

# Lei Fu

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- EDUCATION**
- Pursuing a *PhD.* degree in Geophysics, Rice University 2011-present
- Master of Science* in Geophysics, University of Utah 2009-2011  
Concentration: Forward modeling and inversion of electromagnetic problems
- Bachelor of Science*, University of Science and Technology of China 2005-2009  
Major: Geophysics
- INTERESTS**
- Full waveform inversion, mathematical and computational methods to study seismic theory and data processing, and geology
- SKILLS**
- Programming languages: Unix, C, Matlab, FORTRAN  
Software: ProMAX, SeisWorks 3D, Groundwater Modeling System (GMS)
- RESEARCH**
- 2011 Study of the induced polarization effect in time domain using Cole-Cole and GEMTIP models (M.S. thesis)
- 2010 Computer Simulation of the Marine Horizontal Loop Transmitter CSEM and MT Surveys over the Oil Deposit
- 2009 3-D marine controlled-source electromagnetic (MCSEM) IP Inversion
- 2009 Undergraduate thesis “Fast Marching Method in Polar coordinates”
- 2008 Theoretical Calculation of the Earth’s Toroidal Free Oscillations, Institute of Geodesy and Geophysics, Chinese Academy of Sciences (IGGCAS).

# Induced polarization effect in time domain: theory, modeling and applications

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CEMI, University of Utah

# Outline

1. Introduction
2. Induced polarization effect
3. Cole-Cole model and GEMTIP model
4. Inversion of the time-domain GEMTIP model
  - 4.1 Two-phase model
  - 4.2 Two-phase model with 2% random noise
  - 4.3 Three-phase model
5. Experimental analysis of rock samples
6. Conclusion

# Introduction

# 1. Introduction

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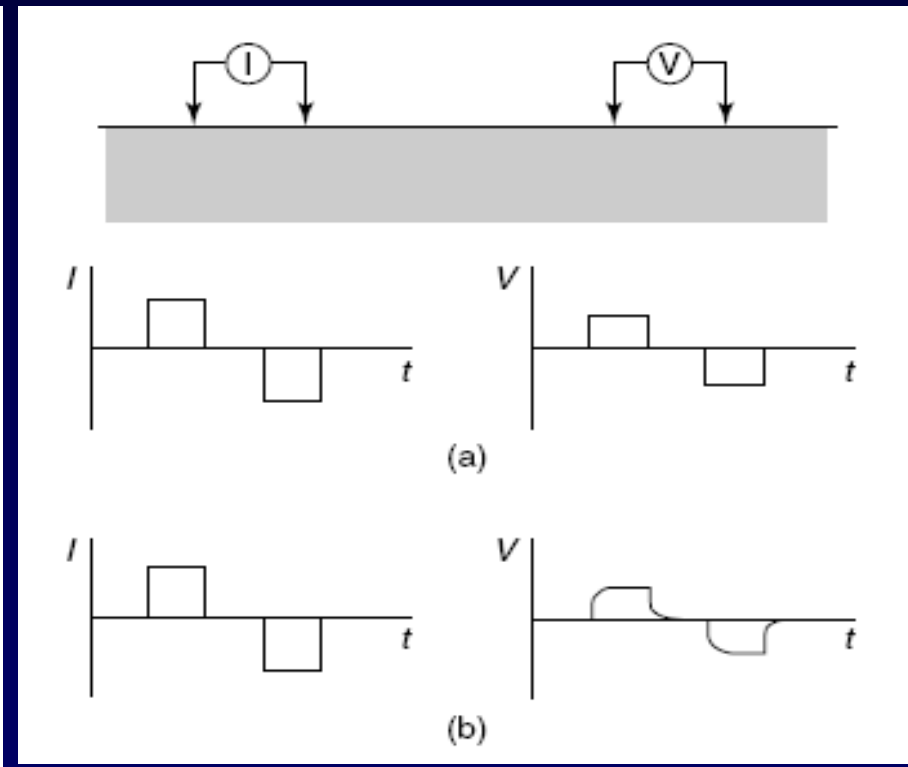
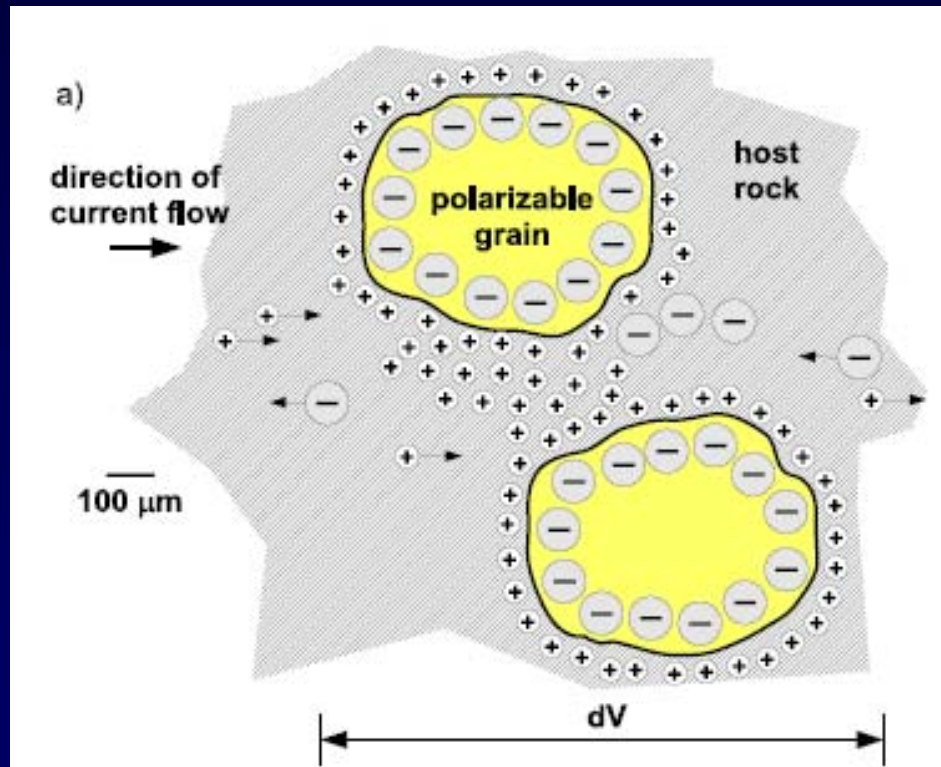
Induced polarization (IP) phenomenon has long been observed in electromagnetic data.

Pelton et al. (1978) empirically introduced the Cole-Cole relaxation model, which has provided a useful framework for the interpretation of EM and IP data over the past 30 years.

Zhdanov (2008) introduced the Generalized Effective Medium Theory for Induced Polarization (GEMTIP).

# Induced polarization effect

IP is caused by complex electrochemical reactions that accompany current flow (Frasier, 1964):



1) Surface polarization of disseminated minerals in a host rock

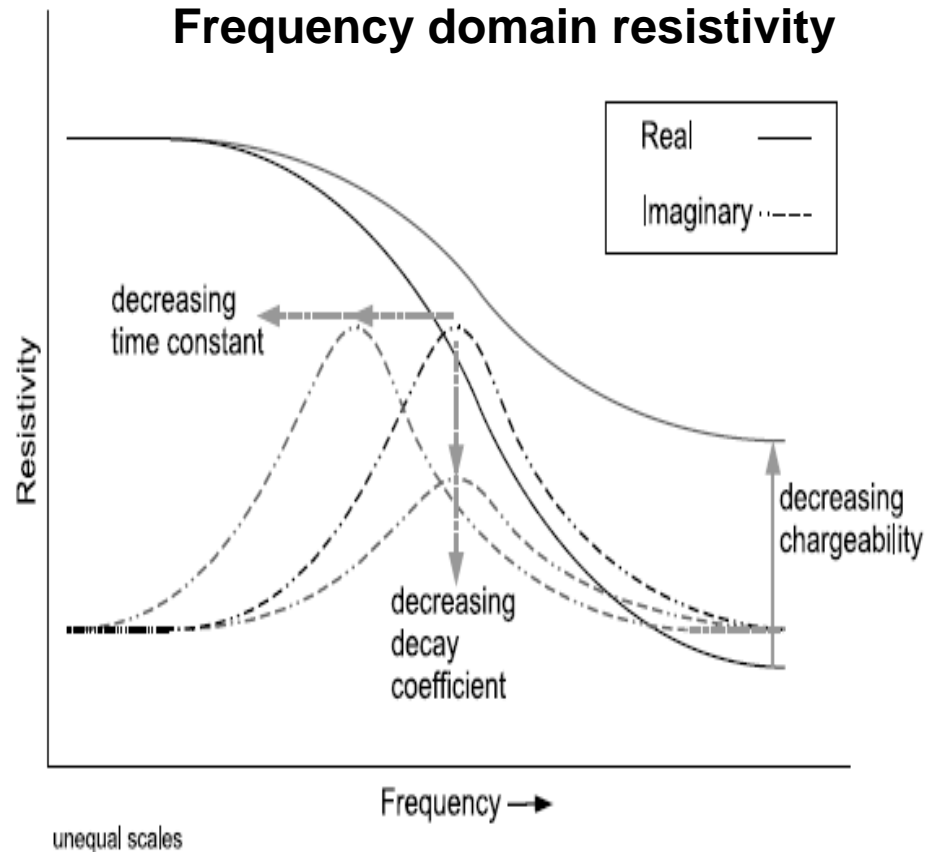
2) Sketch of the potential waveform for a current injected into nonpolarizable and polarizable ground.

# Cole-Cole Model

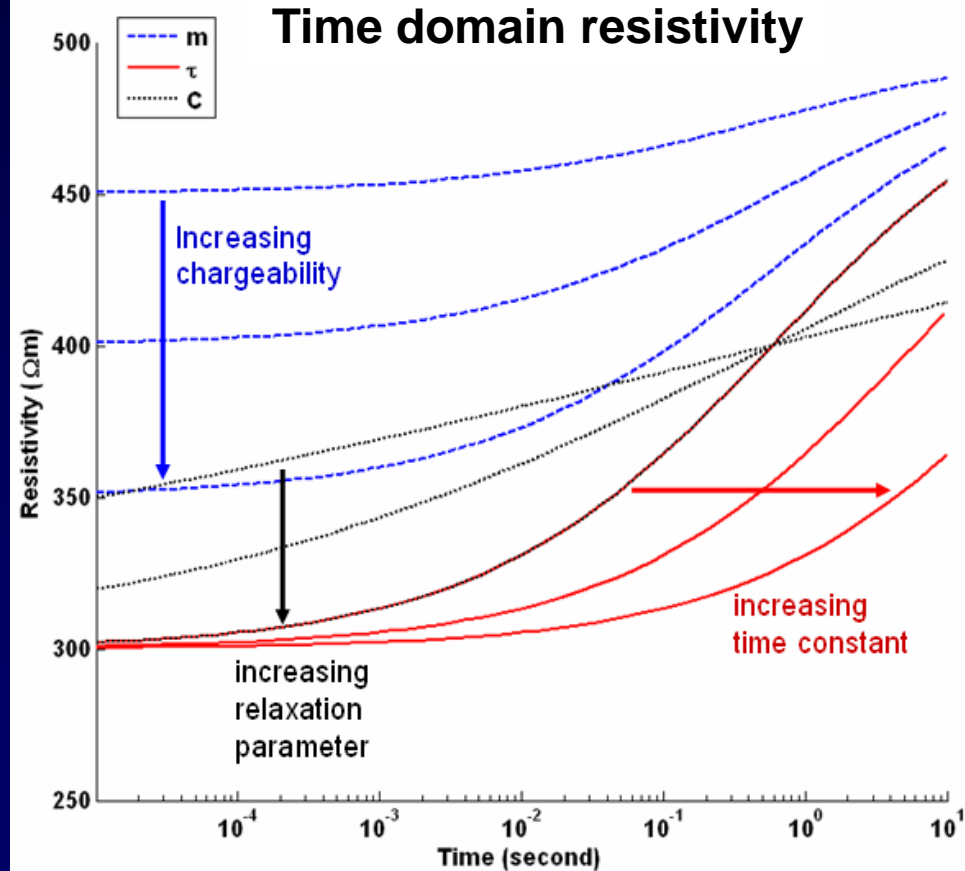
$$\rho(\omega) = \rho_{DC} \left( 1 - m \left( 1 - \frac{1}{1 + (-i\omega\tau)^c} \right) \right)$$

## Time domain resistivity

### Frequency domain resistivity



### Time domain resistivity



(Emond, 2007)

# GEMTIP model

The initial goal is to construct the realistic electrical models of the rocks based on the effective medium theory of the multi-phase composite media.



**Name: quartz monzonite porphyry (QMP)**

**Minerals: quartz, feldspar, pyrite,  
chalcopyrite, (biotite)**

**5% chalcopyrite**



# Two-phase time-domain GEMTIP model

Two-phase frequency domain spherical GEMTIP model:

$$\rho_e(\omega) = \rho_0 \left( 1 + f_1 m_1 \left( 1 - \frac{1}{1 + (i\omega\tau_1)^{c_1}} \right) \right)^{-1}$$
$$m_1 = 3 \frac{\rho_0 - \rho_1}{2\rho_1 + \rho_0}, \quad \tau_1 = \left[ \frac{a_1}{\alpha_1} (2\rho_1 + \rho_0) \right]^{1/c_1}$$

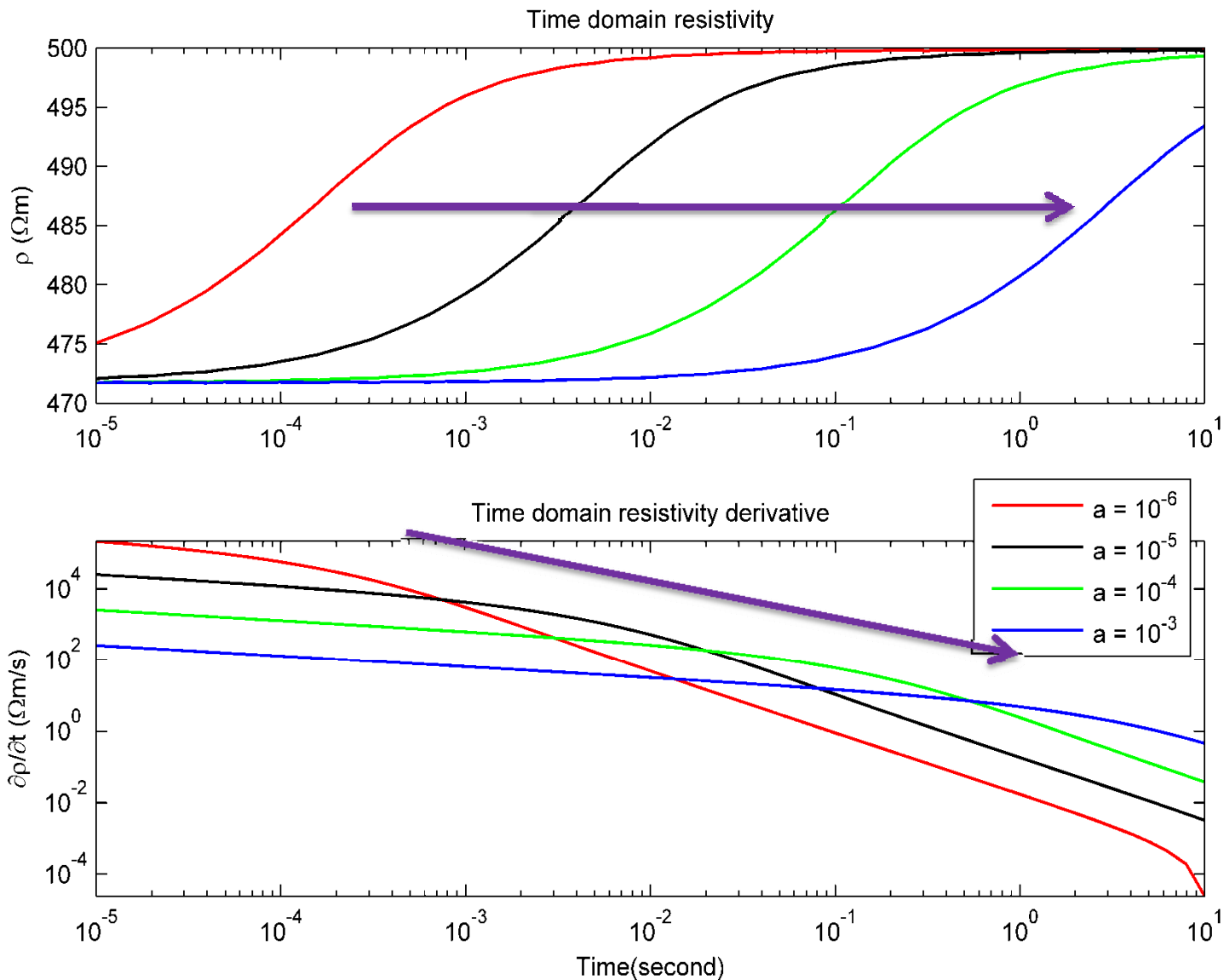
In order to obtain a time domain resistivity curve, one should apply the inverse Fourier transform:

$$\rho_e(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_e(\omega) e^{i\omega t} d\omega$$

# Effects of grain radius (a)

Numerical solution for two-phase heterogeneous model:

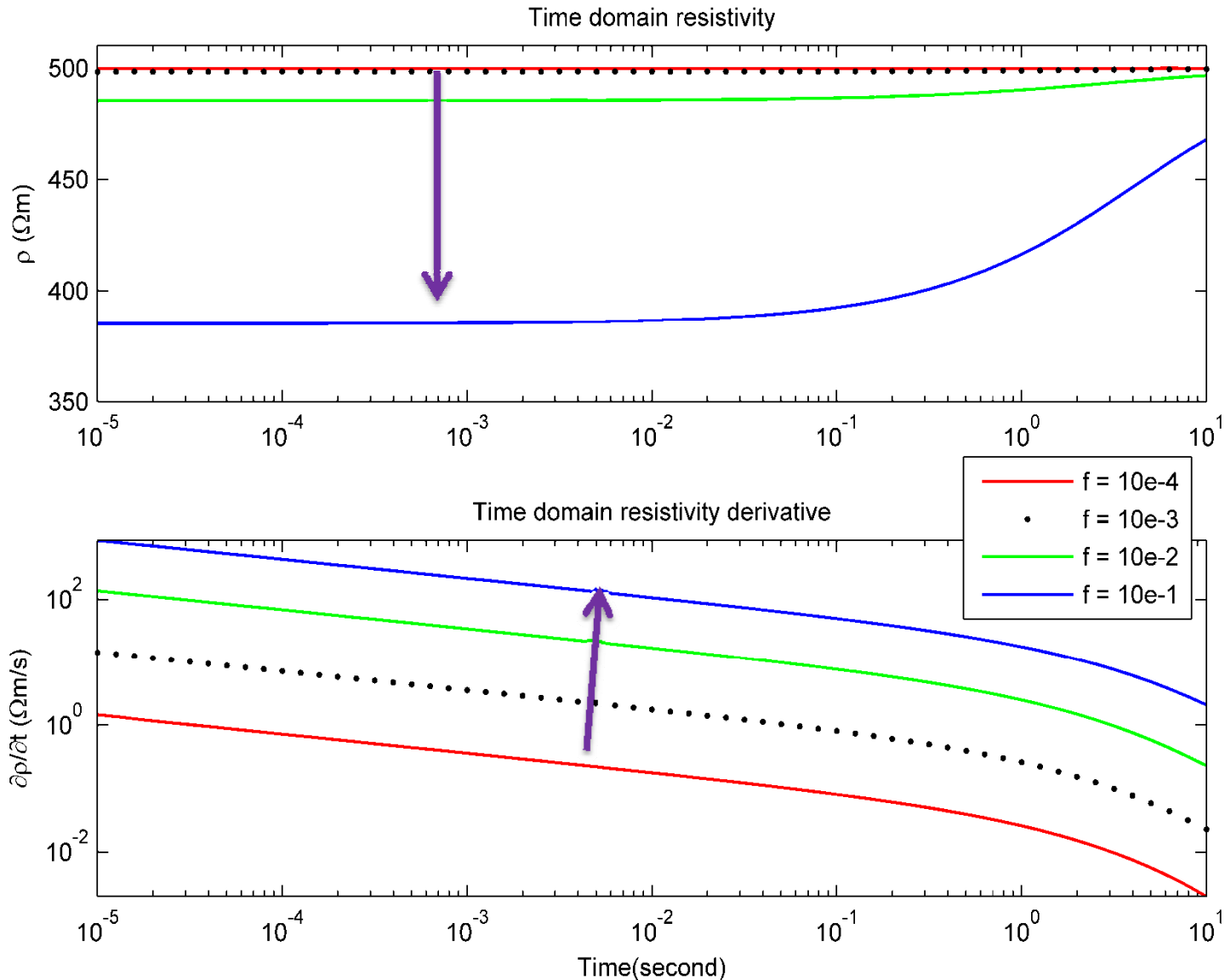
Variable	Unit	Value
$\rho_m$	$\Omega m$	500
f	-	0.02
C	-	0.7
$\rho_1$	$\Omega m$	1
$\alpha$	$\frac{\Omega \cdot m^2}{sec^{cl}}$	0.1



# Effects of volume fraction (f)

Numerical solution for two-phase heterogeneous model:

Variable	Unit	Value
$\rho_m$	$\Omega m$	500
a	mm	0.1
C	-	0.7
$\rho_1$	$\Omega m$	1
$\alpha$	$\frac{\Omega \cdot m^2}{sec^{cl}}$	0.1



# Inversion of the time-domain GEMTIP Model

# Inversion of the time-domain GEMTIP model

$$\vec{d} = A(\vec{m}) \longrightarrow A^{-1}(\vec{d}) = \vec{m}$$

where  $\mathbf{d}$  is observed data;  $\mathbf{m}$  is model parameter;  $A$  is a forward modeling operator.

Tikhonov parametric functional

$$P(\vec{m}) = \phi(\vec{m}) + \beta s(\vec{m}) \Rightarrow \min$$

where

$$\phi(\vec{m}) = \| A\vec{m} - \vec{d} \|^2 \quad \text{is a misfit functional}$$

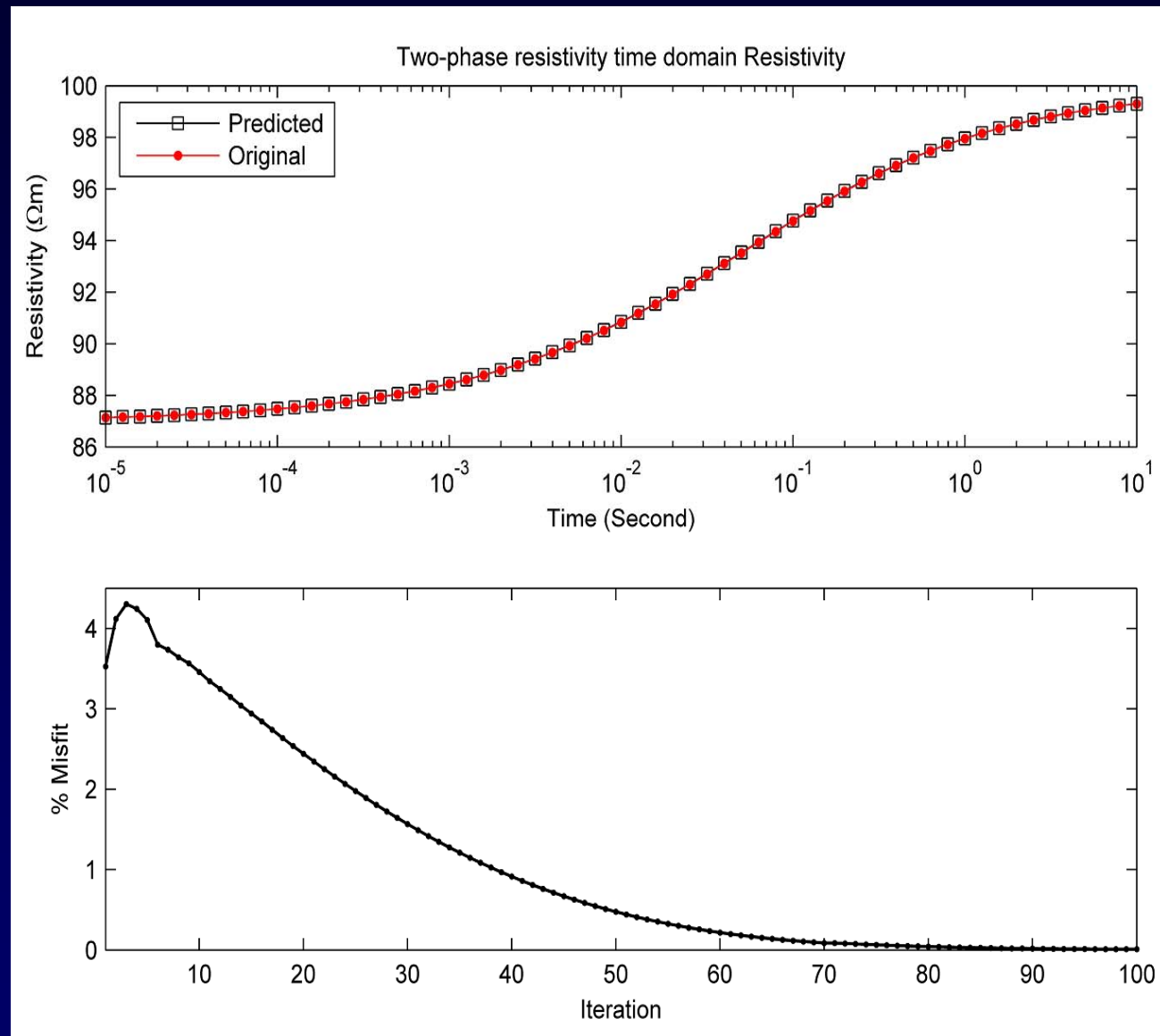
$s(\vec{m})$  is a stabilizing functional

$\beta$  is a regularization parameter

Regularized conjugate gradient (RCG) method

# 4.1 Inversion for two-phase GEMTIP model

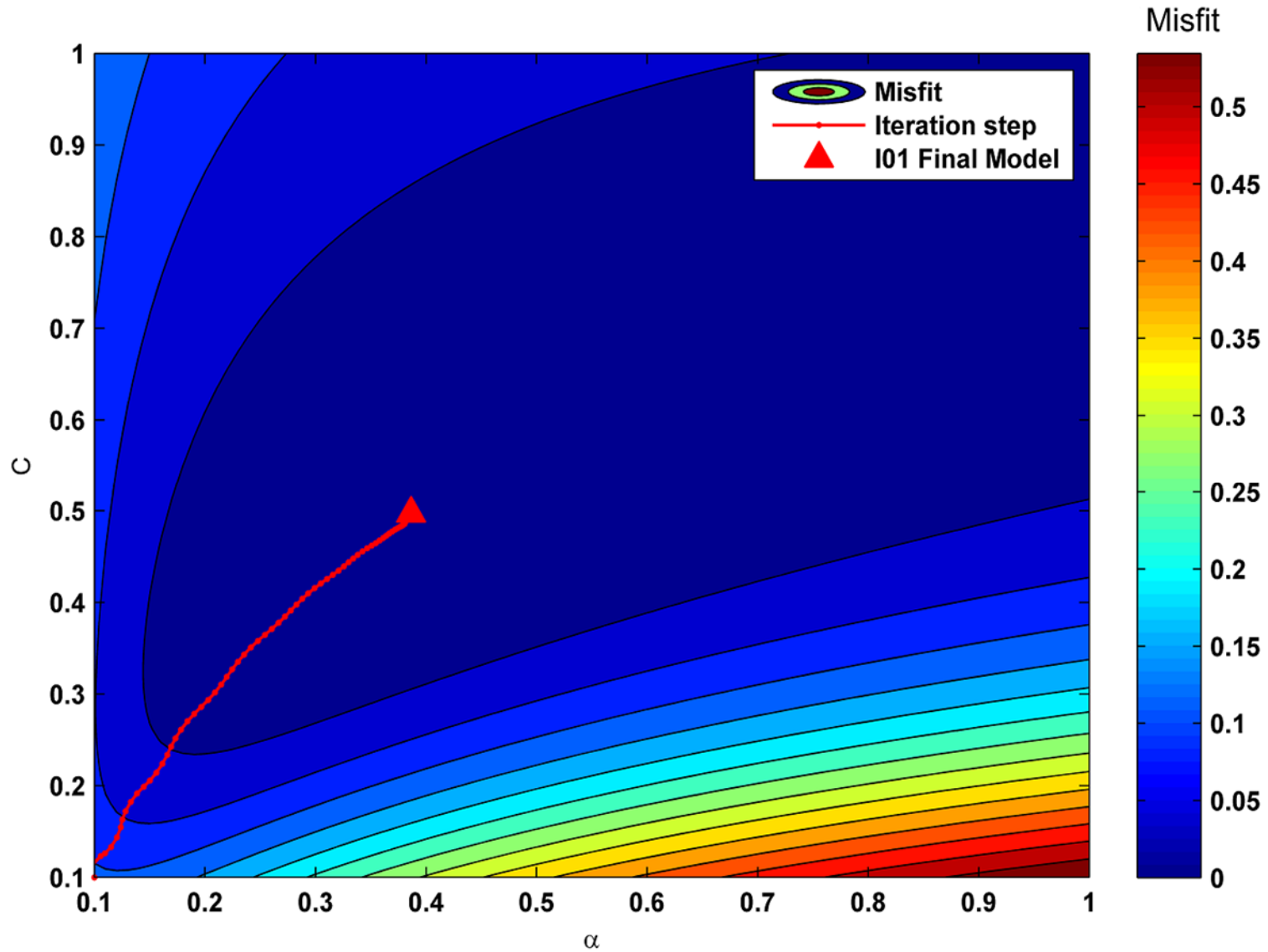
Variable	Units	True model	Initial	Recovered
$\rho_m$	$\Omega m$	100	-	-
f	-	0.05	-	-
C	-	0.5	0.1	0.50
$\rho_1$	$\Omega m$	0.1	-	-
a	mm	2	-	-
$\alpha$	$\frac{\Omega \cdot m^2}{sec^{ct}}$	0.4	0.1	0.40



Iteration number: 100

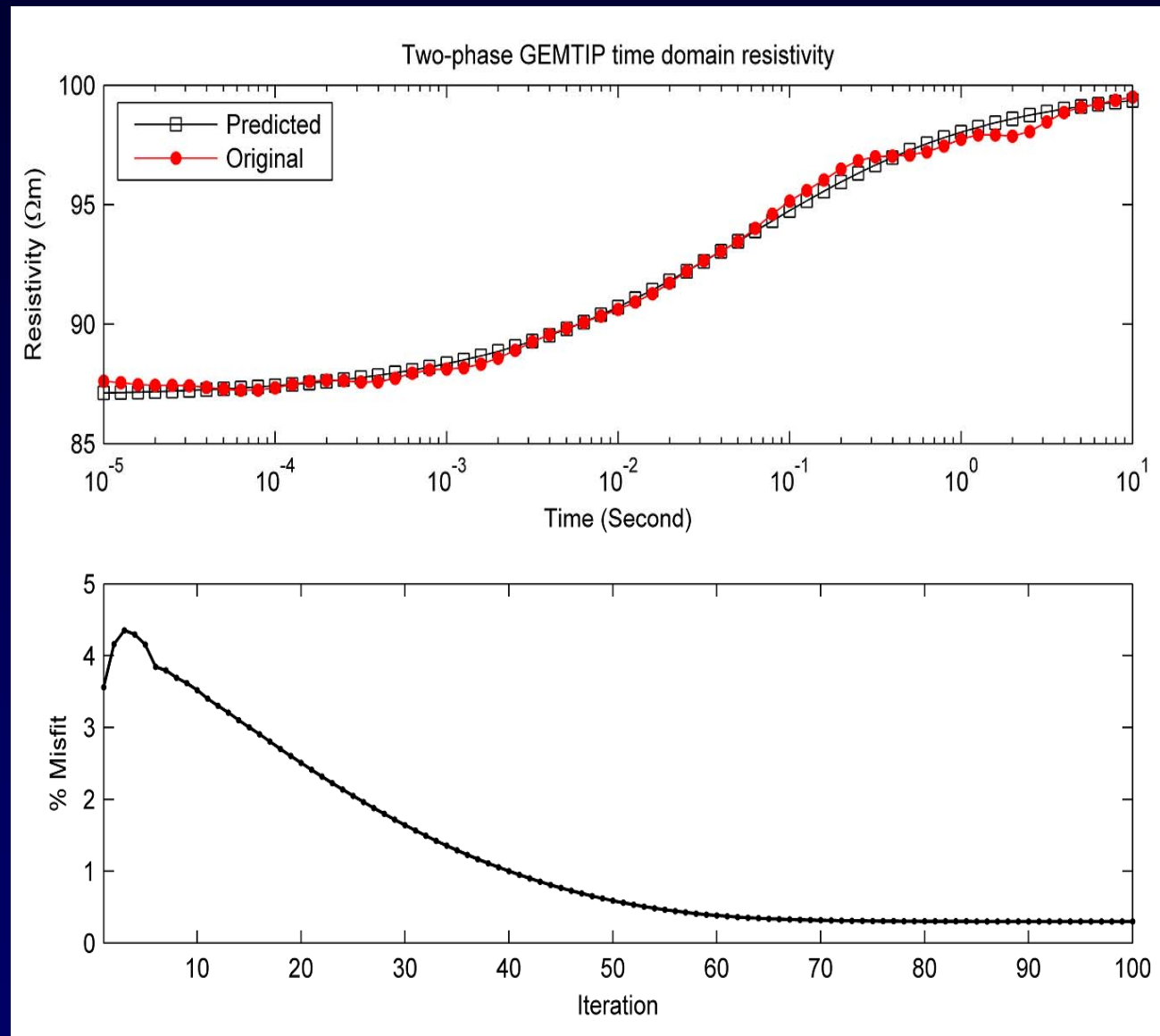
Misfit: 0.1%

# Misfit functional map



# 4.2 GEMTIP (2% random noise)

Variable	Units	True model	Initial	Recovered
$\rho_m$	$\Omega m$	100	-	-
f	-	0.05	-	-
C	-	0.5	0.1	0.52
$\rho_1$	$\Omega m$	0.1	-	-
a	mm	2	-	-
$\alpha$	$\frac{\Omega \cdot m^2}{sec^{ct}}$	0.4	0.1	0.41

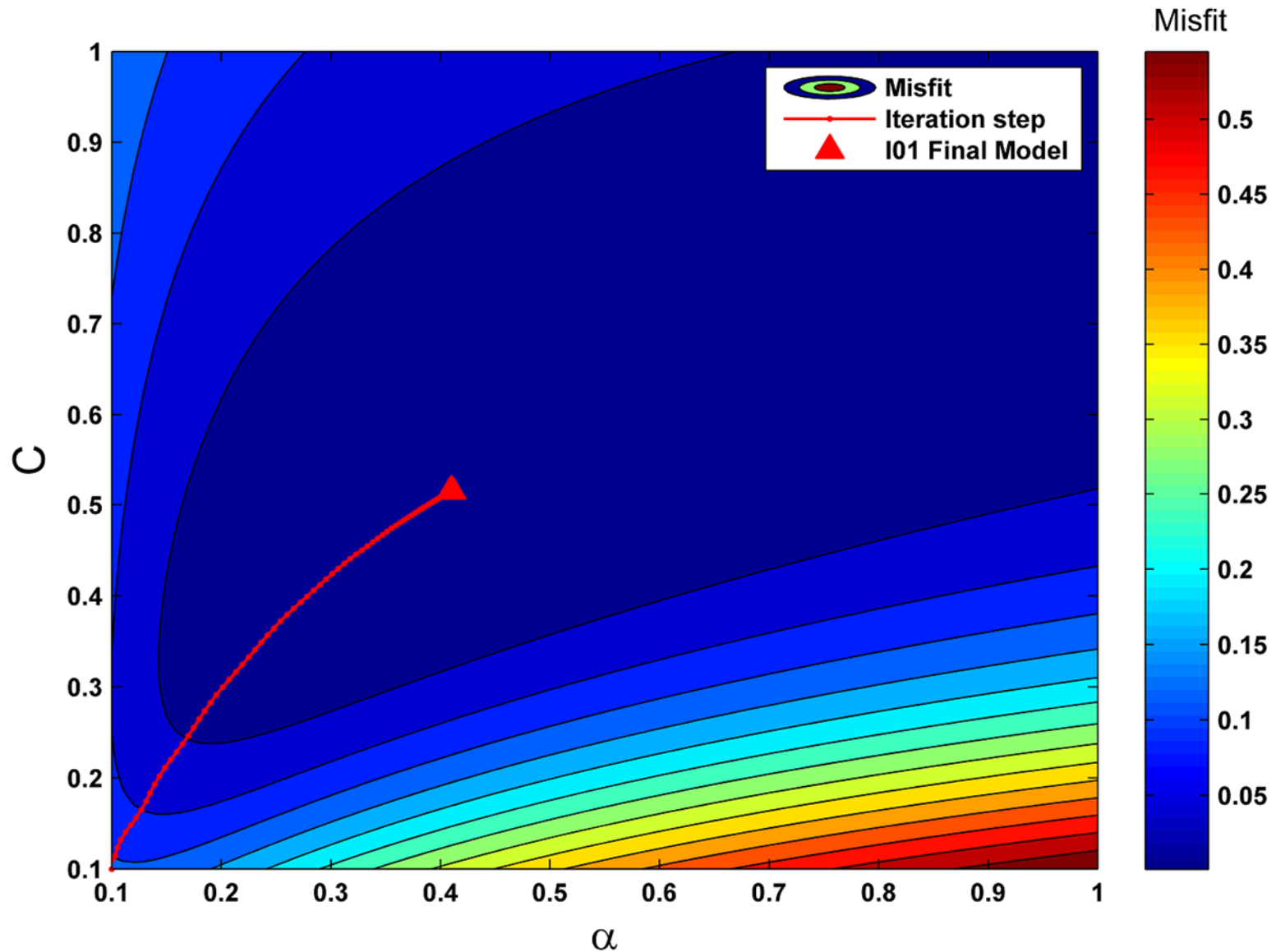


Iteration number: 100

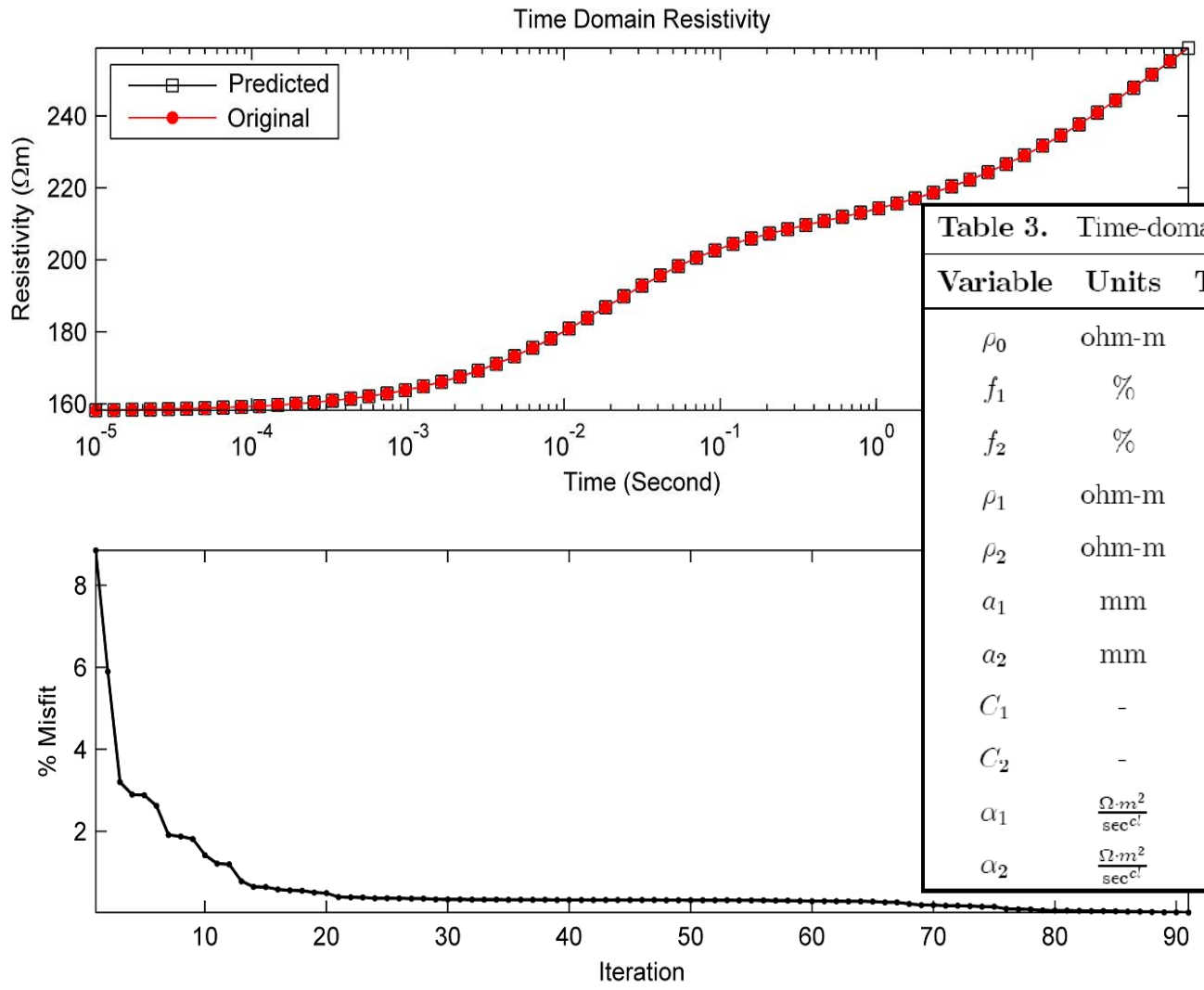
Misfit: 0.3%



# Misfit functional map (2% noise)



# 4.3 Inversion of three-phase GEMTIP model



Iteration: 91

Misfit: 0.02%

Table 3. Time-domain GEMTIP model 3 inversion parameters

Variable	Units	True model	Initial	Recovered
$\rho_0$	ohm-m	300	-	-
$f_1$	%	15	-	-
$f_2$	%	15	-	-
$\rho_1$	ohm-m	0.2	-	-
$\rho_2$	ohm-m	0.004	-	-
$a_1$	mm	0.0002	-	-
$a_2$	mm	0.0008	-	-
$C_1$	-	0.8	1	0.80
$C_2$	-	0.6	1	0.60
$\alpha_1$	$\frac{\Omega \cdot m^2}{sec^{cl}}$	2	3	1.99
$\alpha_2$	$\frac{\Omega \cdot m^2}{sec^{cl}}$	0.04	0.2	0.04

# Frequency-domain inversion of three-phase GEMTIP model

Iteration: 114

Misfit: 1%

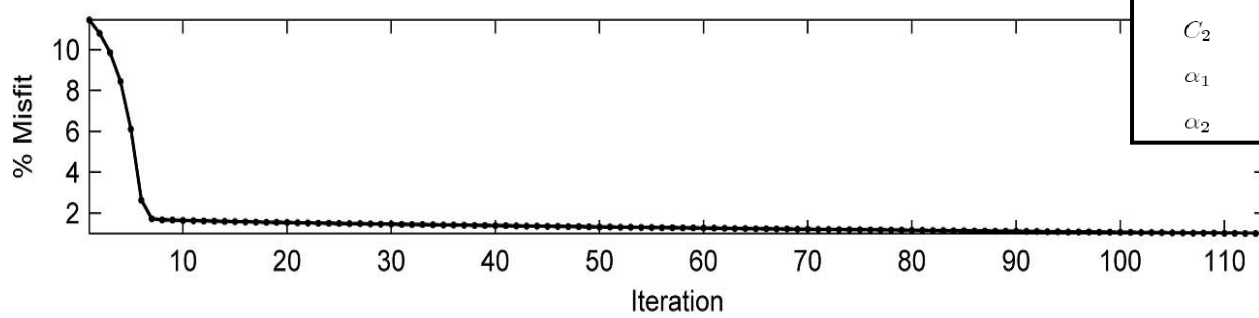
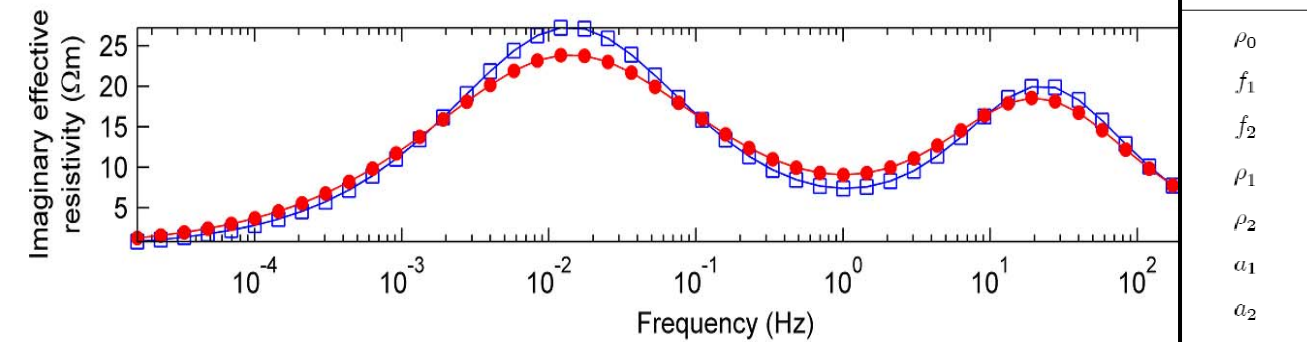
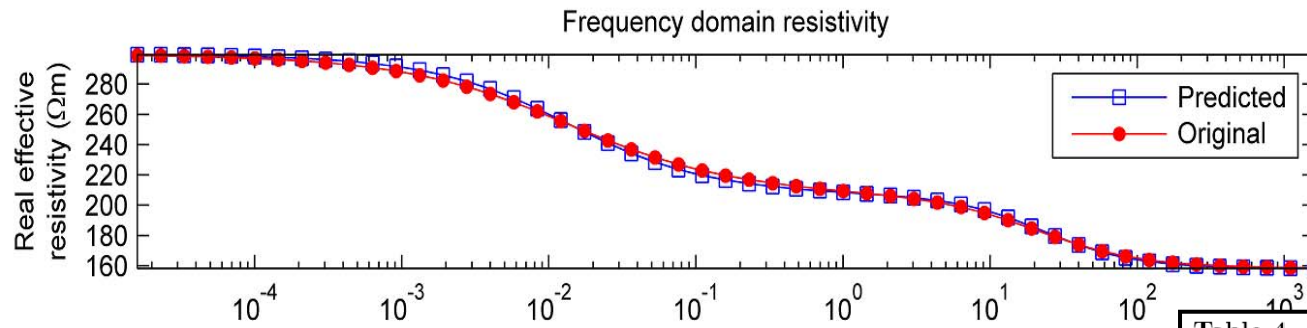


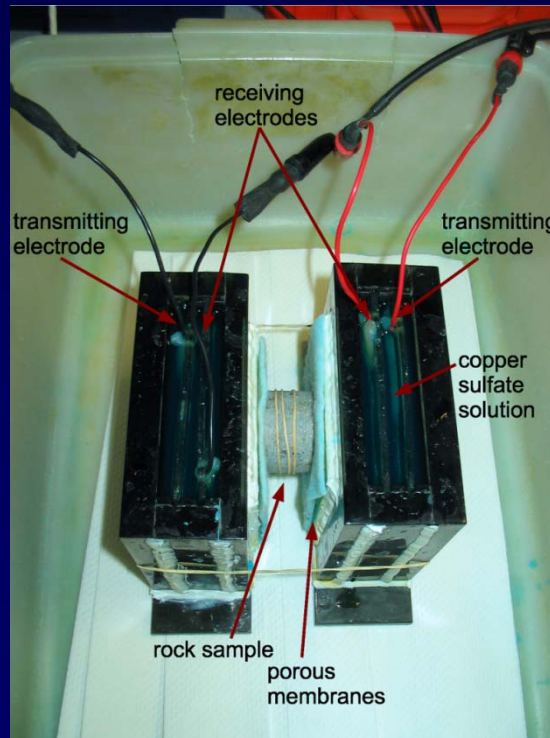
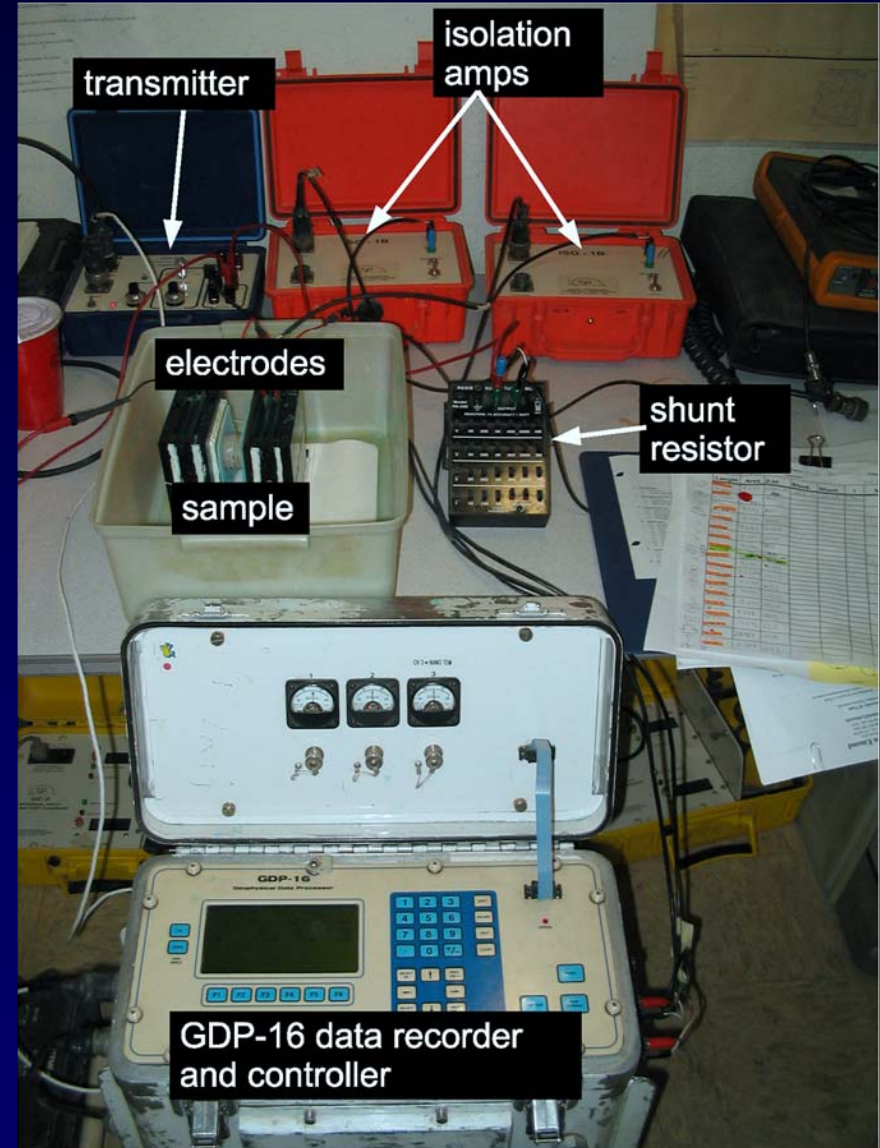
Table 4. Frequency-domain GEMTIP model 3 inversion parameters

Variable	Units	True model	Initial	Recovered
$\rho_0$	ohm-m	300	-	-
$f_1$	%	15	-	-
$f_2$	%	15	-	-
$\rho_1$	ohm-m	0.2	-	-
$\rho_2$	ohm-m	0.004	-	-
$a_1$	mm	0.0002	-	-
$a_2$	mm	0.0008	-	-
$C_1$	-	0.8	1	0.86
$C_2$	-	0.6	1	0.67
$\alpha_1$	$\frac{\Omega \cdot m^2}{sec^{eI}}$	2	3	2.88
$\alpha_2$	$\frac{\Omega \cdot m^2}{sec^{eI}}$	0.04	0.2	0.03

# Experimental analysis of rock samples

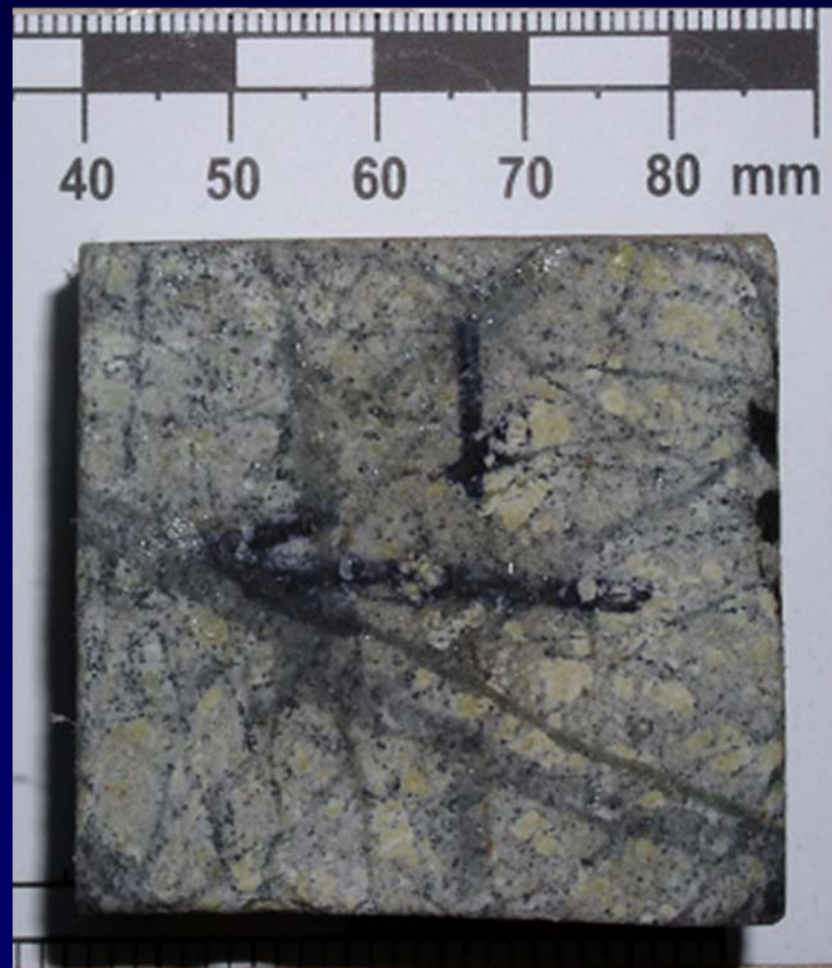
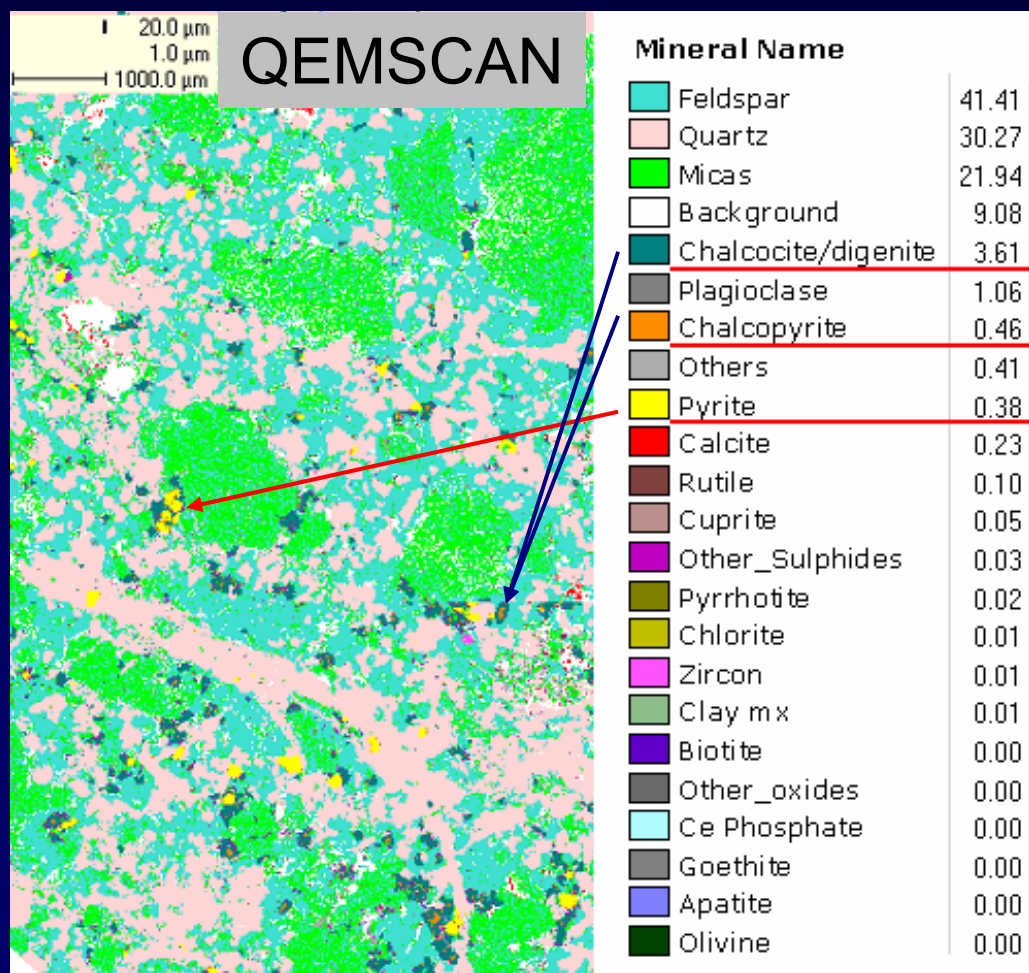
# Zonge CR system:

GDP16 receiver; LTD10 transmitter



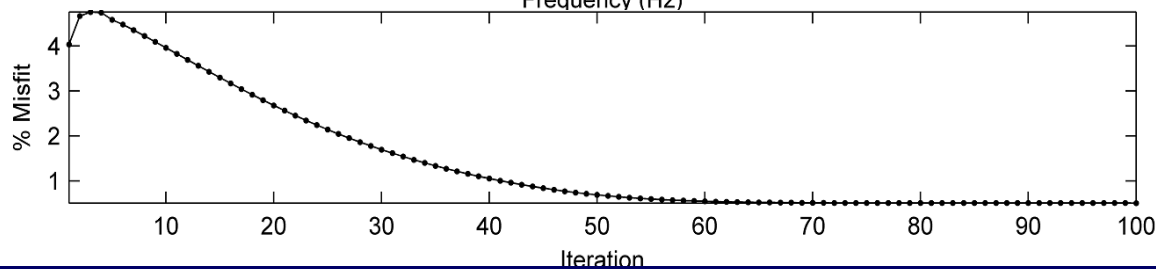
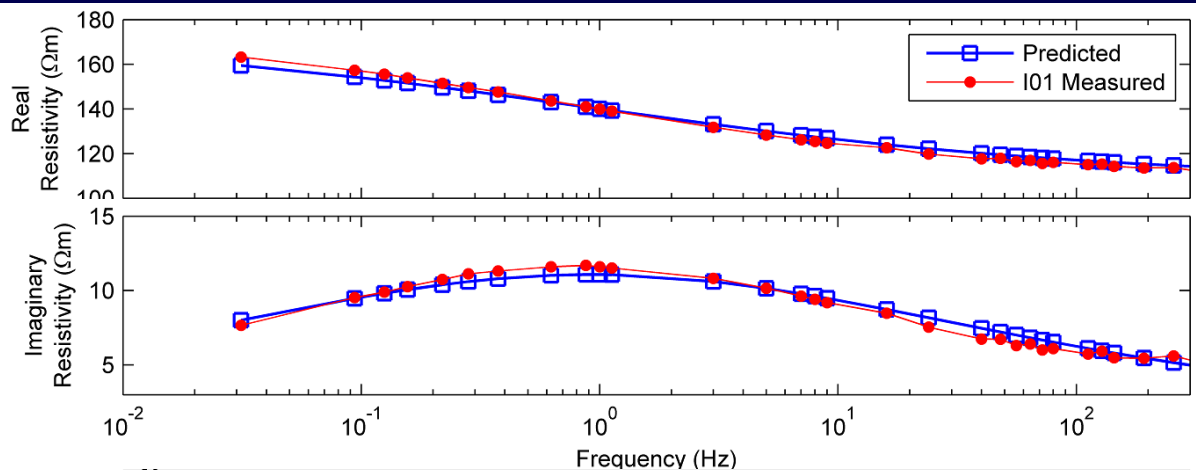
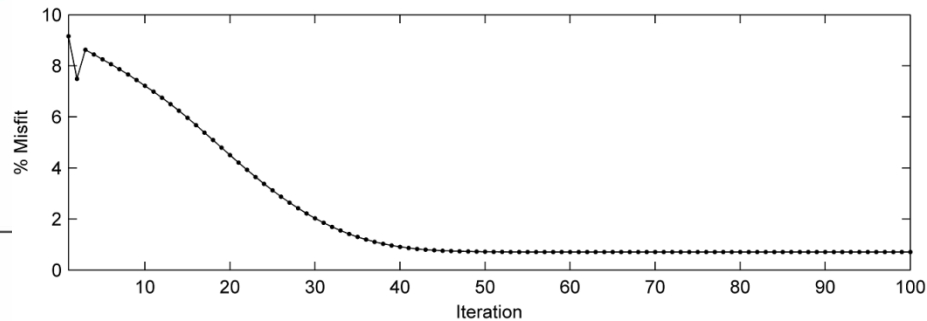
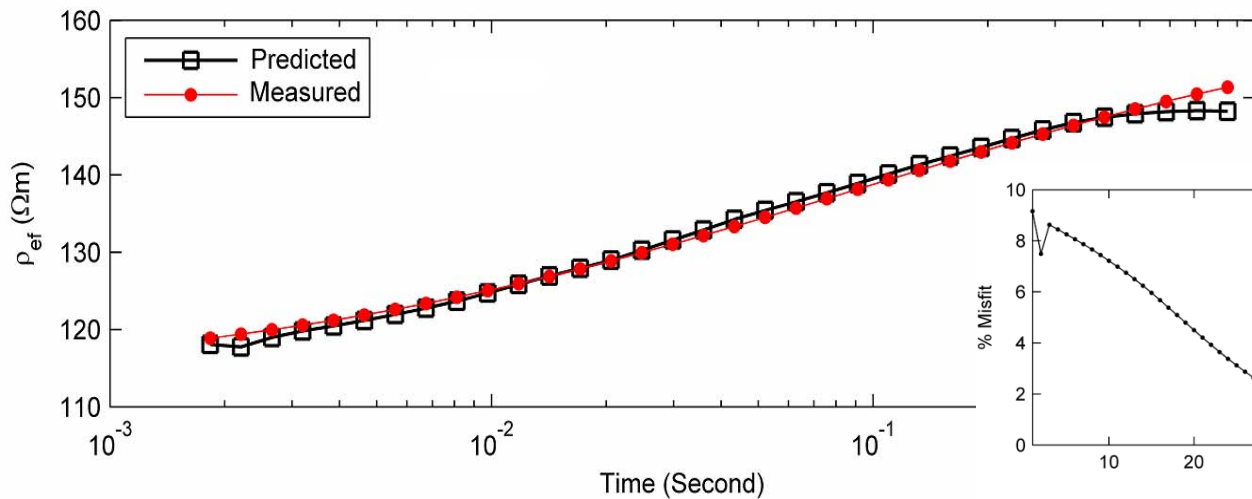
# Sample #13

Sar Cheshmeh copper porphyry deposit in Iran  
The reflective spots are chalcopyrite and pyrite inclusions.  
All sulfides are introduced during vein formation.



# 5.2 Sample #13

Time domain



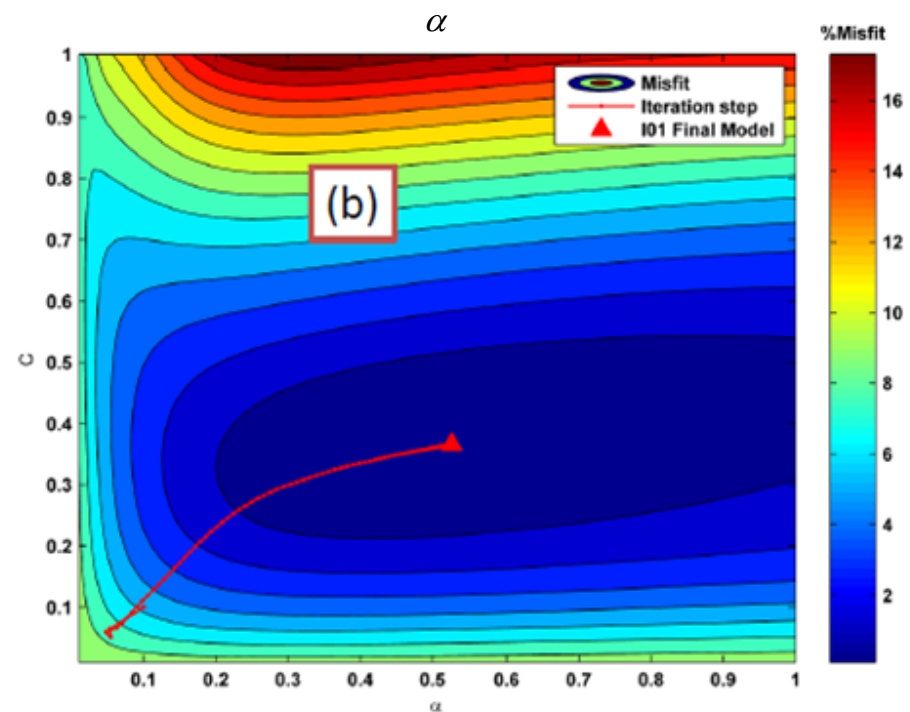
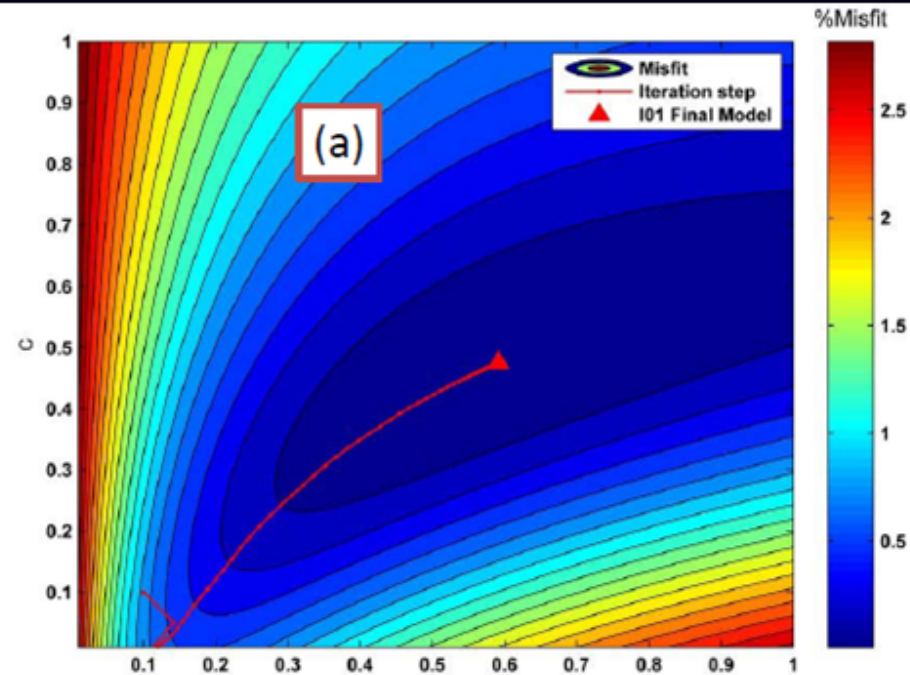
Frequency domain

# 5.2 Sample #13

variable	Units	Initial model	Time result	Freq result	TD & FD difference
$\rho_m$	$\Omega m$	163	-	-	-
f	-	0.05	-	-	-
C	-	0.1	0.58	0.52	10.3%
$\rho_1$	$\Omega m$	0.1	-	-	-
a	mm	0.002	-	-	-
$\alpha$		0.1	0.46	0.39	15.2%

a: misfit functional in time domain

b: misfit functional in frequency domain





# Conclusion

- I have extended the basic principles of the general effective medium theory of induced polarization (GEMTIP) from the frequency domain to the time domain.
- I have simulated the time-domain resistivity responses for two and three phase heterogeneous media. The IP parameters manifest themselves as peaks in the complex resistivity spectra, or as slopes of the resistivity transients.
- The inversion of synthetic resistivity transients demonstrates that it is possible to discriminate multiple parameters for different mineral inclusions.

Thank you!