Deterministic Source Synthesis for Waveform Inversion

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The Rice Inversion Project

Annual Review, 2010

Agenda

Source Synthesis - Why & How

Deterministic source synthesis

Numerical Exploration

Conclusions and Prospects

Source Synthesis - Why & How

Main motivation for this work: More efficient inversion - use fewer sources (ideally, one for entire data set) in each iterative inversion step

- length-1 encoding (weighted zero-lag data stacks - Krebs et al. 2009)
- inversion using source blending, simultaneous shooting (Ayeni et al. 2009, Verschuur & Berkhout 2009)
- random filtering, incoherency

Explicit recovery of individual shots not primary goal - synthetic sources chosen to drive model towards optimal inversion solution

= model which best fits *any* data (so shots are *implicitly* recovered...)

Source Synthesis - Why & How

This talk explores deterministic source synthesis via optimality principle:

best source $\Leftrightarrow$ *worst residual*

- origin in other inversion/imaging technologies
- simple source selection algorithm for acoustic modeling
- a few examples suggest pitfalls, remedies
- many unanswered questions - notably, does it really work? (in FWI)

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Deterministic source synthesis

Introduced into biomedical Electrical Impedance Tomography (EIT) by Isaacson (1986) - similar ideas: array ultrasonics (Fink & Prada 2004), ocean acoustics (Roux & Kuperman 2005), SAR (Borcea & Papanicolaou 2007),...

David Isaacson

EIT: image anomalies interior to body by measuring voltage response to applied current on boundary.

Deterministic source synthesis

Acoustic Model: state $u =$ acoustic potential in model domain R (subsurface), model $m = $ (velocity v, density ρ),

$$\frac{1}{\rho v^2}\frac{\partial^2 u}{\partial t^2} - \nabla \cdot \frac{1}{\rho}\nabla u = f(x, y, t)\delta(z - z_s).$$

Synthetic source $f =$ divergence of force density, confined to source depth plane z_s - must be post-synthesized digitally.

Measured response: pressure at fixed spread receiver locations $\Lambda^d f = \{\partial u/\partial t(\mathbf{x}_r, t)\}$ - linear in f - synthesized from field (point) source data traces.

Predicted response for model m $(= (v, \rho))$: $\Lambda[m]f$, computed by FE or FD or...

Deterministic source synthesis

Isaacson's Distinguishability Principle: seek normalized f so that RMS difference is largest: given estimated model m,

$$\text{maximize } (\Lambda^d f - \Lambda[m]f)^T(\Lambda^d f - \Lambda[m]f) \text{ subject to } f^T f = 1$$

max value $\lambda[m]$ = largest eigenvalue (operator norm) of *distinguishability operator*

$$A[m] = (\Lambda^d - \Lambda[m])^T(\Lambda^d - \Lambda[m])$$

= *largest discrepancy* in response for *any* (normalized) source (applied current pattern).

Deterministic source synthesis

Isaacson's algorithm:

- initialize m, f
- while (not satisfied),
  - fixed m, update f: perform several power method steps: $f \leftarrow A[m]f$, $f \leftarrow (1/\sqrt{f^t f})f$
  - fixed f, update m: perform several quasi-Newton steps with objective function $f^t A[m] f$ (standard output least squares)

Deterministic source synthesis

A few practical points:

1. assuming field data wavelet w known (!), achievable synthetic sources are filters:

$$f(x,y,t) = \sum_{x_s,y_s} \int d\tau \, g(x_s,y_s,t-\tau) w(\tau) \delta(x-x_s) \delta(y-y_s)$$

   Possibilities for g (1) arbitrary length filters (random choice - Romero et al. 00); (2) length-1 filters (amplitude factor) - Krebs et al. 09.

2. transpose operator $\Lambda[m]^T = R\Lambda[m]R$, $R =$ time-reversal op
3. Isaacson's alternating algorithm: Each step of both types involves 2 or 3 simulations (forward and/or reverse time loops), *for single (array) source*

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Setup

2D numerical experiments, using FD modeling/inversion package (IWAVE++).

Two models per experiment:

- Target model m^* - "measured" data $\Lambda^d f = \Lambda[m^*]f$
- Reference model m - "predicted" data $\Lambda[m]f$.

Measure progress in terms of *Rayleigh quotient* ("RQ"):

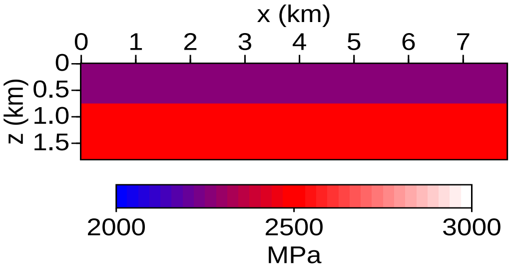
$$RQ = \frac{f^T A[m^*, m] f}{f^T f}$$

involves computing distinguishability operator $A[m^*, m] = (\Lambda[m^*] - \Lambda[m])^T (\Lambda[m^*] - \Lambda[m])$.

Setup

- staggered grid scheme for pressure, particle velocity
- source represented as *constitutive law defect* = RHS in pressure equation
- Models sampled at $\Delta x = \Delta z = 20$ m
- Absorbing BC on all sides of simulation domain
- Source, receiver depth 20 m - source = receiver locations
- 6 km fixed spread sampled at $\Delta x_s = 20$ m, 3 s recording interval
- 25 Hz high-cut imposed uniformly by filtering all sources, sources windowed to 0.0-0.4 s,

Layer over Half Space



Bulk modulus - 2.25 GPa to 0.75 km, 2.5 GPa below

Density is homogeneous = 1 gm/cc

Layer over Half Space

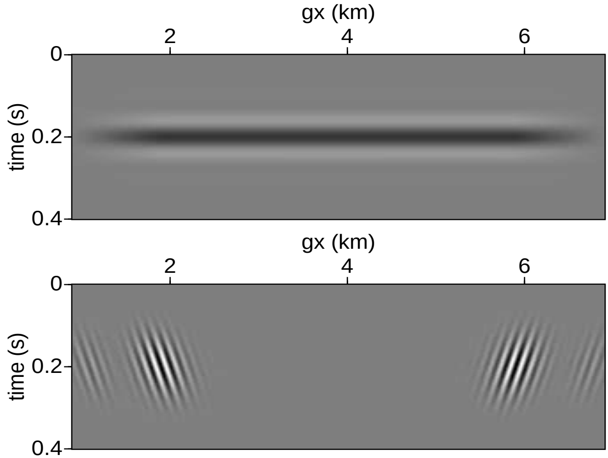
Initial source = truncated normal incidence plane wave

10 iterations of power method:

- initial Rayleigh quotient = 1.27
- final Raleigh quotient = 56.3

Looks great - however...

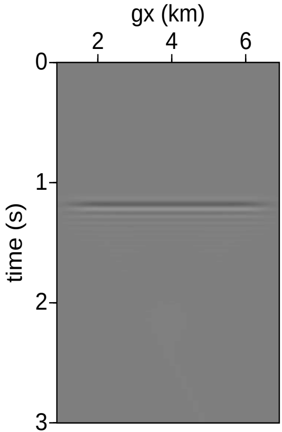
Layer over Half Space



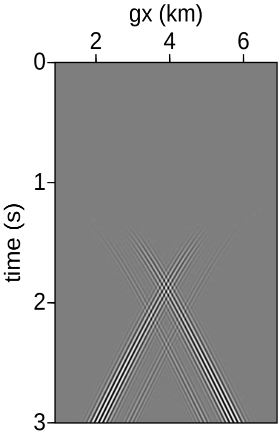
Initial (top), “optimal” (bottom) sources

Layer over Half Space

Data Difference $\Lambda[m^*]f - \Lambda[m]f$



it=0: RMS=1.1



it=10: RMS=7.5

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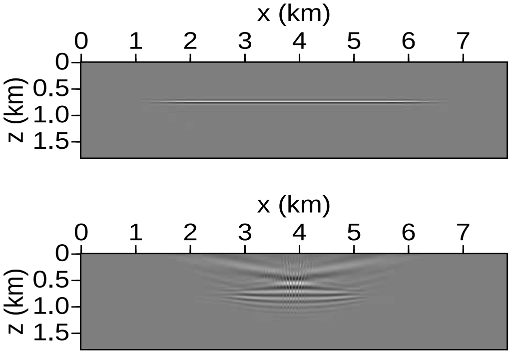
Layer over Half Space

RTM Image = Least Squares gradient



Amplitude of top (it=0) $10^{-2}\times$ amplitude of bottom (it=10).

Theory: Why this happens, what to do

Wave packed data:

$$f(x, y, t) = A(x, y, t)e^{i(k_x x + k_y y + \omega t)}$$

Guess: solution of wave equation

$$\frac{1}{\rho v^2}\frac{\partial^2 u}{\partial t^2} - \nabla \cdot \frac{1}{\rho}\nabla u = f(x, y, t)\delta(z - z_s)$$

takes form for $\pm z > 0$

$$u \simeq B_\pm e^{i(k_x x + k_y y \pm k_z z + \omega t)},$$

where $k_z = \pm\left(\frac{\omega^2}{v^2} - k_x^2 - k_y^2\right)^{\frac{1}{2}}$ and $B_\pm$ solves transport equation.

Theory: Why this happens, what to do

Causality: $\pm k_z > 0$ if $\pm z > 0$. Choose test function $\phi(x,y,z,t)$, then integration by parts gives

$$\int\int\int\int dxdydzdt\, p(x,y,z,t)\left(\frac{1}{\rho v^2}\frac{\partial^2\phi}{\partial t^2} - \nabla\cdot\frac{1}{\rho}\nabla\phi\right)$$

$$= \int\int\int dxdydt\, f(x,y,t)\phi(x,y,t)$$

Since both p and $\partial p/\partial z$ are continuous (normal stress, displacement), can split first integral into $z < 0$ and $z > 0$ pieces and integrate by parts again. Because of wave equation for p, only boundary terms left:

$$= \int\int\int dxdydt\left\{[p]\frac{\partial\phi}{\partial z} - \left[\frac{\partial p}{\partial z}\right]\phi\right\}$$

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Theory: Why this happens, what to do

This identity must hold for any test function (smooth, vanishing for large $\mathbf{x}, t$) - in particular, can choose ϕ to be $= 0$ on $z = 0$ whilst $\partial\phi/\partial z$ takes on arbitrary values. Hence [p]=0. Since ϕ can also take arbitrary values, follows that

$$f(x, y, t) = -\left[\frac{\partial p}{\partial z}\right](x, y, t)$$

First condition implies that $B_- = B_+$ on $z = 0$; second, that

$$f(x, y, t) = -2ik_z B_\pm e^{i(k_x x + k_y y + \omega t)},$$

Thus

$$u \simeq \frac{\tilde{A}}{k_z} e^{i(k_x x + k_y y + k_z z + \omega t)},$$

where $\tilde{A}|_{z=0} = \frac{i}{2}A$, and $\tilde{A}$ solves transport eqns.

Theory: Why this happens, what to do

Upshot: k_z small $\Rightarrow$ energy transfer to acoustic field *extremely efficient* per RMS unit f.

k_z small $\Rightarrow$ most energy propagates near-horizontally - limits imaging aperture, vertical resolution.

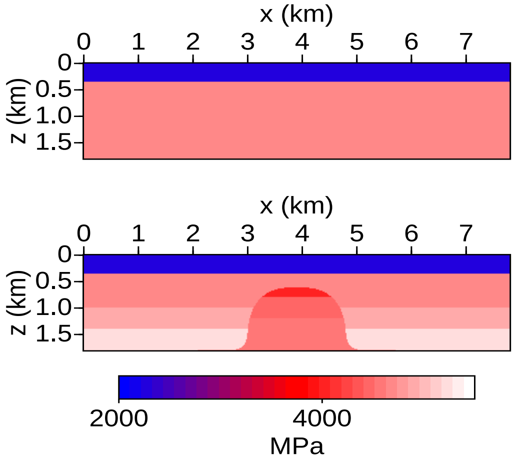
Solution: depress part of spectrum of $A[m]$ corresponding to small k_z by composing $\Lambda^d - \Lambda[m]$ with dip filter.

For water layer near surface: k_z small when $|k_x| \simeq 0.67$ s/km.

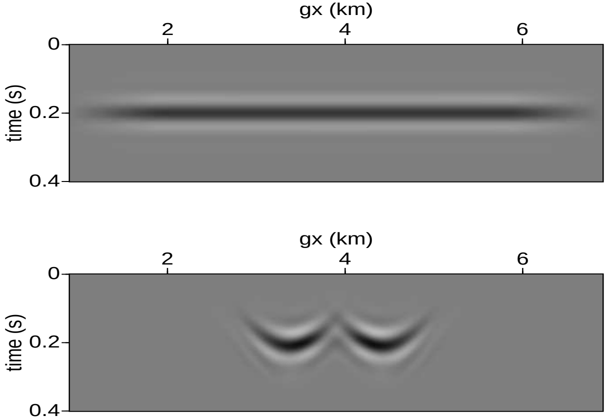
Example: for LOHS example, choose dip filter with corner slope of 0.3 s/km, cut slope of 0.5 s/km - then optimal source is small modification of plane wave source.

Laterally Heterogeneous Example

Bulk moduli - reference (top), target (bottom)



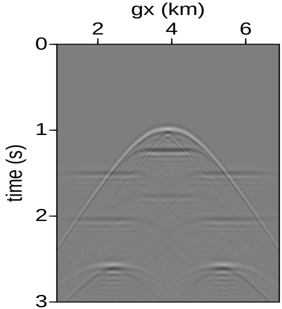
Laterally Heterogeneous Example



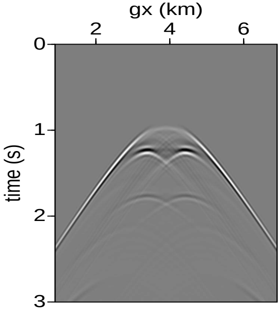
Initial (top), “optimal” (bottom) sources

Laterally Heterogeneous Example

Data Difference $\Lambda[m^*]f - \Lambda[m]f$



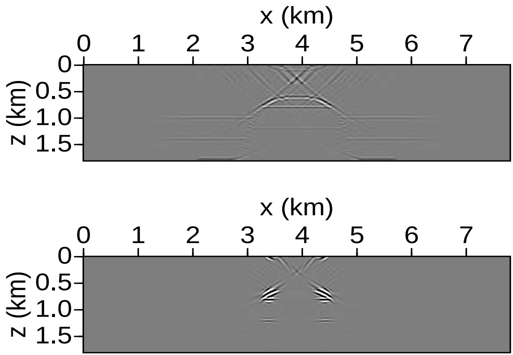
it=0: RMS=2.0, RQ=5.8



it=10: RMS=3.3, RQ=11.2

Laterally Heterogeneous Example

RTM Image = Least Squares gradient



Top (it=0) and bottom (it=10, 10× clip)

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Conclusions and Prospects

Many numerical experiments suggest

- "optimal" source emphasizes largest features in residual data, as intended
- dip filtering effectively controls tendency to produce horizontally traveling energy
- selective illuminates features in gradient (RTM residual image)

Conclusions and Prospects

If anything, illumination is *too* selective - a single source is likely not sufficient

Gao et al. 2010: find all eigenpairs of distinguishability operator above a threshhold, use these collectivley - still much smaller than number of source positions in typical survey (?)

Natural method: Lanczos algorithm - finds segment of spectrum, rather than merely largest eigenvalue.

Next step: use Lanczos implementation in RVL to explore time-domain version of Gao et al. proposal.

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Sponsors of The Rice Inversion Project

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