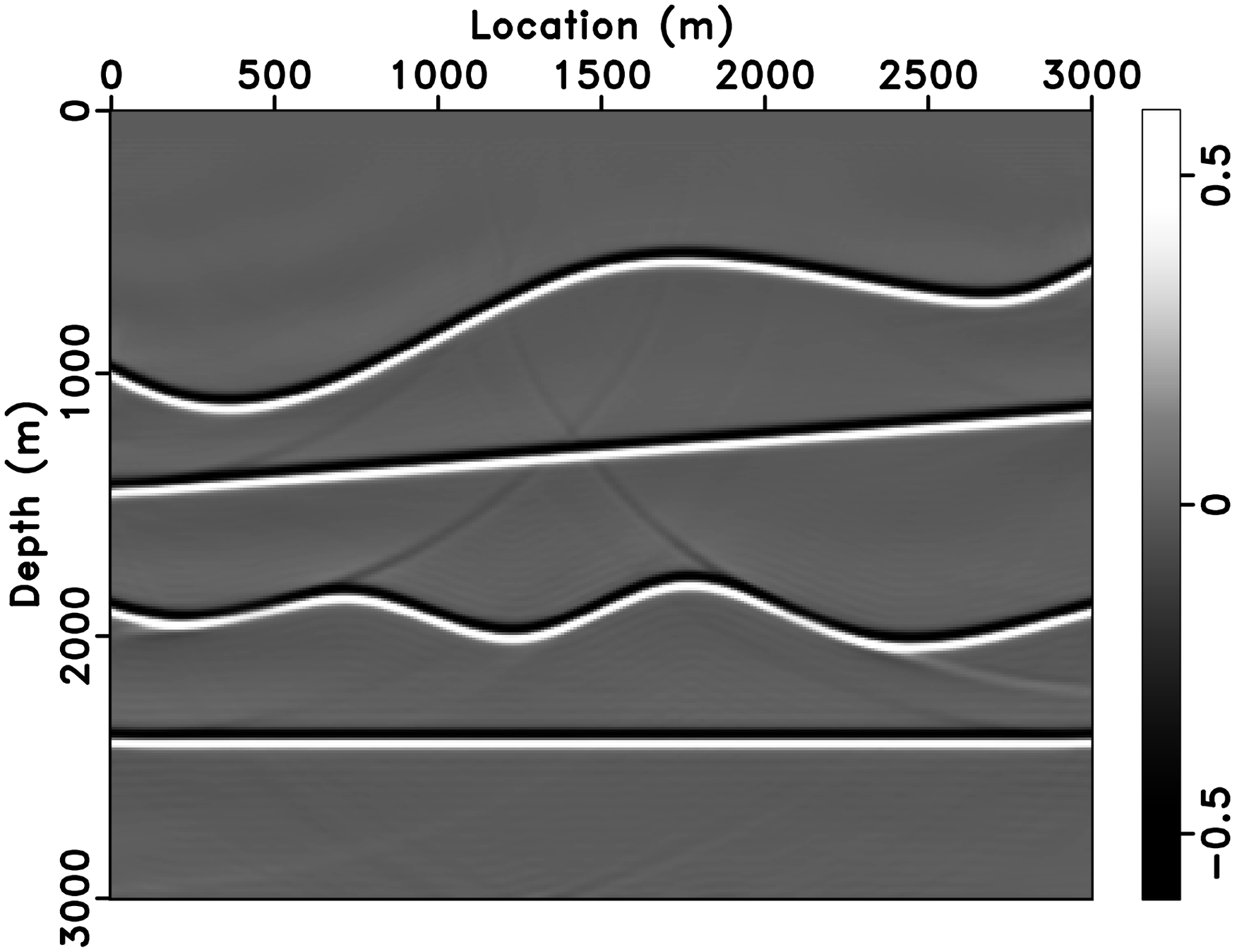
ELSM with 20 iteration WCG

Numerical Example - Simple



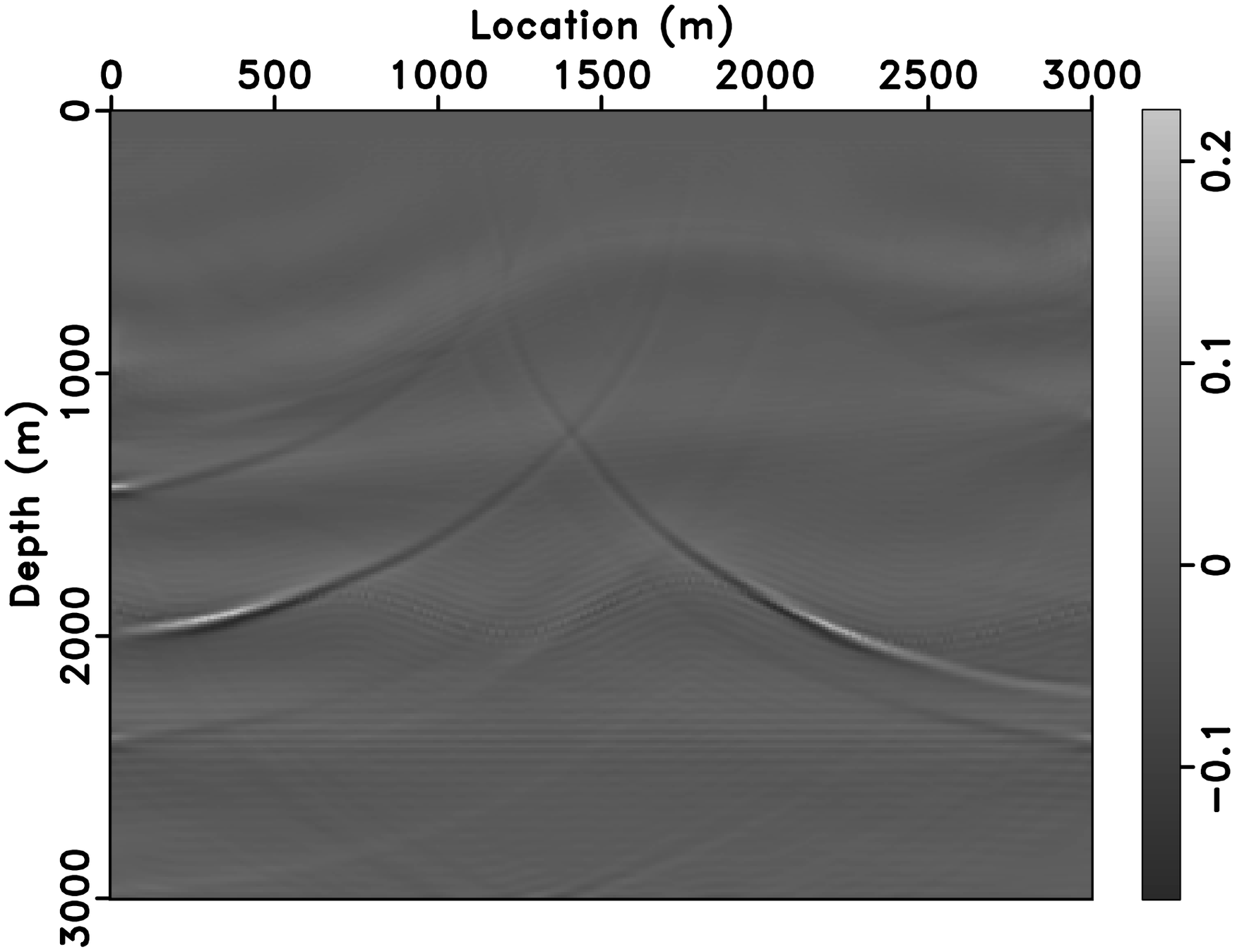
Stacked Image from CG

Numerical Example - Simple



Stacked Image from WCG

Numerical Example - Simple



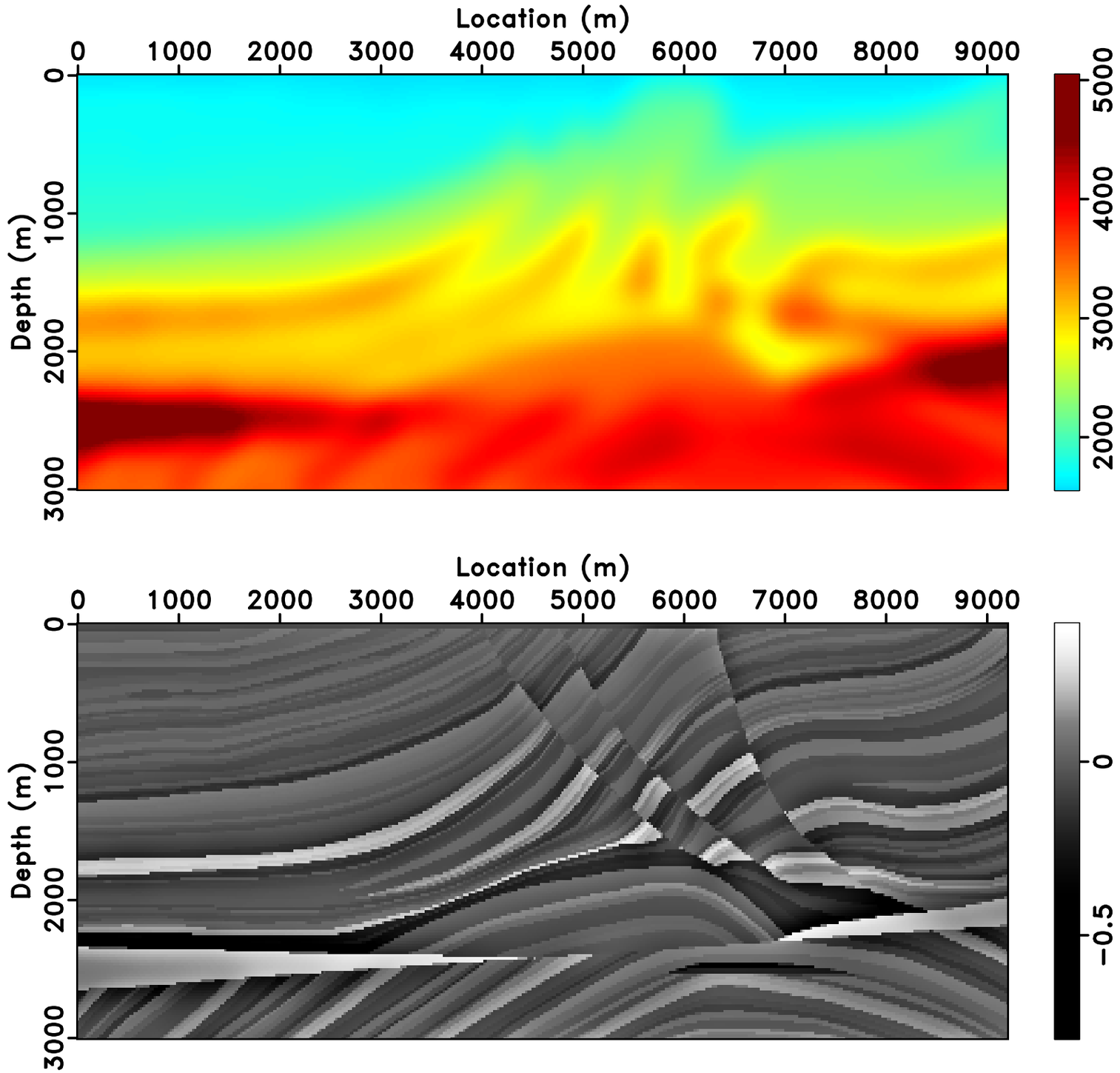
Difference with Reflectivity Model

Numerical Example - Marmousi

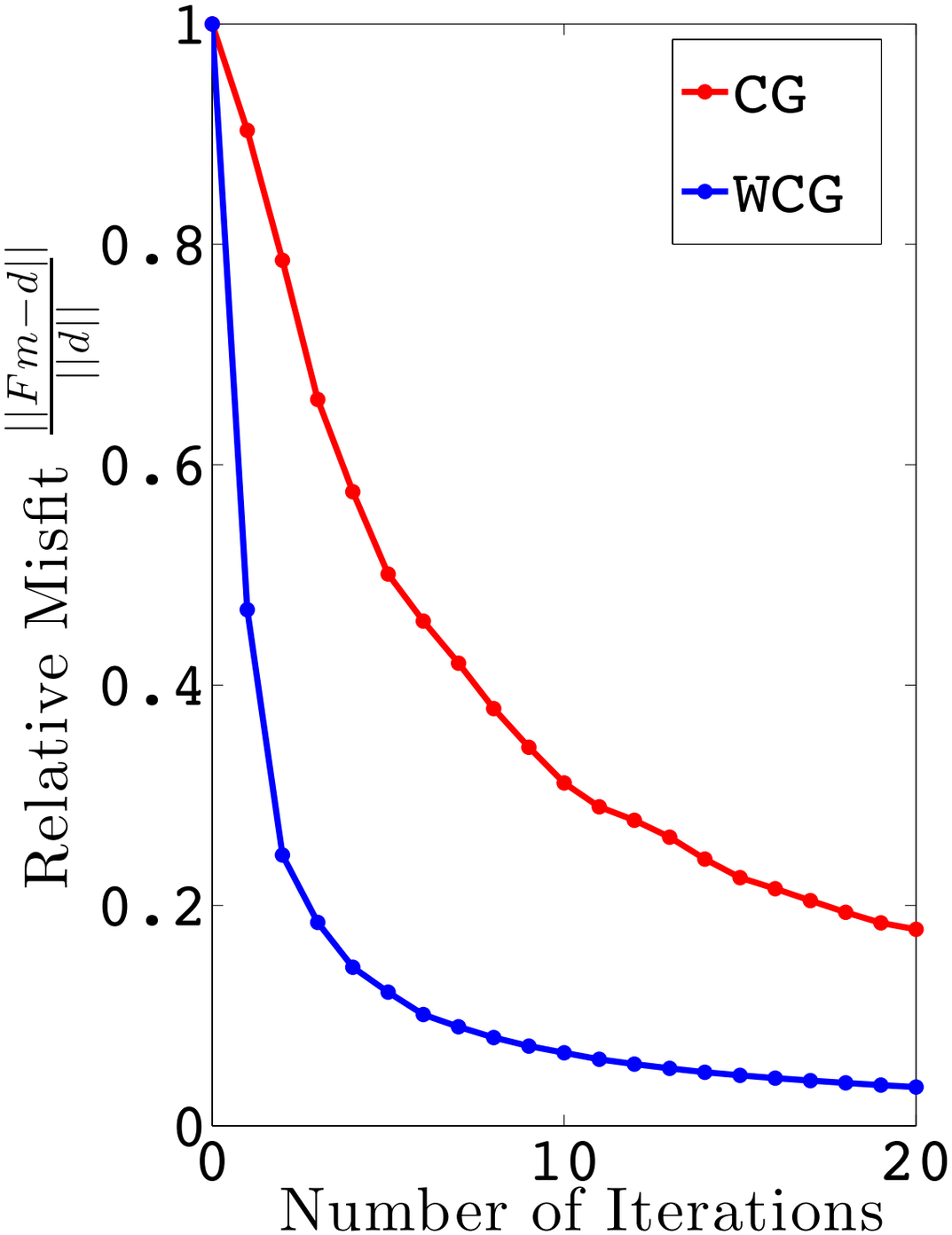
Background Model

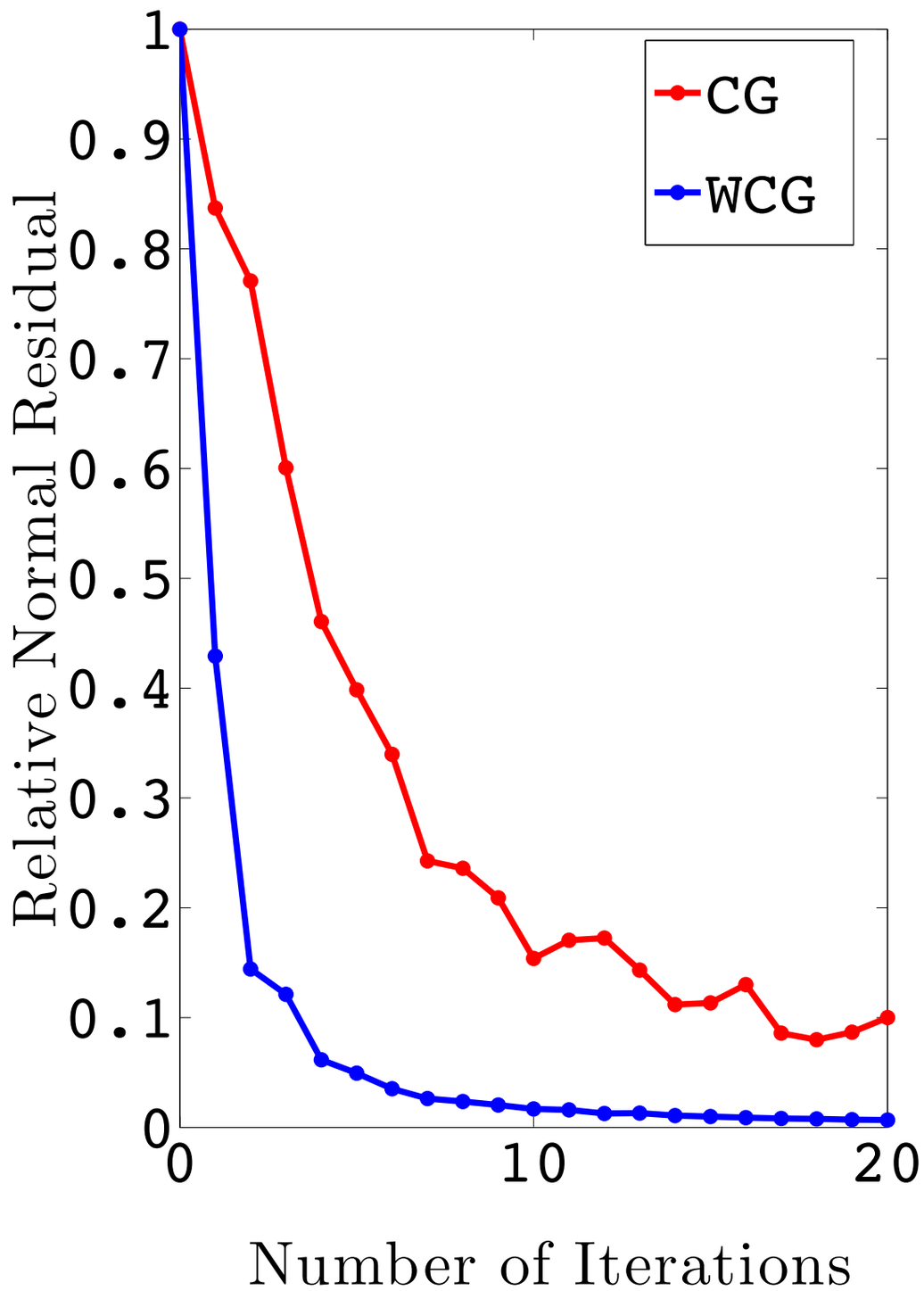
- $dx=20m$
- $dt=2ms$

Reflectivity Model

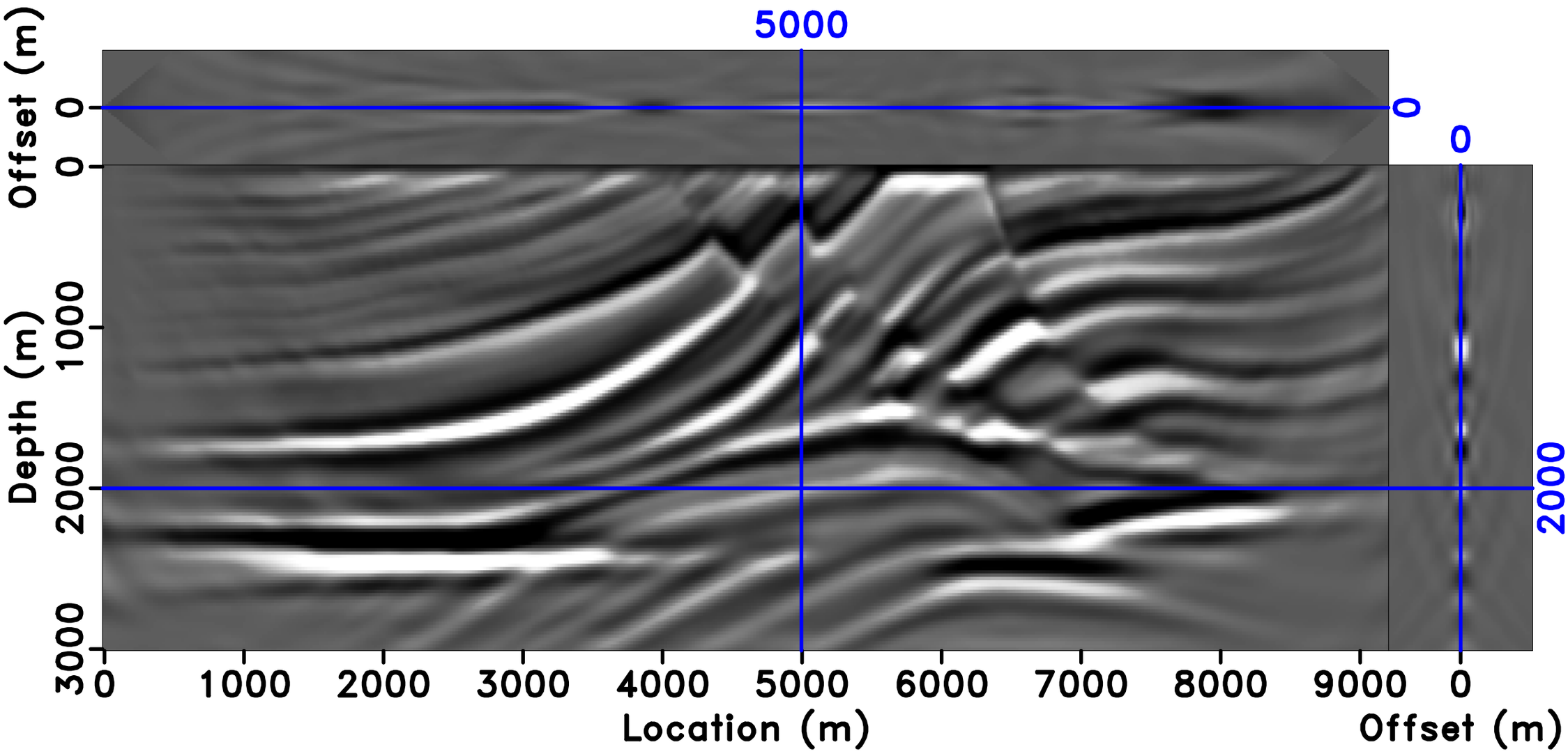


Numerical Example - Marmousi



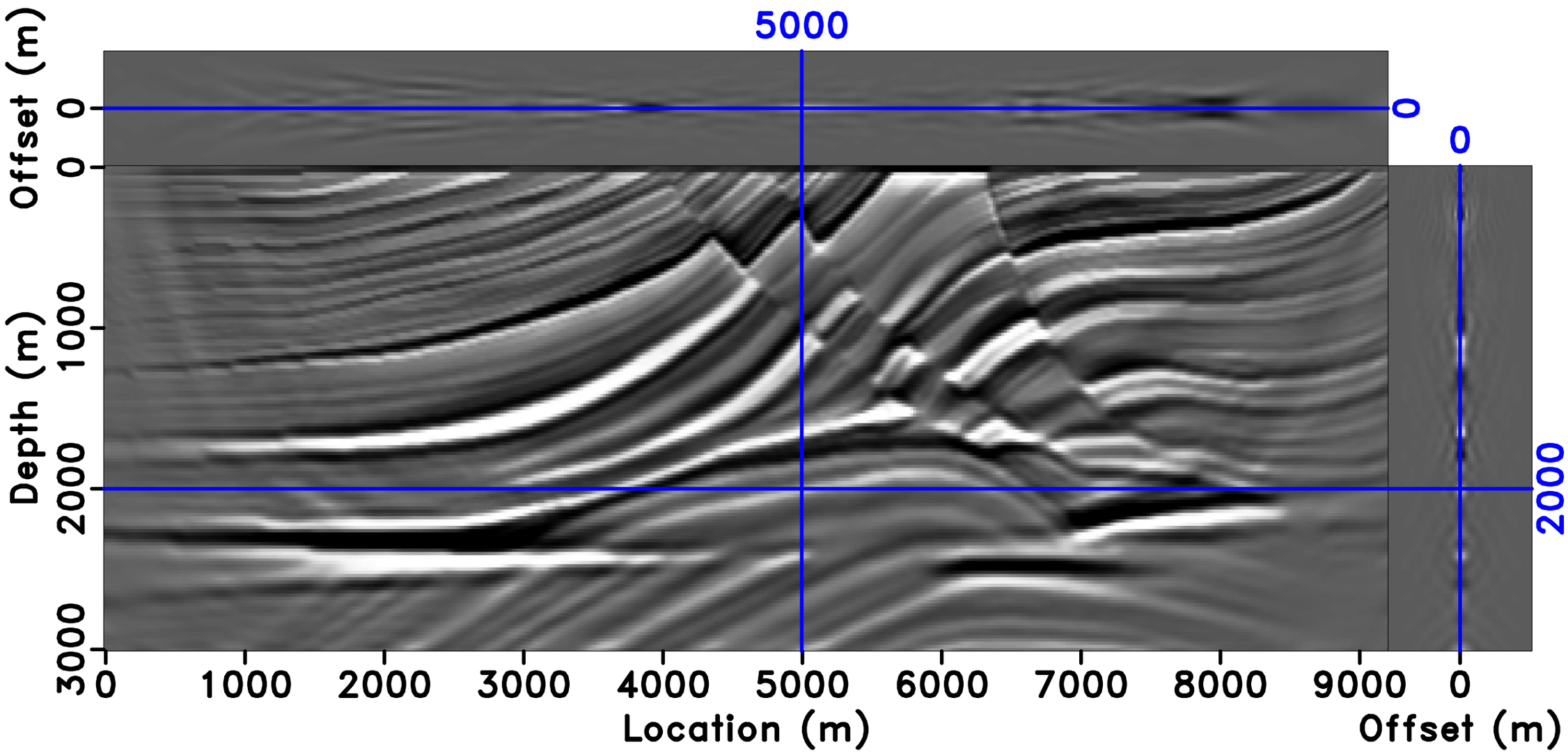


Numerical Example - Marmousi



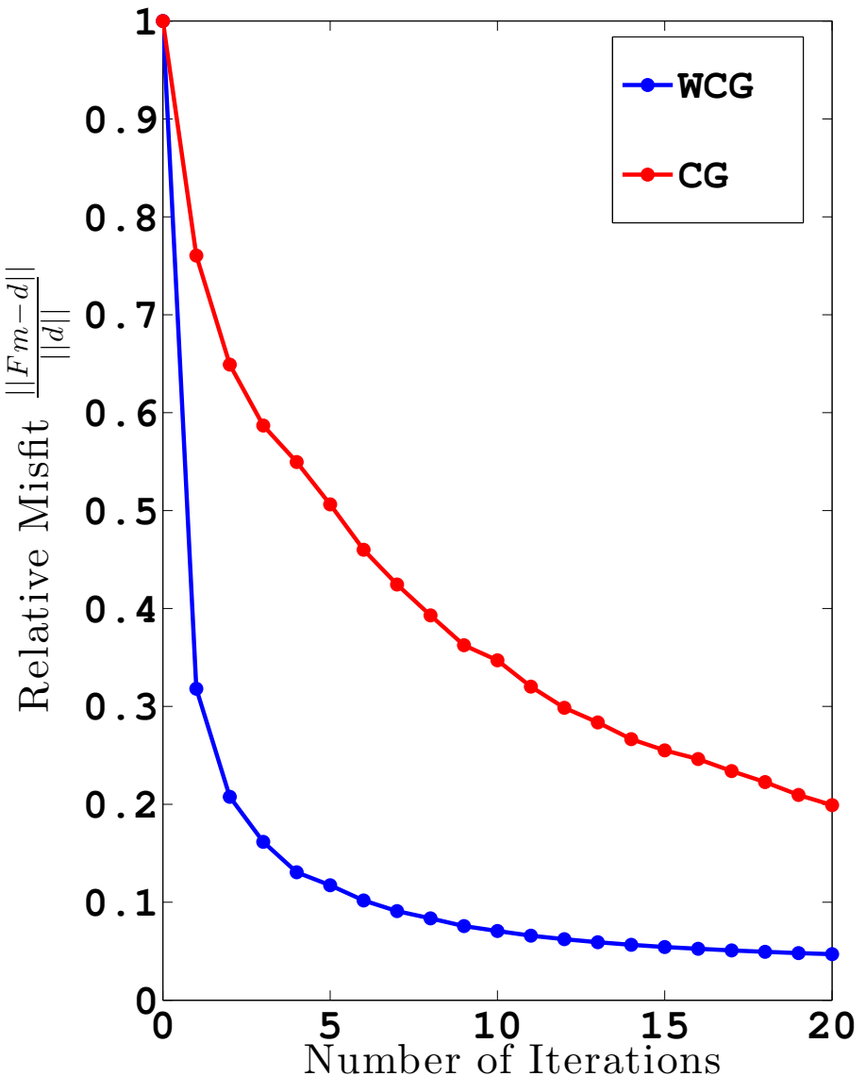
ELSM with 20 iteration CG

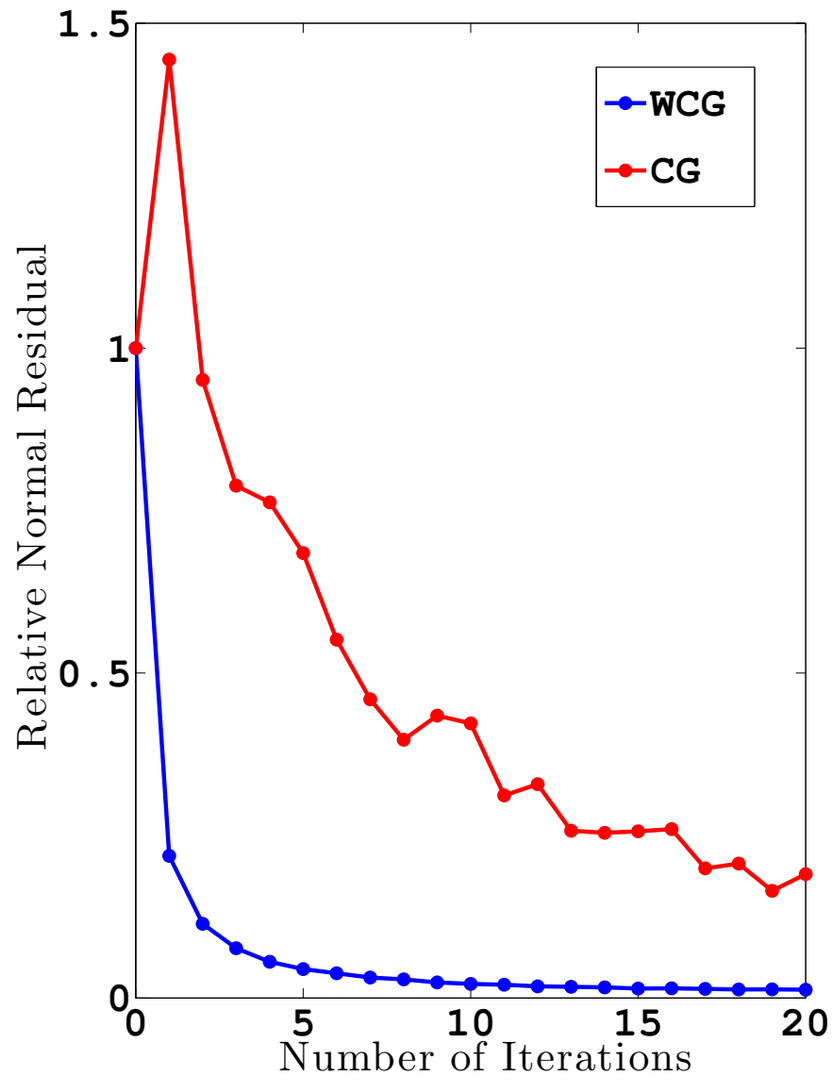
Numerical Example - Marmousi



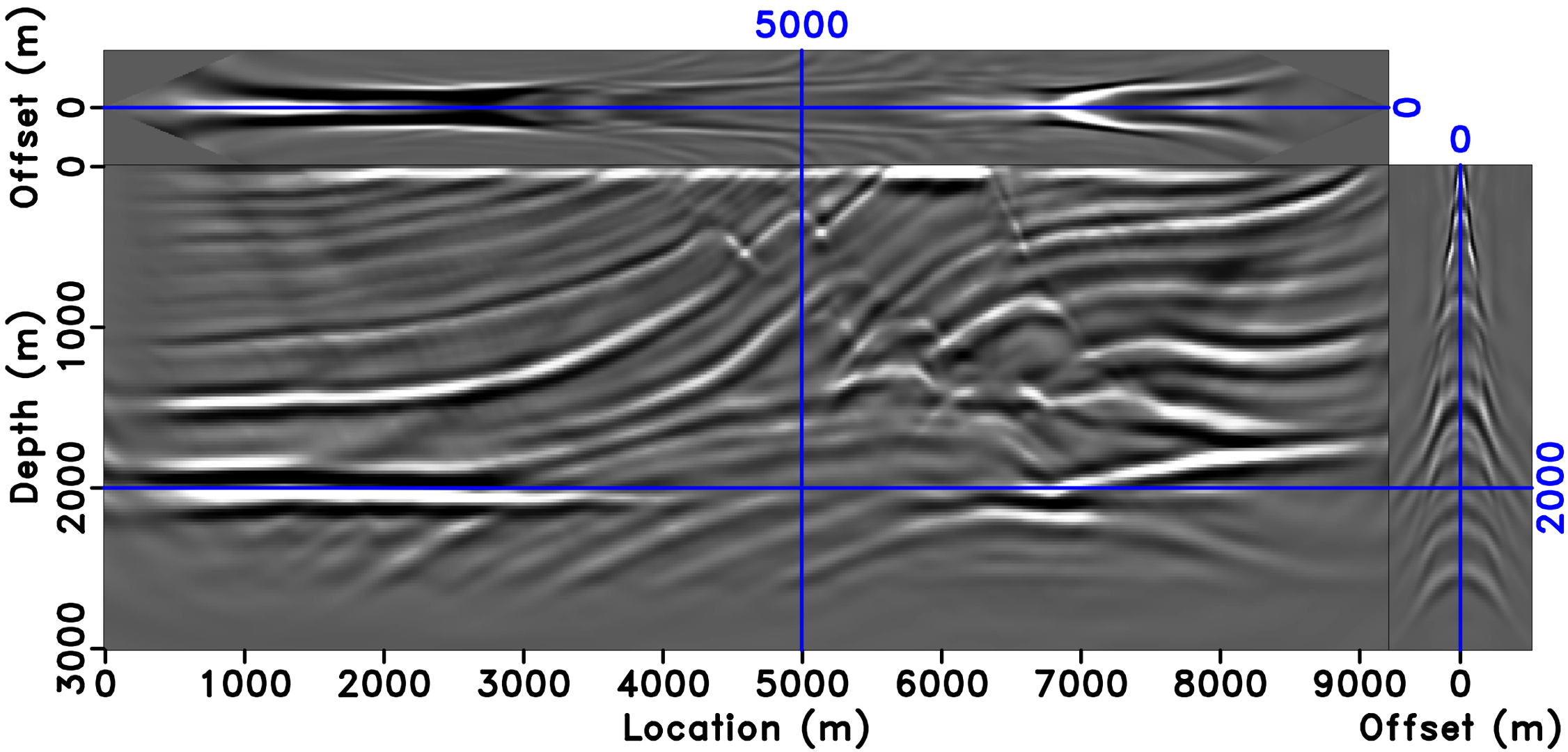
ELSM with 20 iteration WCG

Numerical Example - Wrong Velocity Model





Numerical Example - Wrong Velocity Model



ELSM with 20 iteration CG

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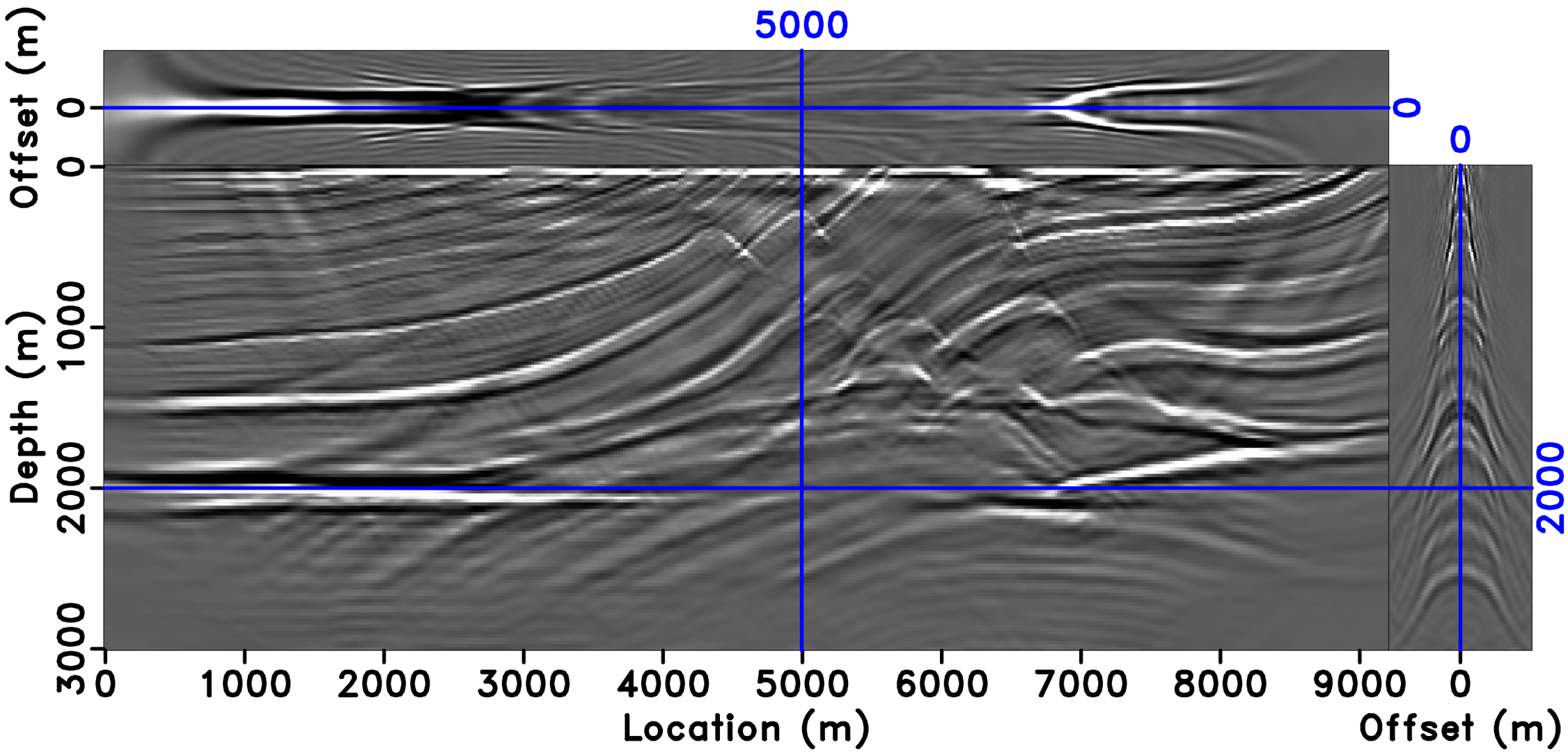
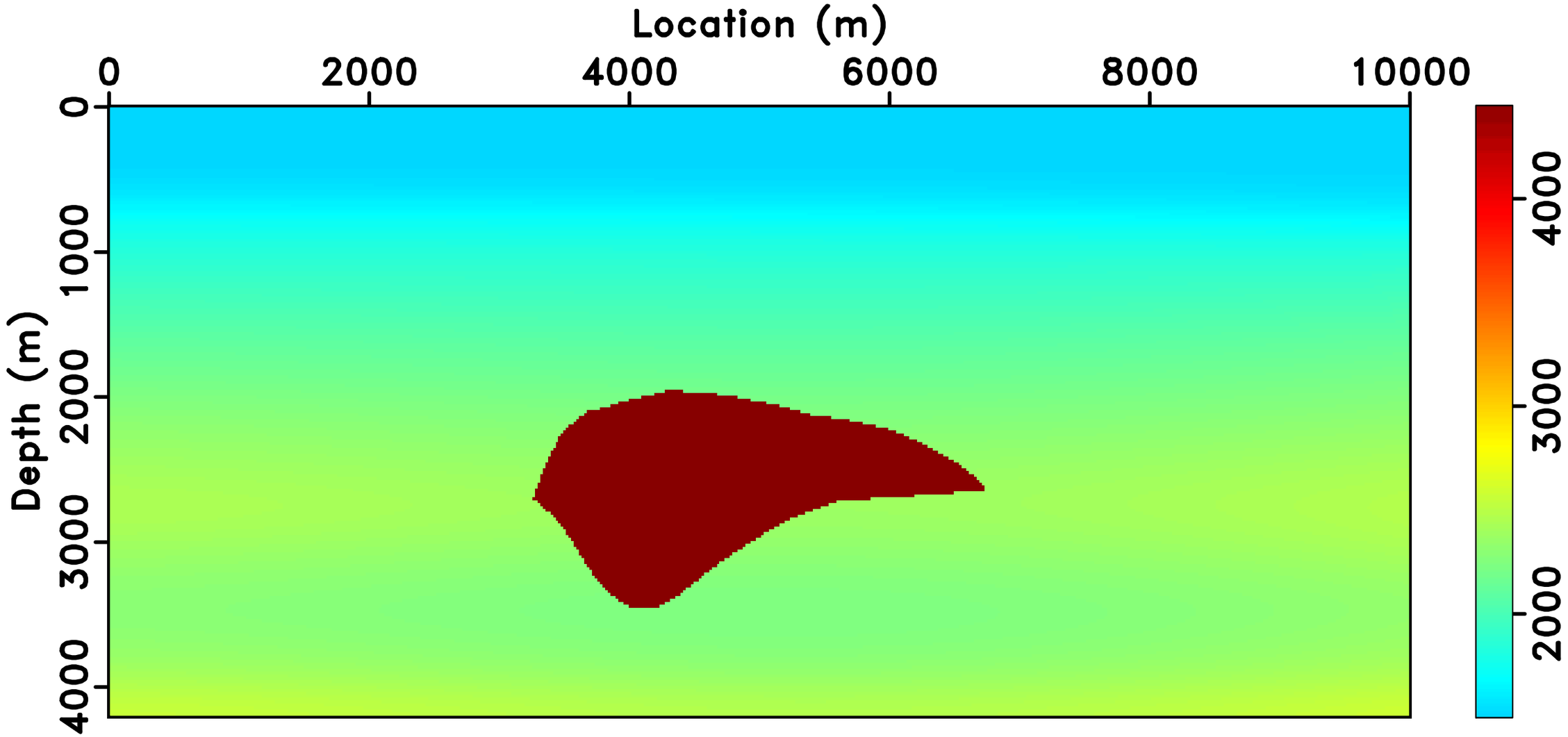


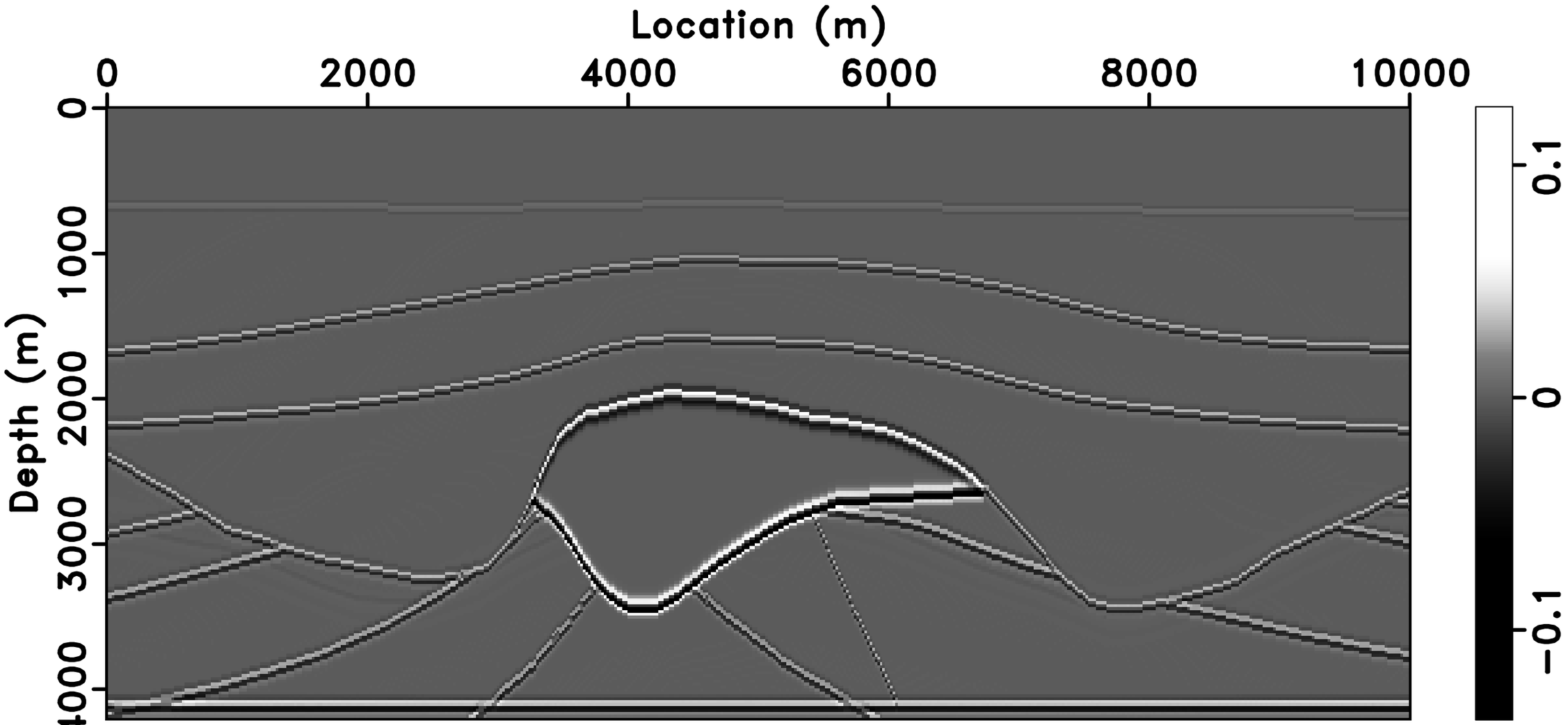
ELSM with 20 iteration WCG

Numerical Example - Salt



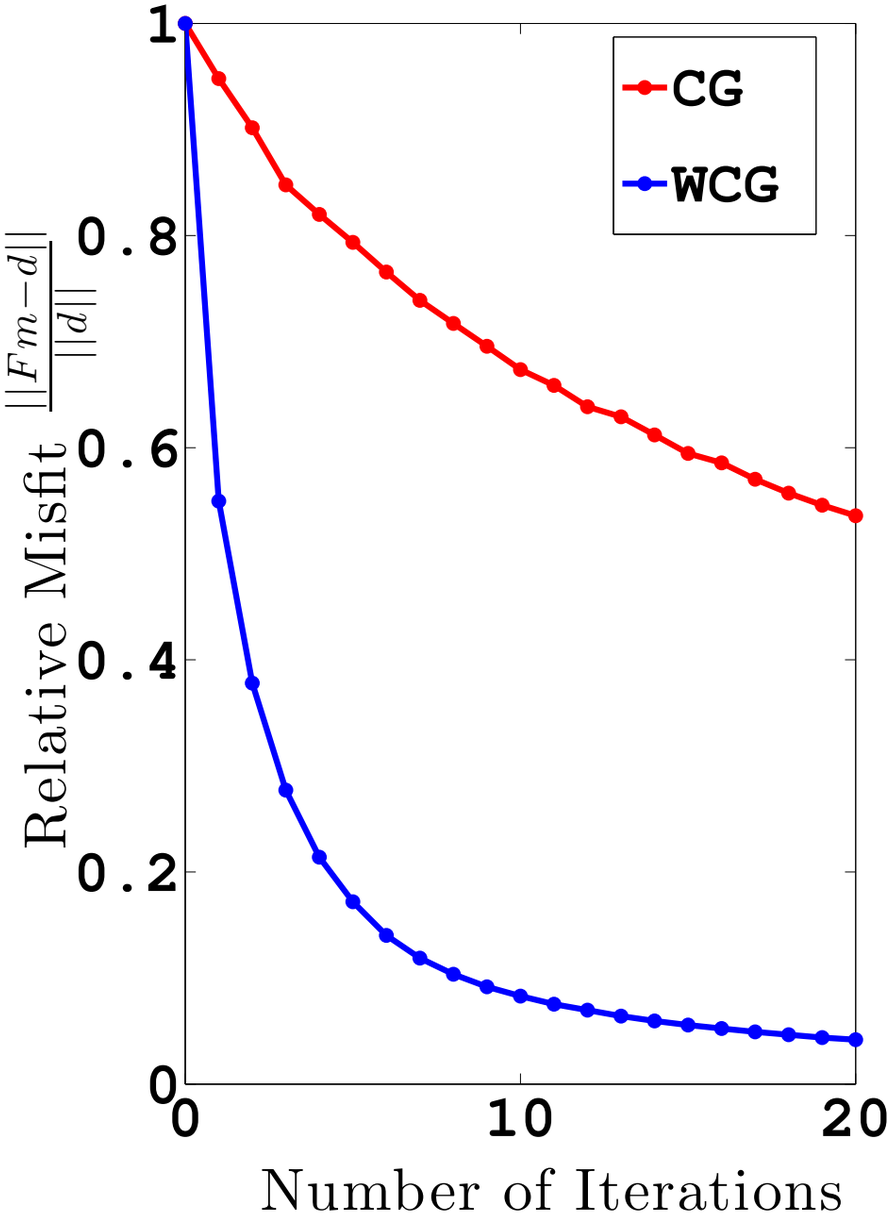
Velocity Model

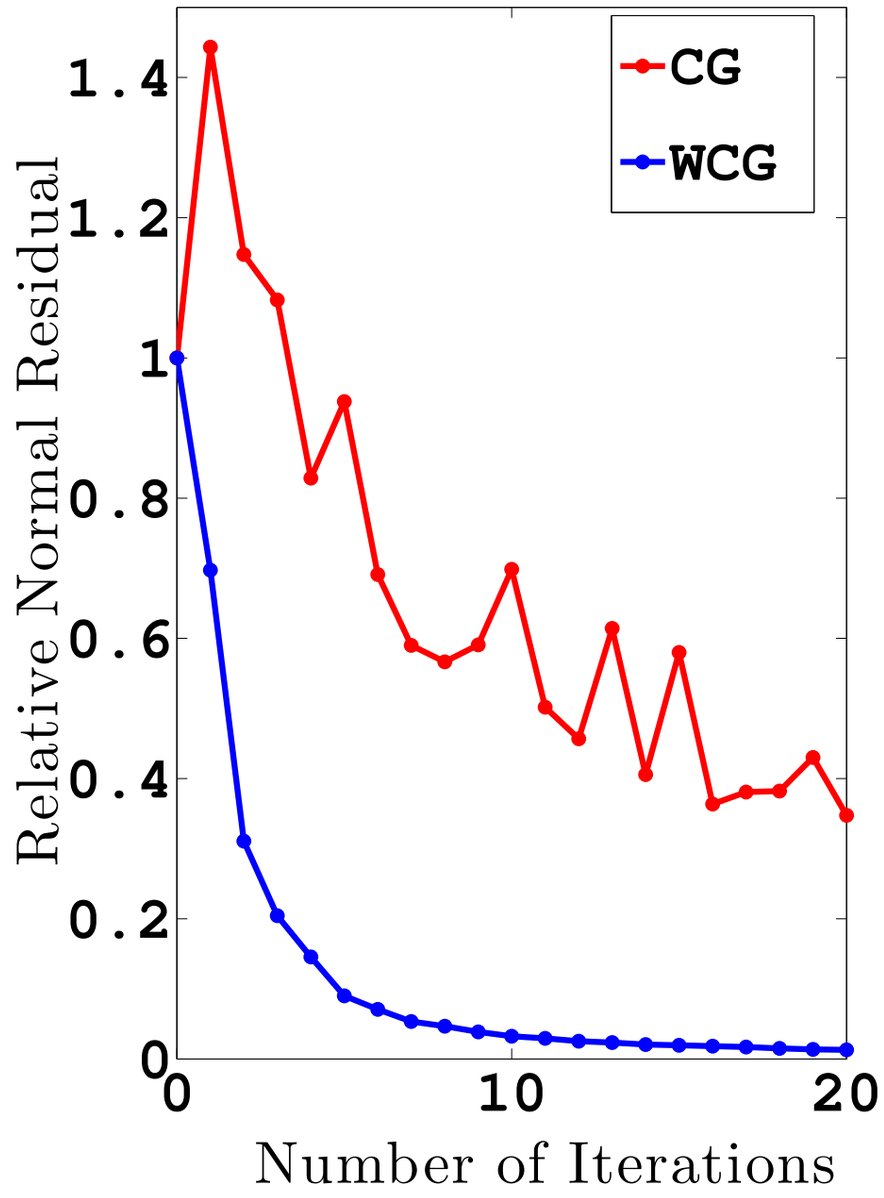
Numerical Example - Salt



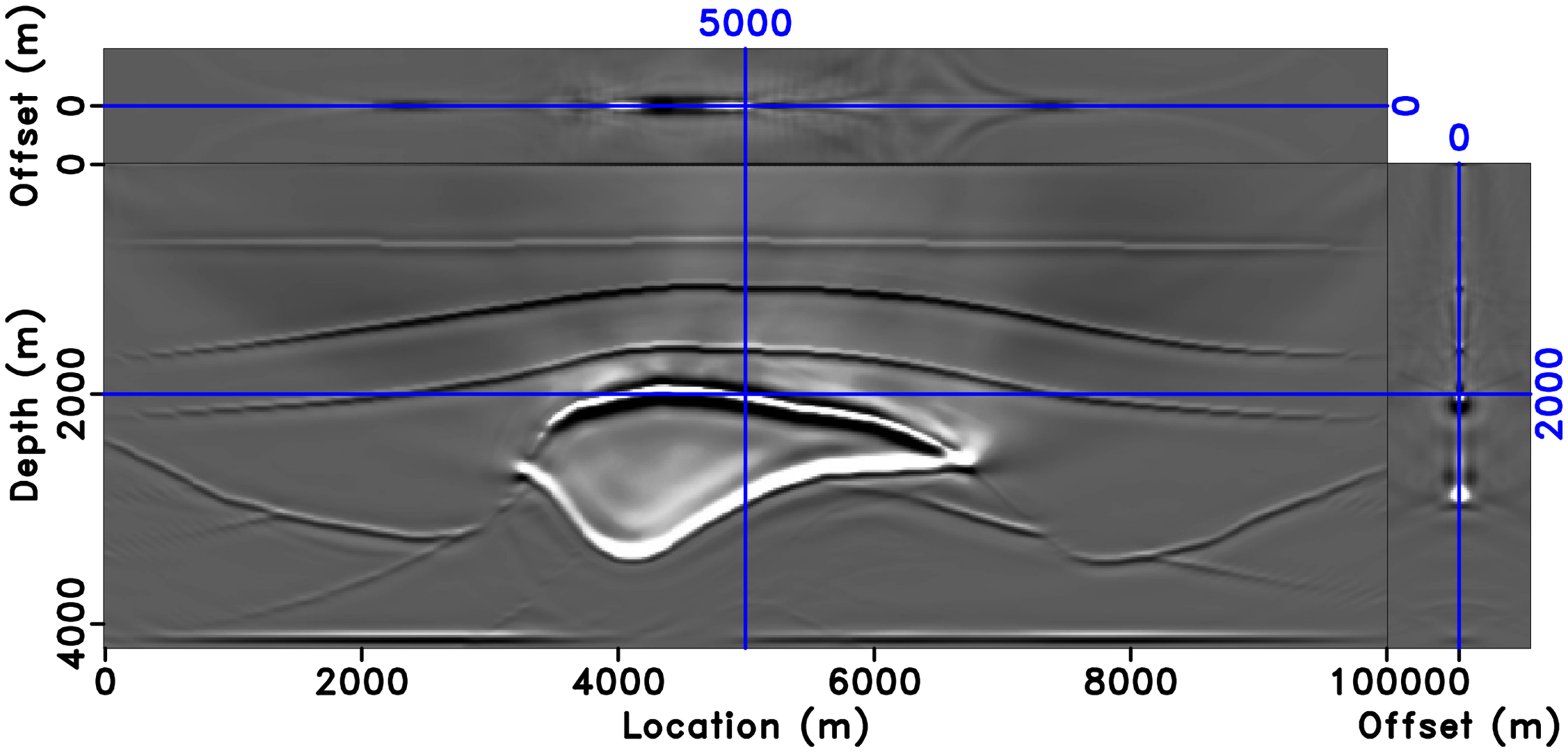
Reflectivity Model

Numerical Example - Salt



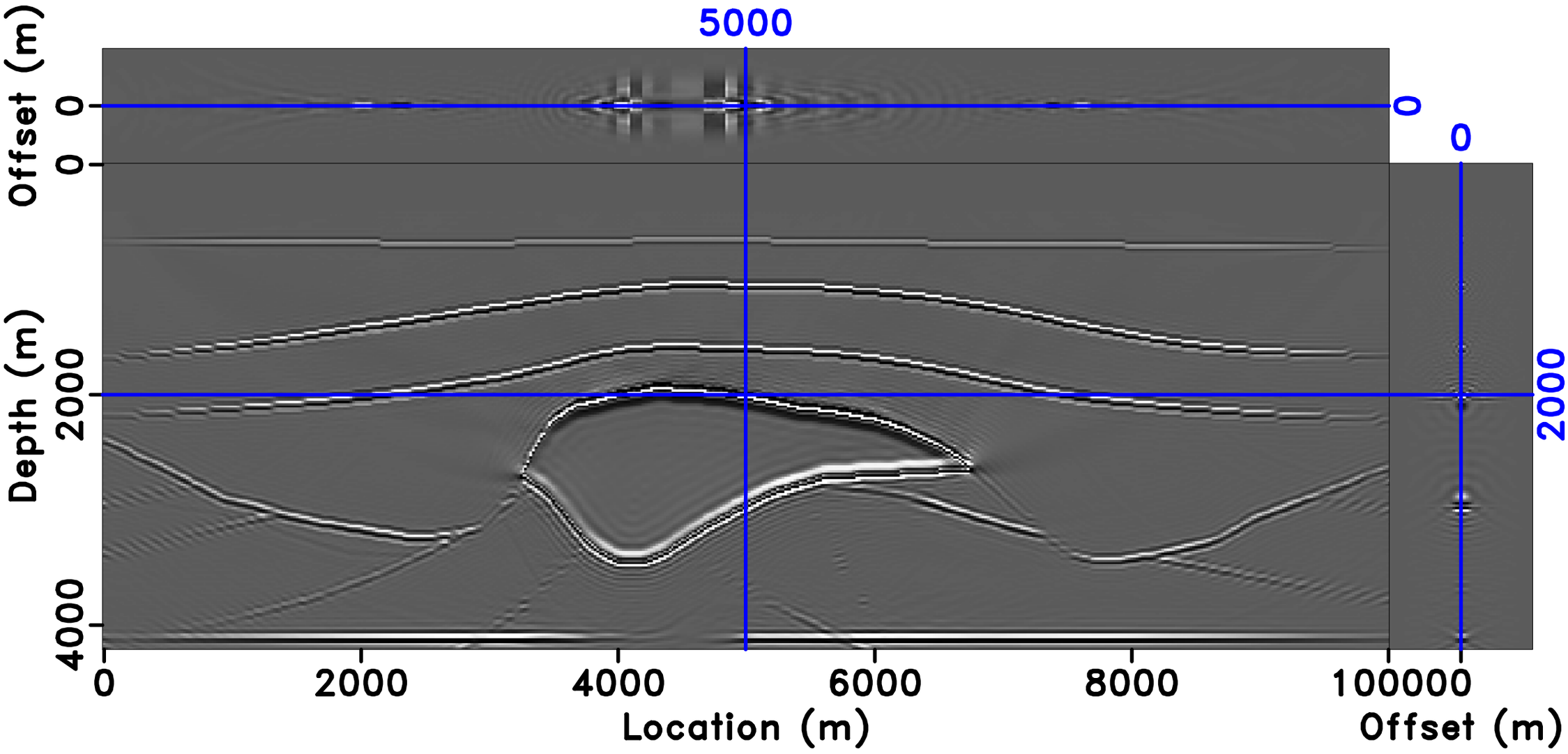


Numerical Example - Salt



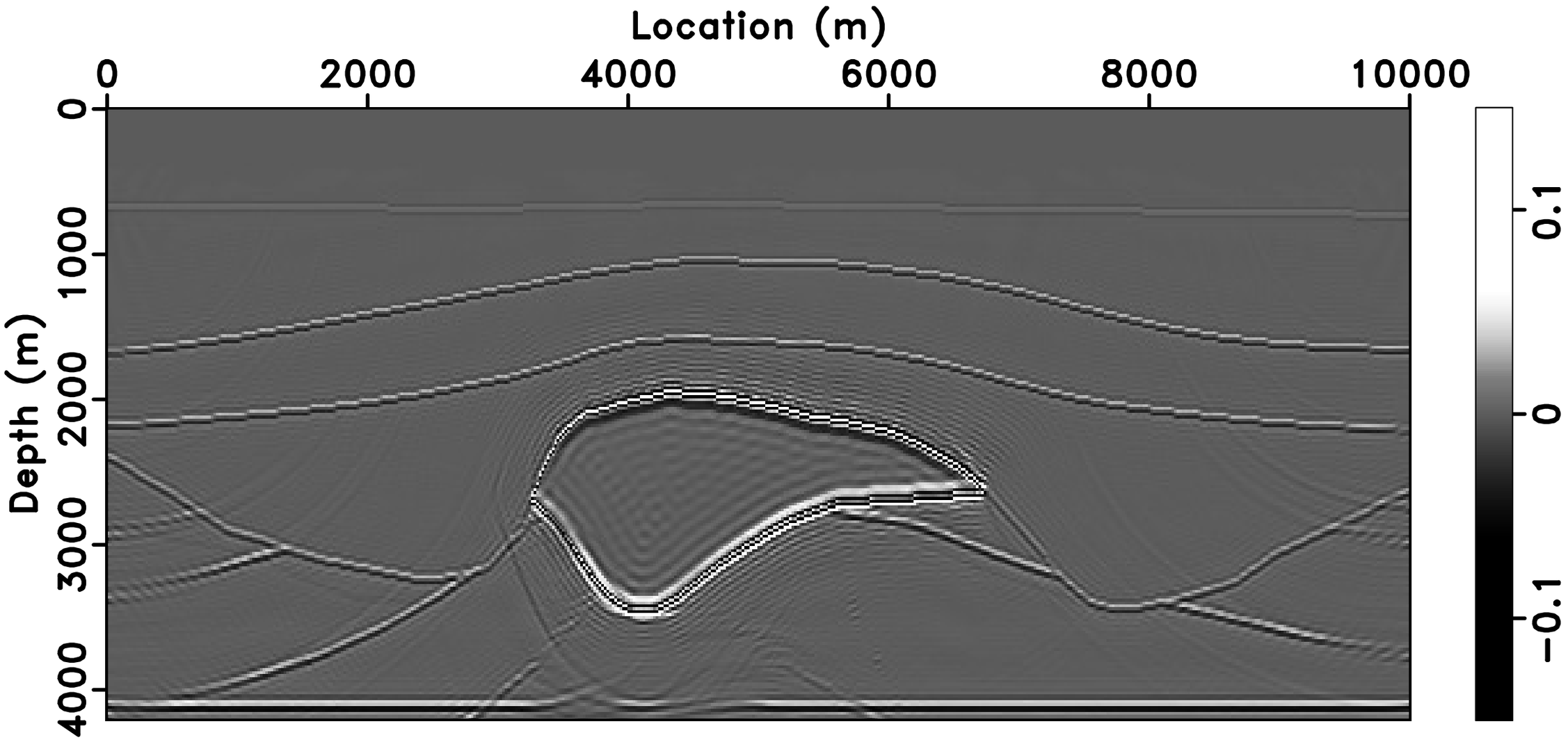
ELSM with 20 iteration CG

Numerical Example - Salt



ELSM with 20 iteration WCG

Numerical Example - Salt



Stacked Image from WCG

Conclusion & Future Work

Conclusion

- ELSM is not sensitive to velocity error
- WCG iteration can accelerate ELSM

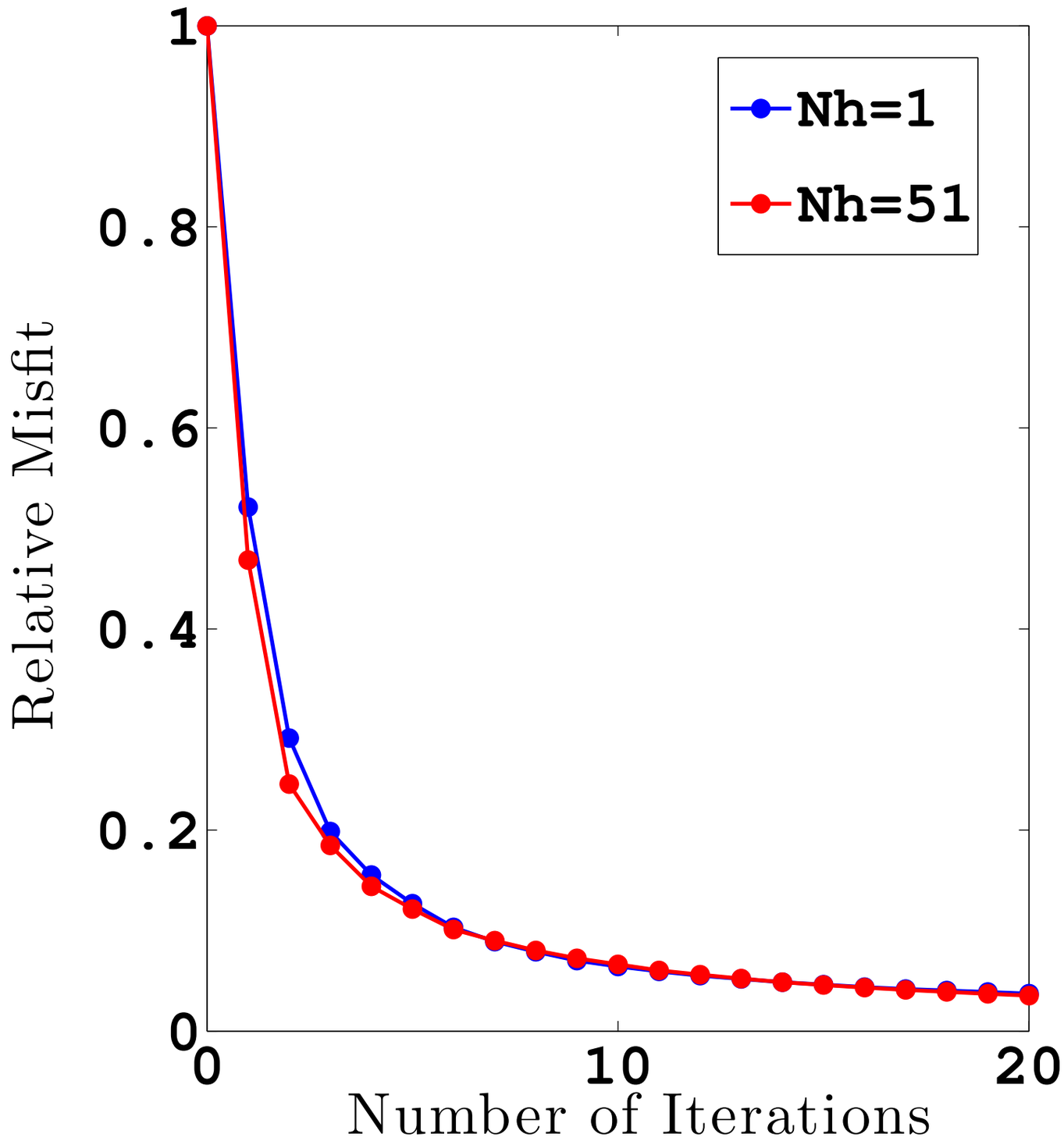
Future Work

- Coarse Sampling,Missing Data
- Inversion Velocity Analysis

Acknowledgement

- Fons ten Kroode for inspiring our work
- Jon Sheiman, Henning Kuehl, Peng Shen
- Shell International Exploration & Production
- TRIP Members and Sponsors
- Seismic Unix and Madagascar
- TACC and RCSG
- Thank you for listening

Numerical Example - Offset Range

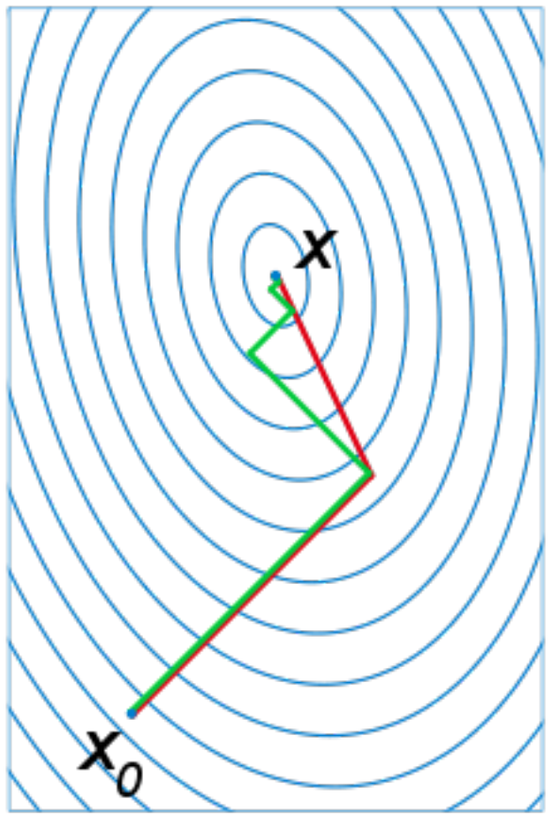


Conjugate Gradient

Algorithm 1 Conjugate Gradient

$z_0 \leftarrow F^T(d - Fx_0)$
$p_0 \leftarrow z_0$
$k \leftarrow 0$
repeat
  $\alpha_k \leftarrow \frac{<z_k, z_k>}{<Fp_k, Fp_k>}$
  $x_{k+1} \leftarrow x_k + \alpha_k p_k$
  $z_{k+1} \leftarrow z_k - \alpha_k F^T F p_k$
  $\beta_{k+1} \leftarrow \frac{<z_{k+1}, z_{k+1}>}{<z_k, z_k>}$
  $p_{k+1} \leftarrow z_{k+1} + \beta_{k+1} p_k$
  $k \leftarrow k + 1$
until Error is sufficiently small

$$F^T F m = F^T d$$



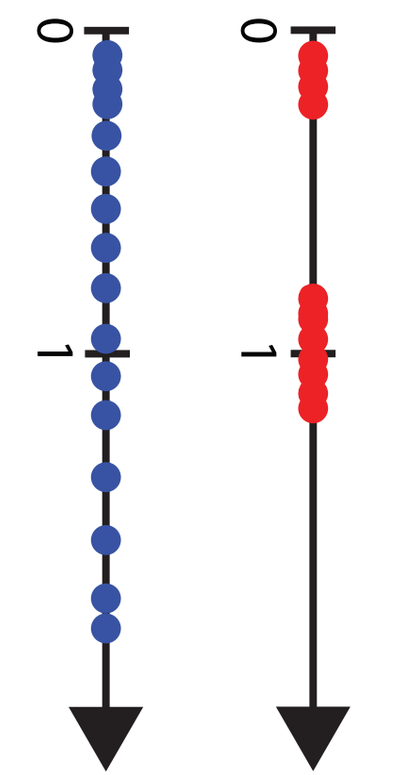
$$F^\dagger = W_{model}^{-1} F^T W_{data}$$

Weighted Conjugate Gradient

Algorithm 2 Weighted Conjugate Gradient

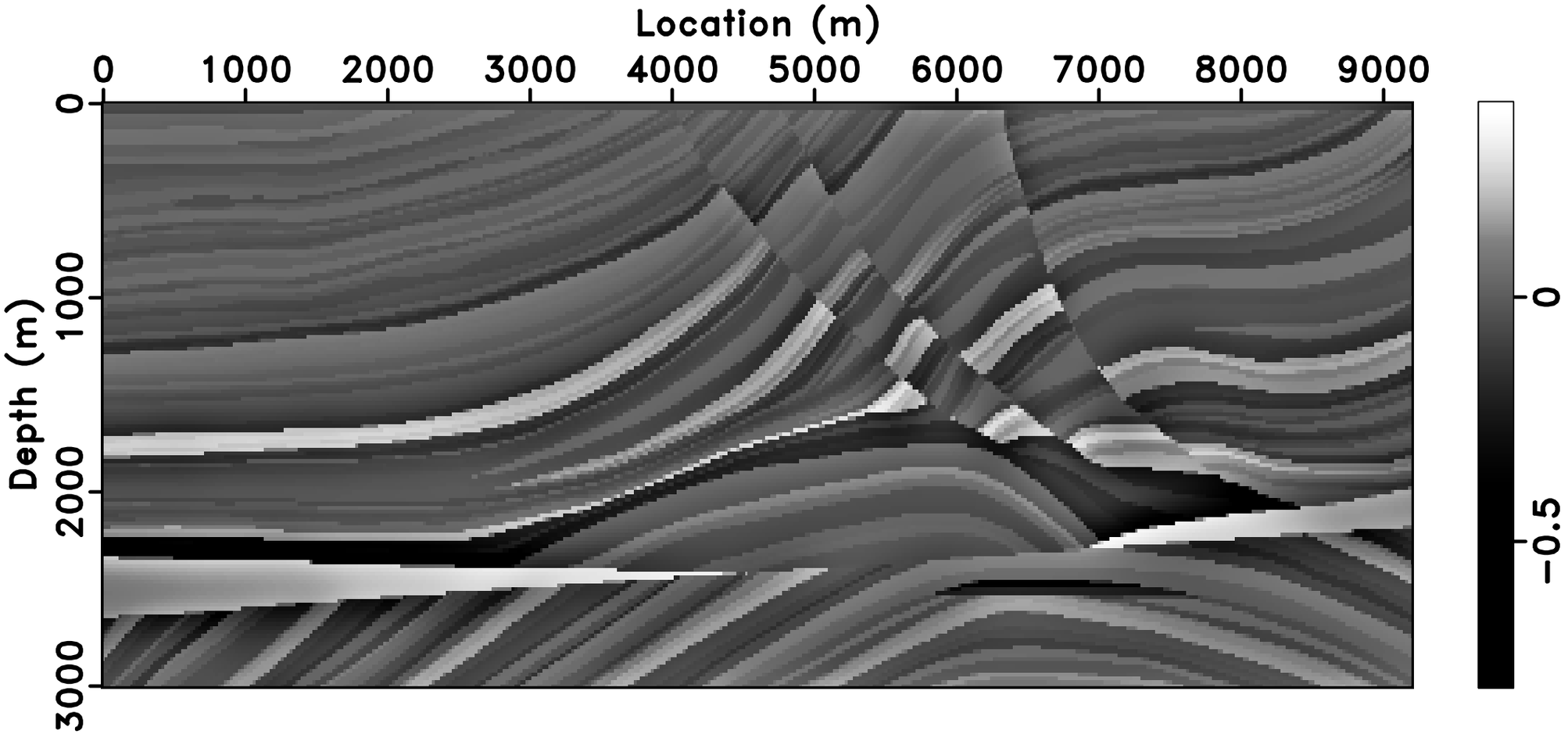
$z_0 \leftarrow F^{\dagger}(d - Fx_0)$
$p_0 \leftarrow z_0$
$k \leftarrow 0$
repeat
  $\alpha_k \leftarrow \frac{<z_k, z_k>_{model}}{<Fp_k, Fp_k>_{data}}$
  $x_{k+1} \leftarrow x_k + \alpha_k p_k$
  $z_{k+1} \leftarrow z_k - \alpha_k F^{\dagger} F p_k$
  $\beta_{k+1} \leftarrow \frac{<z_{k+1}, z_{k+1}>_{model}}{<z_k, z_k>_{model}}$
  $p_{k+1} \leftarrow z_{k+1} + \beta_{k+1} p_k$
  $k \leftarrow k + 1$
until Error is sufficiently small

$$F^{\dagger}Fm = F^{\dagger}d$$



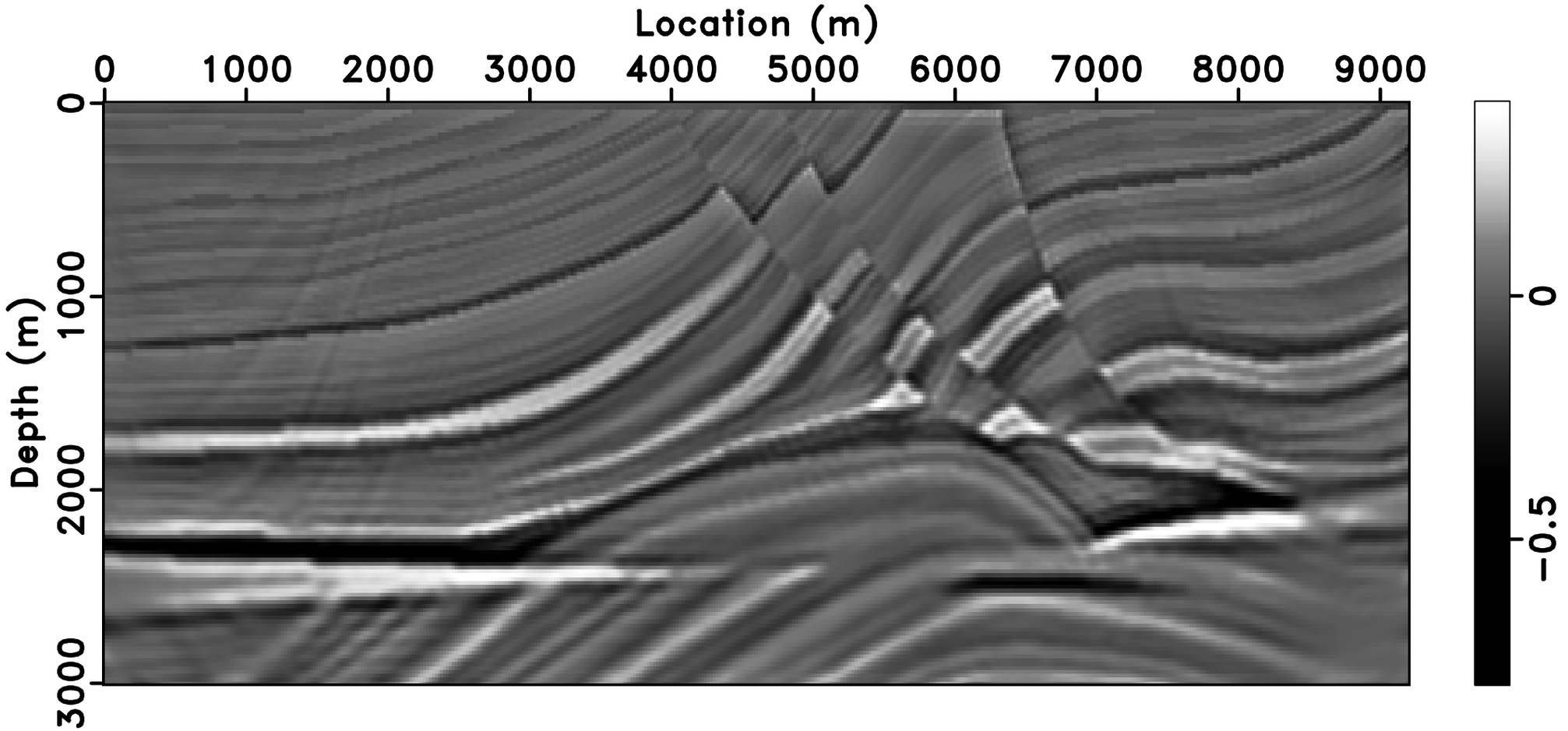
$$< m, m >_{model} = m^T W_{model} m \qquad < d, d >_{data} = d^T W_{data} d$$

Numerical Example - Marmousi



Reflectivity Model

Numerical Example - Marmousi



Stacked Image from WCG