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Education

- **University of Chinese Academy of Sciences**, Beijing, China
  Ph.D. in Computational Mathematics 09/2009 - 07/2014
  Thesis: *Reverse time migration for inverse scattering problems*
  Thesis supervisor: Professor Zhiming Chen
- **Central South University**, Changsha, China
  B.S. in Information and Computing Science 09/2005 - 07/2009

Research Interests

- Acoustic/Electromagetic/Elastic wave inverse scattering problem
- Phaseless data imaging and inversion
- Waveform inversion

Reverse Time Migration for Inverse Scattering Problems

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May 1, 2015, TRIP Annual Meeting

Outline

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Direct Scattering Problem in the Half Space

We consider acoustic wave propagating in the half space with Neumann condition,

$$\Delta u + k^2 u = -\delta_{x_s}(x) \quad \text{in } \mathbb{R}^2_+ \backslash \bar{D}, \tag{1}$$

$$u = 0 \quad \text{on } \Gamma_D, \quad \frac{\partial u}{\partial x_2} = 0 \ \text{ on } \Gamma_0, \tag{2}$$

$$r^{1/2}\left(\frac{\partial u}{\partial r} - \mathbf{i} k u\right) \to 0 \ \text{ as } r = |x| \to \infty, \tag{3}$$

Here $k = \frac{\omega}{c}$ is the wavenumber. Let $N(x,y) = \Phi(x,y) + \Phi(x,y')$ be the Neumann Green function satisfying the homogeneous Neumann condition on Γ_0, and $\Phi(x,y)$ be the Green function in the free space.

Outline

Half Space Reverse Time Migration

Given the data $u^s(x_r, x_s)$ which is the measurement of the scattered field at $x_r = (x_1(x_r), x_2(x_r))^T$ when the source is emitted at $x_s = (x_1(x_s), x_2(x_s))^T$, $s = 1, \ldots, N_s$, $r = 1, \ldots, N_r$.

- Back-propagation: For $s = 1, \ldots, N_s$, compute the back-propagation field

$$v_b(z, x_s) = \frac{|\Gamma_0^d|}{N_r} \sum_{r=1}^{N_r} \frac{\partial \Phi(x_r, z)}{\partial x_2(x_r)} \overline{u^s(x_r, x_s)}, \qquad \forall\, z \in \Omega.$$

- Cross-correlation: For $z \in \Omega$, compute

$$I_d(z) = \mathrm{Im} \left\{ \frac{|\Gamma_0^d|}{N_s} \sum_{s=1}^{N_s} \frac{\partial \Phi(x_s, z)}{\partial x_2(x_s)} v_b(z, x_s) \right\}.$$

Relation with Yu Zhang's True Amplitude Imaging Condition

In Yu Zhang's paper (First Break, 2009), the forward source wavefield is changed into

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right)p_F = 0 \tag{4}$$

$$p_F(x, y, z = 0) = \delta(x - x_s)\int_{-\infty}^{t} f(t')dt'. \tag{5}$$

And the backpropagated received wavefield is given by

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right)p_B = 0 \tag{6}$$

$$p_B(x, y, z = 0; t) = Q(x, y; x_s, y_s; t). \tag{7}$$

where Q is the received data.

Relation with Yu Zhang's True Amplitude Imaging Condition

The imaging condition is the conventional cross-correlation condition,

$$I(x) = \int_{\Gamma_s} \int p_F(x,t;x_s) p_B(x,t;x_s) dt dx_s \tag{8}$$

By integral representation, the forward source wavefield can be written as in the frequency domain,

$$\hat{p}_F = \int_{\Gamma_s} \frac{\partial G(\xi,x)}{\partial \nu} \delta(\xi - x_s) \frac{1}{\mathbf{i}\omega} \hat{f}(\omega) d\xi = \frac{2}{\mathbf{i}\omega} \frac{\partial \Phi(x_s,x)}{\partial x_2(x_s)} \hat{f}(\omega)$$

Also the backproprogation field is given by

$$\hat{p}_B = \int_{\Gamma_r} \frac{\partial \overline{G(x_r,x)}}{\partial \nu} \hat{Q}(x_r,x_s) dx_r = 2 \int_{\Gamma_r} \frac{\partial \overline{\Phi(x_r,x)}}{\partial x_2(x_r)} \hat{Q}(x_r,x_s) dx_r$$

Relation with Yu Zhang's True Amplitude Imaging Condition

Recall the Parseval's identity

$$\begin{aligned}\int_{-\infty}^{+\infty} g(t)h(t)dt &= \int_{-\infty}^{+\infty} \hat{g}(\omega)\overline{\hat{h}}(\omega)d\omega \\ &= 2\mathrm{Re}\int_{0}^{+\infty} \hat{g}(\omega)\overline{\hat{h}}(\omega)d\omega \quad \text{(as } g,h \text{ are real)}\end{aligned}$$

Hence, Yu Zhang's imaging result is now given by

$$\begin{aligned}I(x) &= 2\mathrm{Re}\int_{\Gamma_s}\int \hat{p}_F(x,x_s)\overline{\hat{p}}_B(x,x_s)d\omega dx_s \\ &= 8\mathrm{Re}\int_{\Gamma_s}\int_0^{+\infty}\frac{1}{\mathrm{i}\omega}\frac{\partial\Phi(x_s,x)}{\partial x_2(x_s)}\hat{f}(\omega)\int_{\Gamma_r}\frac{\partial\Phi(x_r,x)}{\partial x_2(x_r)}\overline{\hat{Q}(x_r,x_s)}dx_r dx_s d\omega \\ &= 8\int_0^{+\infty}\frac{1}{\omega}\mathrm{Im}\left(\int_{\Gamma_s}\int_{\Gamma_r}\frac{\partial\Phi(x_s,x)}{\partial x_2(x_s)}\hat{f}(\omega)\frac{\partial\Phi(x_r,x)}{\partial x_2(x_r)}\overline{\hat{Q}(x_r,x_s)}dx_r dx_s\right)d\omega\end{aligned}$$

Single Frequency vs Time domain RTM

Our method:

$$I_d(z) = \mathrm{Im}\left\{\sum_{s=1}^{N_s}\sum_{r=1}^{N_r}\frac{\partial\Phi(x_s,z)}{\partial x_2(x_s)}\frac{\partial\Phi(x_r,z)}{\partial x_2(x_r)}\overline{u^s(x_r,x_s)}\right\}.$$

Yu Zhang's method:

$$I(x) = 8\int_0^{+\infty}\frac{1}{\omega}\mathrm{Im}\left(\int_{\Gamma_s}\int_{\Gamma_r}\frac{\partial\Phi(x_s,x)}{\partial x_2(x_s)}\hat{f}(\omega)\frac{\partial\Phi(x_r,x)}{\partial x_2(x_r)}\overline{\hat{Q}(x_r,x_s)}dx_r dx_s\right)d\omega.$$

Outline

Point Spread Function

Let the point spread function be

$$J(z,y) = \int_{\Gamma_0} \frac{\partial G(x,z)}{\partial x_2} \overline{N(x,y)} ds(x), \quad \forall z,y \in \mathbb{R}^2_+.$$

Lemma

For any $z,y \in \mathbb{R}^2_+$, $J(z,y) = F(z,y) + R(z,y)$, *where*

$$F(z,y) = -\frac{\mathbf{i}}{2\pi} \int_0^{\pi} e^{\mathbf{i}k(z_1-y_1)\cos\theta + \mathbf{i}k(z_2-y_2)\sin\theta} d\theta,$$

$$R(z,y) = \frac{1}{\pi} \int_k^{+\infty} \frac{e^{-\sqrt{\xi_1^2-k^2}(z_2+y_2)}}{\sqrt{\xi_1^2-k^2}} \cos(\xi_1(z_1-y_1)) d\xi_1.$$

Moreover, $F(y,y) = -\mathbf{i}/2$ *and* $|F(z,y)| \le C(\sqrt{k|z-y|})^{-1}$, $|R(z,y)| + k^{-1}|\nabla_y R(z,y)| \le \frac{1}{\pi k(z_2+y_2)}$ *uniformly for* $z,y \in \mathbb{R}^2_+$.

For any $z \in \Omega$, let $\psi(y,z)$ be the scattering solution to the following problem:

$$\Delta_y \psi(y,z) + k^2\psi(y,z) = 0 \quad \text{in } \mathbb{R}^2\backslash\bar{D},$$
$$\psi(y,z) = -\overline{F(z,y)} \ \text{ on } \Gamma_D.$$

Theorem

We have

$$I_d(z) = \frac{1}{4}\,\mathrm{Im}\left\{\int_{\Gamma_D} \frac{\partial(F(z,y)+\psi(y,z))}{\partial\nu(y)}\overline{F(z,y)}ds(y)\right\} + W_{I_d}(z),$$

where $|W_{I_d}(z)| \le C(1+kd_D)^4((kh)^{-1/2} + h/d)$ *uniformly for* z *in* Ω.

Physical Interpretation

Definition

For any unit vector $\eta \in \mathbb{R}^2$, let $v^i = e^{\mathbf{i}kx\cdot\eta}$ be the incident wave and $v^s = v^s(x,\eta)$ be the radiation solution of the Helmholtz equation:

$$\Delta v^s + k^2 v^s = 0 \ \text{ in } \mathbb{R}^2\backslash\bar{D}, \quad v^s = -e^{\mathbf{i}kx\cdot\eta} \ \text{ on } \Gamma_D.$$

The scattering coefficient $R(x,\eta)$ for $x \in \Gamma_D$ is defined by the relation

$$\frac{\partial(v^s + v^i)}{\partial\nu} = \mathbf{i}kR(x,\eta)e^{\mathbf{i}kx\cdot\eta} \quad \text{on } \Gamma_D.$$

Note that there are some differences between scattering coefficient and reflection coefficient.

Physical Interpretation

Now we consider the physical interpretation of the imaging function $\hat{I}_d(z)$ when $z \in \Gamma_D$. Since

$$\overline{F(z,y)} = \frac{\mathbf{i}}{2\pi}\int_0^\pi e^{\mathbf{i}k(y-z)\cdot\eta_\theta}d\theta, \quad \eta_\theta := (\cos\theta, \sin\theta)^T,$$

We obtain from the previous theorem and the definition of scattering coefficient that

$$\begin{aligned} I_d(z) &= -\frac{k}{8\pi}\mathrm{Im}\int_{\Gamma_D}\int_0^\pi \overline{F(z,y)}R(y,\eta_\theta)e^{\mathbf{i}k(y-z)\cdot\eta_\theta}d\theta ds(y) \\ &\quad +O\left(\frac{1}{\sqrt{kh}}+\frac{h}{d}\right). \end{aligned}$$

Physical Interpretation

For the strictly convex D, the scattering coefficient can be approximated by

$$R(x,\eta)=\begin{cases}2\nu(x)\cdot\eta & \text{If } x\in\partial D_\eta^- := \{x\in\Gamma_D:\nu(x)\cdot\eta<0\},\\ 0 & \text{If } x\in\partial D_\eta^+ := \{x\in\Gamma_D:\nu(x)\cdot\eta>0\}.\end{cases}$$

Hence we have

$$I_d(z)\approx\left(\frac{k}{8\pi}\right)^{1/2}\mathrm{Im}\int_0^\pi\frac{F(z,y_-(\eta_\theta))}{\sqrt{\kappa(y_-(\eta_\theta))}}e^{\mathbf{i}k(y_-(\eta_\theta)-z)\cdot\eta_\theta-\mathbf{i}\frac{\pi}{4}}d\theta.$$

Outline

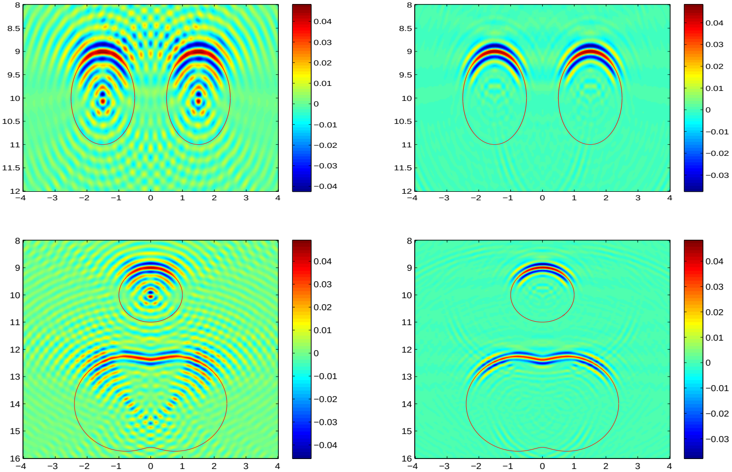

Figure: Left: $k = 4\pi$; Right: nine wavenumbers from $k = 4\pi$ to $k = 6\pi$

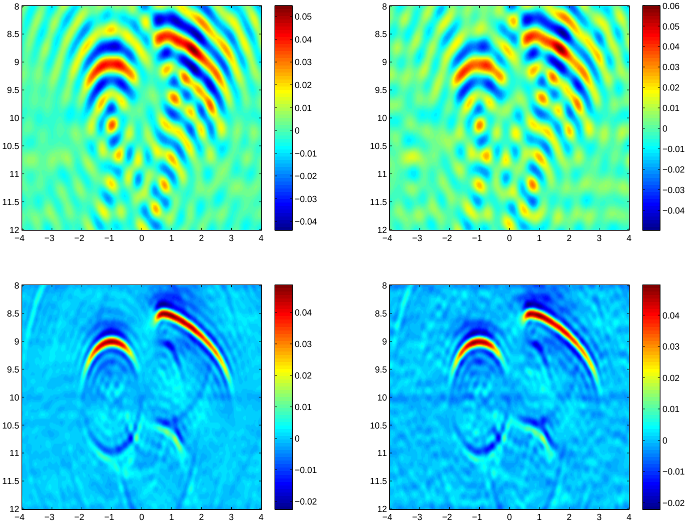

Figure: Noise level 10% (left) and 20% (right); Top row: $k = 2\pi$; Bottom row: nine wavenumbers from $k = \pi$ to $k = 5\pi$

Related Works

- J. Chen, Z. Chen and G. Huang. Reverse Time Migration for Extended Obstacles: Acoustic Waves. Inverse Problem, 29 (2013) 085005 (17pp);
- Z. Chen and G. Huang. Reverse Time Migration for Reconstructing Extended Obstacles in the Half Space. Inverse Problem, 31 (2015) 055007 (19pp);
- Z. Chen and G. Huang. Reverse Time Migration for Reconstructing Extended Obstacles in Planar Acoustic Waveguide. to apppear in Sci. China Math.;
- J. Chen, Z. Chen and G. Huang. Reverse Time Migration for Extended Obstacles: Electromagnetic Waves. Inverse Problem, 29 (2013) 085006 (17pp);
- Z. Chen and G. Huang. Reverse Time Migration for Extended Obstacles: Elastic Waves (in Chinese). Sci. Sin. Math. 2014.

Thanks

- Total E&P USA, for funding me
- The Rice Inversion Project, for supporting me
- TRIP members, for helping me
- All of you, for listening to me