Introduction

An accurate estimation of velocity model is indispensable for obtaining an accurate image of earth's reflectivity by prestack depth migration, but velocity estimation is always a challenging task in exploration seismology. Wave-equation Migration Velocity Analysis (WEMVA) (Sava, 2004; Shen, 2004) uses band-limited wavefields as carriers of information instead of infinite-frequency rays, so it has the potential to accurately and robustly estimate subsurface velocities even in complex geological settings. It's often accomplished via solving a non-linear inverse problem on a quantitative semblance criterion defined in terms of the common image gather, or extended image space. Recently, It has been applied successfully in a number of real data cases where ray-based velocity analysis methods might fail (Sava and Biondi, 2004; Shen and Symes, 2008).

To estimate velocities, WEMVA extracts unfocused energy or unflattened components in the common image gather and projects these information into velocity space to get the velocity information. Among different variants of WEMVA, Differential semblance (DS) based method is appealing because it does not require manual picking and the objective has been shown to be convex theoretically and numerically (Stolk and Symes, 2003). While a number of successful applications of DS-based MVA have been reported (Mulder and Ten Kroode, 2002; Albertin et al., 2006; Shen and Symes, 2008; Fei et al., 2009), it seems to suffer from some drawbacks. Firstly, DSO generates unwanted gradient artifacts when there are reflector truncations and other singularities, which might slows down the convergent rate. The second problem related with general WEMVA methods, not just DSO, is uneven illumination caused by limited acquisition geometry and complex overlying bed such as high-velocity salt. It might create defocussing in common image gathers even when the velocity model is correct. The third problem is specific to two-way wave-equation based velocity analysis method. The scattering wave produces undesired artifacts in the RTM migrated image, which is also sensitive to velocity error, thus affecting the results of the optimization problem.

In this paper, we develop a new velocity analysis method, namely inversion velocity analysis (IVA), to handle these three problems simultaneously. Recalling that the original formulation of differential semblance operator is designed to the inverted extended reflectivity, not to the migrated image volume, we adopt the conventional two-stage scheme in migration velocity analysis but replace the migration step with linearized inversion. With theoretical and numerical analysis, we show that all of the previous difficulties diminish, or sometimes even disappear in our proposed method. We also point out that our proposed IVA method may be extended, at least conceptually, to nonlinear case, that is full waveform inversion.

Theory

With linearized Born approximation and model extension, extended waveform inversion can be rearranged as solving the following inverse problem (Symes, 2008),

$$min_{m_0,\delta\bar{m}} J_{IVA}[m_0,\delta\bar{m}] = \| D\bar{F}[m_0]\delta\bar{m} - F_d \|^2 + \sigma \| A\delta\bar{m} \|^2 \quad (1)$$

where m_0 and $\delta\bar{m}$ is the background model and extended reflectivity (model perturbation) respectively. $D\bar{F}$ is the first derivative of full-wave modeling operator with respect to the extended model (which contains m_0 and $\delta\bar{m}$). $F_d = d - F[m_0]$ is the observed seismic reflection data after muting the direct and diving wave. A is the regularization operator, which is specific to differential semblance operator in this abstract, other choice of A is also possible. σ is the penalty parameter, when $\sigma \to 0$, equation 1 limits to the problem of migration velocity analysis (MVA), when $\sigma \to \infty$, Equation 1 limits to the problem of least-squares migration (LSM).

Liu et al. (2013) have proposed linearized extended waveform inversion (LEWI) method to solve the above problem theoretically and numerically. In this abstract, we also adapt the two-stage scheme but make some modifications to make LEWI method more practical, that is, the first stage minimizes the first term with respect to $\delta\bar{m}$ while the second stage minimizes the second term with respect to m_0. In most case, WEMVA method approximates the solution of the first stage by migration.

The first inverse problem is quadratic and the corresponding gradient can be derived easily as follows,

$$\nabla_{\delta\bar{m}} J_{IVA}[m_0, \delta\bar{m}] = D\bar{F}^T[m_0](D\bar{F}[m_0]\delta\bar{m} - F_d) \tag{2}$$

Set the gradient to zero gives the normal equation, i.e.

$$D\bar{F}^T[m_0]D\bar{F}[m_0]\delta\bar{m} = D\bar{F}^T[m_0]F_d \tag{3}$$

which can be re-written as:

$$N[m_0]\delta\bar{m} = M[m_0]F_d \tag{4}$$

where $N[m_0]$ is normal (or Hessian) operator and $M[m_0]$ is migration operator. Any iterative scheme for $\delta\bar{m}$ of equation 4 gives the following approximation,

$$\delta\bar{m} = P(N[m_0])M[m_0]F_d \tag{5}$$

where $P(N[m_0])$ is a polynomial of normal operator $N[m_0]$, it can be approximated to the inverse of normal operator. Equation 5 is also the explicit solution of Least-Squares Extended Reverse-Time Migration (LSERTM). If we ignore $P(N[m_0])$, it reduces to Extended Reverse-Time Migration (ERTM).

Take the derivative of $\delta\bar{m}$ with respect to m_0, we can get:

$$D_{m_0}\delta\bar{m}dm_0 = D_N P(N[m_0])DN[m_0]dm_0 M[m_0]F_d + P(N[m_0])D(M[m_0]F_d)dm_0 \tag{6}$$

It has been demonstrated that, in the case of depth-oriented model extension, when the background model is changed, the migration operator will dramatically change the phase of input image while the normal operator will not (Stolk et al., 2009), so $DN[m_0]dm_0$ is relatively small compared with $D(M[m_0]F_d)dm_0$ and we can drop the first term of equation 6, that is,

$$D_{m_0}\delta\bar{m}dm_0 = P(N[m_0])D(M[m_0]F_d)dm_0 = P(N[m_0])D(DF^T[m_0]F_d)dm_0 \tag{7}$$

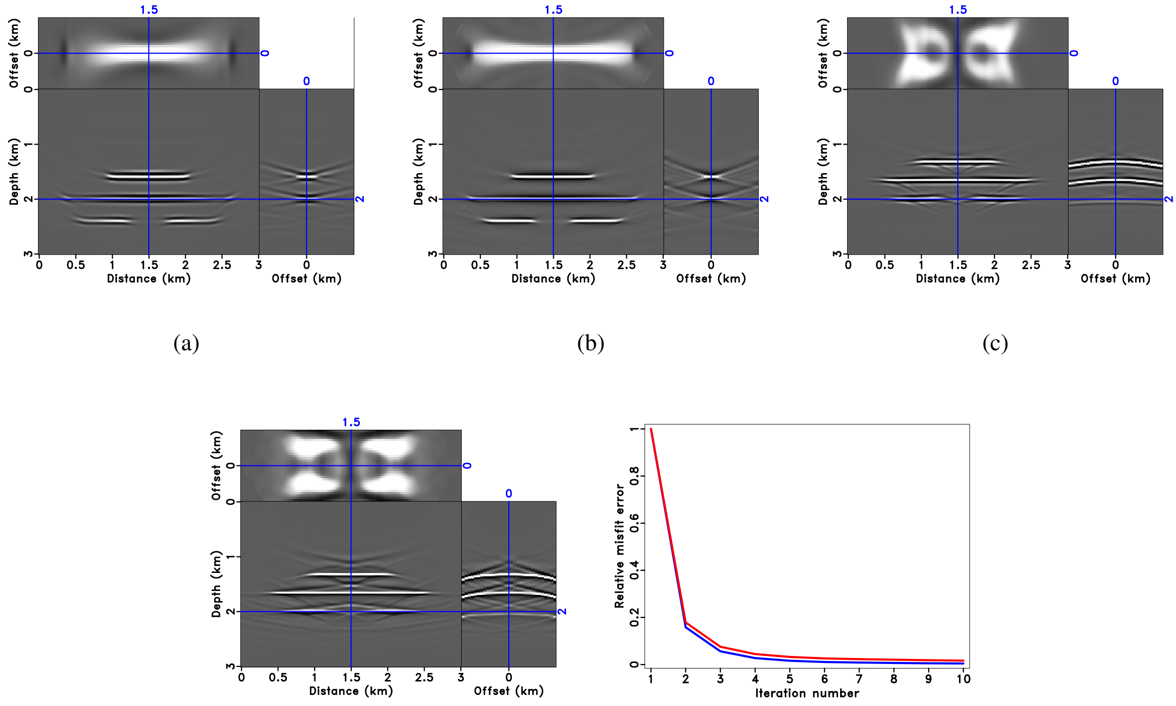
Then, the gradient with respect to long scales can be derived as follows,

$$\nabla_{m_0} J_{IVA}[m_0, \delta\bar{m}[m_0]] = A^T A D_{m_0}\delta\bar{m}dm_0 = B[P(N[m_0])A^T A\delta\bar{m}_k, F_d] \tag{8}$$

Where B is bilinear (or WEMVA) operator, which is the adjoint of second derivative of F with respect to m_0, $\delta\bar{m}_k$ is the inverted reflectivity after k iterations at the first stage. If the first input of equation 8 is migration result, it changes to MVA gradient. The derivation of WEMVA operator based on reverse-time migration can be found in the papers (Shen, 2012; Weibull and Arntsen, 2013).

Numerical tests

A simple model is used to demonstrate the advantages of IVA over MVA. The reflectivity model contains three layers with the bottom layer truncated in the middle part. The background velocity is constant ($v = 3km/s$). Figure 1 (a) and figure 1 (b) shows the extended image using RTM and LSERTM when the velocity is correct, we can see that both of them have most of the energy focused at zero subsurface offset. Inversion doesn't help focus the energy at zero offset but promotes the image resolution and compensate the uneven illumination. Figure 1 (c) and figure 1 (d) shows the extended image using RTM and LSERTM when the velocity is wrong ($v_0 = 2.5km/s$), we can see that the reflector is not at the correct position and the energy in the subsurface offset gather is unfocused. LSERTM doesn't correct the position of reflectors but it also promotes the image resolution and retrieves the amplitude information. Figure 1 shows the relative misfit error curve of LSERTM with correct velocity and wrong velocity, we can both of them decreases dramatically after several iterations, which means extended modeling



(a) (b) (c)

(d) (e)

Figure 1 *Image of extended reflectivity (a) ERTM with correct velocity; (b) LSERTM with correct velocity after 10 iterations; (c) ERTM with wrong velocity; (d) LSERTM with wrong velocity after 10 iterations; (e) Relative misfit error of LSERTM with correct (blue line) and wrong (red line) velocity.*

operator is a surjective mapping process so LSERTM will be convergent to global minimum no matter the background velocity is correct or wrong. Then we apply WEMVA operator to the LSRTM result with different iterations. Figure 2 show the gradient using extended image after one iteration (which is equivalent to RTM), 3 iterations and 10 iterations respectively. From these figures, we can see that the artifacts well-known in DSO method has been suppressed gradually along with increased iterations.

Conclusions and Discussions

In this abstract, we propose a new velocity estimation method, namely inversion velocity analysis, to tackle the problems of WEMVA based on RTM. From the numerical tests, we can see that IVA can successfully suppress the artifacts in DS-MVA. Other advantages of IVA gained from LSERTM, such as illumination compensation and low-frequency noise suppression, is obvious and well demonstrated in in other publications about LSM. The efficiency of IVA method can be improved by the strategies already developed in LSM, such as phase encoding, preconditioning and so on.

We adopt subsurface horizontal offset model extension in our example. In fact, the theory of IVA is general and can be extended to other model extension, such as shot index, surface offset, scattering angle and so on. It's also possible to extend this idea into non-linear case, which is still on the research.

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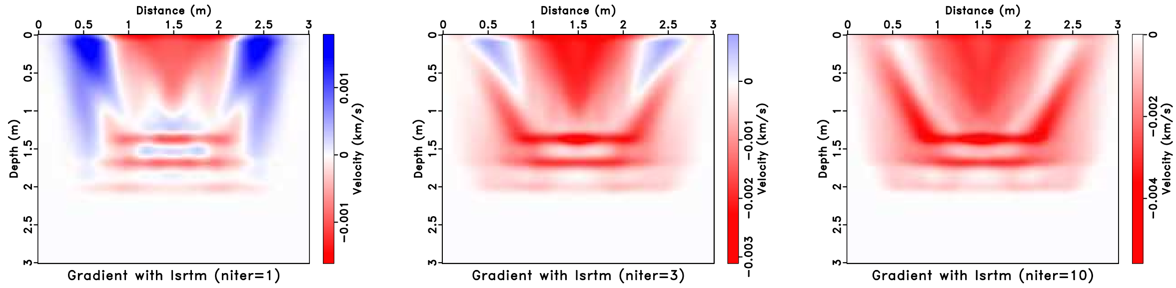

(a) (b) (c)

Figure 2 *Background velocity gradient using extended reflectivity image by linearized inversion after different iteration number (a) iteration 1; (b) iteration 3; (c) iteration 10.*

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