Differential Semblance Migration Velocity Analysis via Reverse Time Migration: Gradient Computation

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Outline

Introduction

- Differential Semblance Velocity Analysis
  - Smoothness
  - Convexity
- Reverse Time Migration
  - Dip limitation
- DSMVA-RTM

Acoustic Model

Acoustic wave equation with constant density

$$\left(\frac{1}{c^2(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \nabla^2\right) p(\mathbf{x}, t; \mathbf{x}_s) = f(t)\delta(\mathbf{x} - \mathbf{x}_s)$$

$\mathbf{x}$ = position vector
$\mathbf{x_s}$ = position of the point source
$c(\mathbf{x})$ = velocity
$p(\mathbf{x}, t; \mathbf{x_s})$ = pressure
Source time function $f(t) = \delta(t)$
Two-way wave equation operator $\mathbf{L} := \frac{1}{c^2(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \nabla^2$

Wavefields

Source Wavefields (forward in time):

$$\mathbf{L}S(\mathbf{x},t;\mathbf{x}_s) = \delta(t)\delta(\mathbf{x}-\mathbf{x}_s)$$
$$S \equiv 0 \text{ for } t < 0$$

Receiver Wavefields (backwards in time):

$$\mathbf{L}R(\mathbf{x},t;\mathbf{x}_s) = \int d(\mathbf{x}_r,t;\mathbf{x}_s)\delta(\mathbf{x}-\mathbf{x}_r)\,d\mathbf{x}_r$$
$$R \equiv 0 \text{ for } t > t_{max}$$

Write

$$c(\mathbf{x}) = v(\mathbf{x}) + \delta v(\mathbf{x}).$$

Forward Born Modeling:

$$F[v]\delta v = \delta S|_{surface}$$

Adjoint of Forward Modeling:

$$F^* d = I$$

RTM formula and DSMVA objective function

Migration formula:

$$I(\mathbf{x}, h) = \int S(\mathbf{x} + h, t; \mathbf{x}_s) R(\mathbf{x} - h, t; \mathbf{x}_s) \, d\mathbf{x}_s \, dt$$

DSMVA-RTM objective function:

$$J[v] = \frac{1}{2} \| PI \|^2$$

Gradient Computation

For $u = P^*PI$, DI^*u gives the gradient.
Introducing g_r solving

$$\begin{aligned} \mathbf{L}g_r(\mathbf{x}, t; \mathbf{x}_s) &= \int R(\mathbf{x} - 2h, t; \mathbf{x}_s) u(t - h, h)\, dh \\ g_r &\equiv 0 \text{ for } t > t_{max}, BC \end{aligned}$$

Introducing g_s solving

$$\begin{aligned} \mathbf{L}g_s(\mathbf{x}, t; \mathbf{x}_s) &= \int S(\mathbf{x} + 2h, t; \mathbf{x}_s) u(t + h, h)\, dh \\ g_s &\equiv 0 \text{ for } t < 0, BC \end{aligned}$$

The gradient of DSMVA-RTM is

$$\nabla_v J = \int \frac{2}{v^3} \left(\frac{\partial^2 S}{\partial t^2} g_r + \frac{\partial^2 R}{\partial t^2} g_s \right) d\mathbf{x}_s\, dt$$

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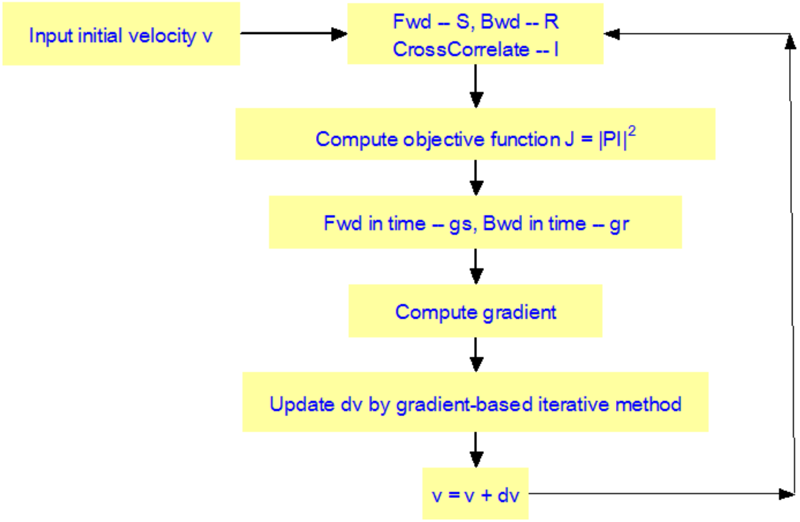
Inversion procedure



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Comparison between one-way and two-way WEMVA

One-way wave equation migration:

- Pros: low computational cost
- Cons: dip limitation

Two-way wave equation migration:

- Pros: no dip limitation
- Cons: high computational cost
- Cons: amplitude correction required

Future Work

- Amplitude Correction (see Rami's work)
- Modification of existing RTM code based on SEAMX, TSOpt to compute objective function and to verify its convexity
- Future modification to create gradient
- Comparison with DS based on downward continuous extrapolation

Perturbation of objective function:

$$\delta J = \frac{1}{2}\langle \delta(PI), PI\rangle + \frac{1}{2}\langle PI, \delta(PI)\rangle$$

$$\Longrightarrow \delta J = \langle \delta c, Re((DI)^* P^* PI)\rangle$$

Gradient of objective function:

$$\nabla_c J = Re\{(DI)^*(P^* PI)\}$$

For arbitrary $u(x, h)$, we have

$$\langle \delta I, u\rangle = \langle DI\delta c, u\rangle = \langle \delta c, DI^* u\rangle$$

When $u = P^* PI$, $DI^* u$ gives the gradient.

$$
\begin{aligned}
\langle \delta I, u\rangle &= \int \overline{\delta I} u(x,h)\, dx\, dh \\
&= \int \delta S(x+h, x_s, t)\overline{R}(x-h, x_s, t) u(x,h)\, dx\, dx_s\, dt\, dh \\
&+ \int S(x+h, x_s, t)\overline{\delta R}(x-h, x_s, t) u(x,h)\, dx\, dx_s\, dt\, dh \\
&= \int \delta S(x, x_s, t)\{\int \overline{R}(x-2h, x_s, t) u(x-h,h)\, dh\}\, dx\, dx_s\, dt \\
&+ \int \overline{\delta R}(x, x_s, t)\{\int S(x+2h, x_s, t) u(x+h,h)\, dh\}\, dx\, dx_s\, dt
\end{aligned}
$$

Introducing g_r solving

$$\begin{aligned}
\mathbf{L}g_r(\mathbf{x},t;\mathbf{x}_s) &= \int R(\mathbf{x}-2h,t;\mathbf{x}_s)u(t-h,h)\,dh \\
\int_\Sigma \nabla S g_r - S\nabla g_r &= 0 \\
\frac{\partial S}{\partial t}g_r - S\frac{\partial g_r}{\partial t}(\mathbf{x},0;\mathbf{x}_s) &= 0 \\
\frac{\partial S}{\partial t}g_r - S\frac{\partial g_r}{\partial t}(\mathbf{x},t_{max};\mathbf{x}_s) &= 0
\end{aligned}$$

Introducing g_s solving

$$\begin{aligned}
\mathbf{L}g_s(\mathbf{x},t;\mathbf{x}_s) &= \int S(\mathbf{x}+2h,t;\mathbf{x}_s)u(t+h,h)\,dh \\
\int_\Sigma \nabla R g_s - R\nabla g_s &= 0 \\
\frac{\partial R}{\partial t}g_s - R\frac{\partial g_s}{\partial t}(\mathbf{x},0;\mathbf{x}_s) &= 0 \\
\frac{\partial R}{\partial t}g_s - R\frac{\partial g_s}{\partial t}(\mathbf{x},t_{max};\mathbf{x}_s) &= 0
\end{aligned}$$

Then,

$$
\begin{aligned}
\langle \delta I, u \rangle &= \int (\delta S \mathbf{L} g_r + \delta R \mathbf{L} g_s)\, dx\, dx_s\, dt \\
&= \int ((\mathbf{L}\delta S) g_r + (\mathbf{L}\delta R) g_s)\, dx\, dx_s\, dt \\
&= \int \frac{2\delta c}{c^3} \frac{\partial^2 S}{\partial t^2} g_r + \frac{2\delta c}{c^3} \frac{\partial^2 R}{\partial t^2} g_s\, dx\, dx_s\, dt
\end{aligned}
$$

Thus

$$
\nabla_c J = \int \frac{2}{c^3} \left(\frac{\partial^2 S}{\partial t^2} g_r + \frac{\partial^2 R}{\partial t^2} g_s \right) dx_s\, dt
$$