Effective Waveform Inversion = Nonlinear MVA

William W. Symes

Agenda

MVA and Semblance

Extended Modeling: MVA + WI

Conclusions

Surface-oriented, Depth-oriented Image Volumes

MVA based on prestack depth migration - two major variants. Both produce *image volume* $I(\mathbf{x}, \mathbf{h})$ depending on image point $\mathbf{x}$, half-offset $\mathbf{h}$.

(I) Surface oriented: $\mathbf{h} = 0.5$(receiver - source), usually computed by diffraction sum ("Kirchhoff common offset migration"); binwise: offset bin $I(\cdot, \mathbf{h})$ depends only on data traces with offset $\mathbf{h}$.

(II) Depth oriented: $2\mathbf{h} =$ difference between subsurface scattering points, $\mathbf{x} =$ their midpoint. Every point in image volume depends on all data traces. Has diffraction sum rep, but usually computed by one-way (shot profile or DSR) or two-way (RTM) wave extrapolation.

Semblance

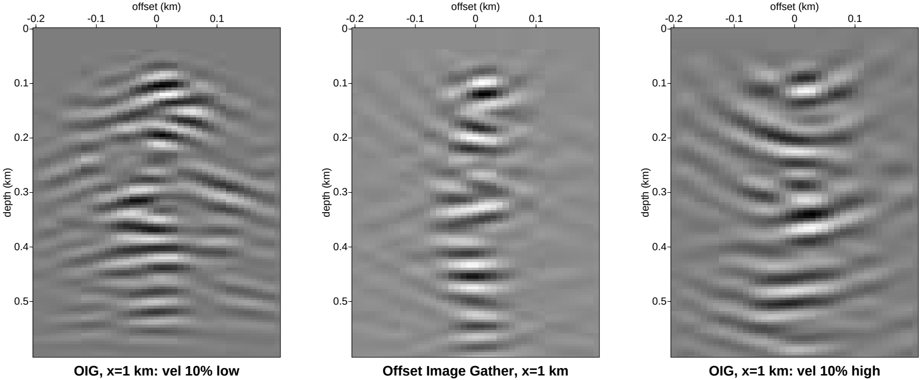
Semblance condition: expresses consistency between data, velocity model in terms of image volume.

(I) Surface oriented: velocity-data consistency when $I(\mathbf{x}, \mathbf{h})$ independent of $\mathbf{h}$ (at least in terms of phase), i.e. image gathers are flat.

(II) Depth oriented: velocity-data consistency when $I(\mathbf{x}, \mathbf{h})$ concentrated near $\mathbf{h} = 0$, i.e. image gathers are focused [or flat, when converted to scattering angle].

Main principle of MVA: adjust velocity until image volume satisfies semblance condition.

Semblance



RTM space shift image gathers ($I_D(\mathbf{x},\mathbf{h})$) from velo model $v+\delta v$, $v=$ const., $\delta v=$ randomly distributed point diffractors. Left to Right: migration velocity $=90\%, 100\%, 110\%$ of true velocity.

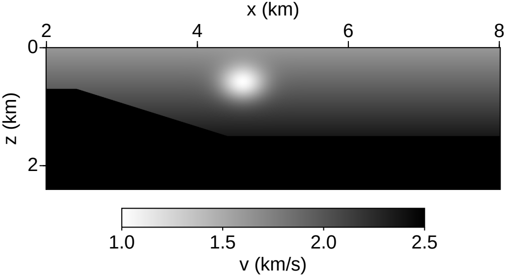
Surface-Oriented vs. Depth-Oriented, MVA

Nolan & S. 97, Stolk & S. 04, deHoop & Brandsberg-Dahl 03: multipathing (multiple rays connecting source, receiver, and image points, caustics) leads to kinematic artifacts in surface oriented image volume.

Artifact = coherent event in wrong place, of strength comparable to correct events.

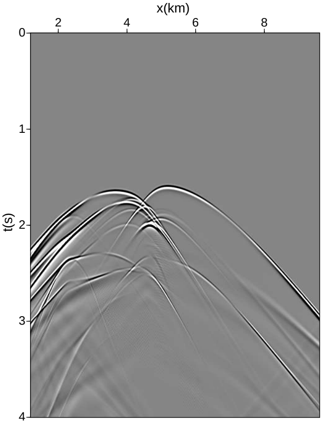
Consequences for velociy analysis: artifacts $\Rightarrow$ semblance condition not satisfied even if velocity is correct!: Nolan and S. 97, Xu SEG 07.

Surface-Oriented vs. Depth-Oriented, MVA



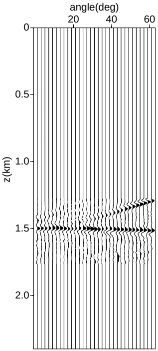
Velocity model after Valhall field, North Sea. Note sloping reflector at left, large low-velocity lens (modeling gas accumulation) in center. Both tend to produce multipathing. (Thanks: M. de Hoop, A. Malcolm)

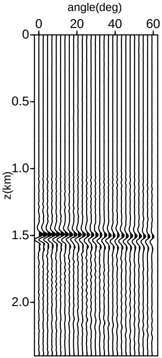
Surface-Oriented vs. Depth-Oriented, MVA



Typical shot gather over center of model, exhibiting extensive multipathing.

Surface-Oriented vs. Depth-Oriented, MVA





Angle Domain CIGs at same horizontal position from surface-oriented (Kirchhoff) and depth-oriented (DSR) migrated image volumes. Left: ADCIG from Kirchhoff migration: kinematic artifacts clearly visible. Right: ADCIG from DSR migration: no artifacts!

Surface-Oriented vs. Depth-Oriented, MVA

Stolk & deHoop 01, S. 02, deHoop, Stolk & S. 05: depth-oriented image volume generally free of artifacts, even with strong multipathing.

So the two types of image volume are not even kinematically equivalent!

Accounts for perceived superiority of "wave equation migration".

Suggests: depth-oriented volume possibly better domain for MVA in complex, refracting subsurface.

Surface-Oriented vs. Depth-Oriented, MVA

MVA via Optimization:

- form measure of deviation of image volume from semblance condition - function of velocity model; all energy not conforming to semblance condition contributes.
- optimize numerically: gradient = backprojection of semblance-inconsistent energy into velocity update.

Inherently uses all events in data, weighted by strength.

Example: for depth-oriented, minimize $J[v] = \sum |\mathbf{h} I(\mathbf{x}, \mathbf{h})|^2$ - penalizes energy at $\mathbf{h} \neq 0$. Apparently: no local mins!

Recent contributions: Shen 03, 05, Li & S. 05, Foss 06, Albertin 06, Khoury 06, Verm 06, Kabir SEG 07, Shen & S TRIP08.

Agenda

MVA and Semblance

Extended Modeling: MVA + WI

Conclusions

Extended Modeling

Extended model $\bar{\mathcal{F}} : \bar{\mathcal{M}} \rightarrow \mathcal{D}$, where $\bar{\mathcal{M}}$ is a *bigger model space*= models depending on $\mathbf{x}$ *and* $\mathbf{h}$, i.e. $\bar{v}(\mathbf{x}, \mathbf{h})$.

Physical (normal) model identified with extended model: for depth oriented modeling, $v(\mathbf{x}) \mapsto v(\mathbf{x})\delta(\mathbf{h}) = \bar{v}(\mathbf{x}, \mathbf{h})$ (satisfies semblance condition!).

Extension property: $\mathcal{F}[v] = \bar{\mathcal{F}}[\bar{v}]$.

Extended Modeling

Lailly, Tarantola, Claerbout (80's): migration operator (producing image) is *adjoint* or transpose $D\mathcal{F}[v]^T$.

True amplitude migration is (pseudo)inverse $D\mathcal{F}[v]^{-1}$. Same for extended modeling $\bar{\mathcal{F}}[\bar{v}]$:

$$D\bar{\mathcal{F}}[\chi[v]]^T d(\mathbf{x},\mathbf{h}) = I(\mathbf{x},\mathbf{h}),\; D\bar{\mathcal{F}}[\chi[v]]^{-1} d(\mathbf{x},\mathbf{h}) = \delta\bar{v}(\mathbf{x},\mathbf{h}).$$

Extended Modeling

(1) MVA (with true amplitude) solves "partially linearized" problem: find reference velocity v and perturbation δv so that $D\mathcal{F}[v]\delta v \simeq d - \mathcal{F}[v]$.

Proof: successful true amplitude MVA produces image volume satisfying imaging condition ($\delta\bar{v}(\mathbf{x},\mathbf{h}) \simeq \delta v(\mathbf{x})\delta(\mathbf{h})$) and fitting data, that is,

$$\min_{v,\delta\bar{v}\in M\times\bar{M}} \|h\delta\bar{v}\|^2 \ \mathrm{subj}\ D\bar{\mathcal{F}}[v]\delta\bar{v} \simeq d - \bar{\mathcal{F}}[v].$$

(2) Nonlinear MVA, or WI based on semblance:

$$\min_{\bar{v}\in\bar{M}} \|h\bar{v}\|^2 \ \mathrm{subj}\ \bar{\mathcal{F}}[\bar{v}] \simeq d.$$

Extended Modeling

- $\mathcal{F}$ can be *any modeling* operator - acoustic, elastic, ... - So: MVA extended to elastic modeling with multiples, for instance.
- For surface oriented extension, nonlinear bin-by-bin modeling - cf. Dong Sun.
- For depth-oriented extension, $\bar{\mathcal{F}}$ expresses action at a distance: elastic moduli are nonlocal, stress at $\mathbf{x} + \mathbf{h}$ results from strain at $\mathbf{x} - \mathbf{h}$. So Claerbout's semblance principle is actually Cauchy's no-action-at-a-distance hypothesis! [Thanks: Scott Morton]

Extended Modeling

Example: an acoustic extended model.

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p, \;\; \frac{\partial p}{\partial t} = -K[\nabla \cdot \mathbf{v}],$$

where K is a bulk modulus *operator* expressing action-at-a-distance.

Key question: how to choose classes of ops K (nonphysical degrees of freedom) so that application of K is em cheap - else time-stepping becomes prohibititve.

Extended Modeling

Must find sparse representation for K!

Possible answer, based on experience with PSDM:

- K is nonlinear analogue of *prestack image volume*;
- prestack image volume sparse in phase space;
- sparsity in phase space expressible via multiresolution frames - eg. curvelets, see Candes-Donoho, de Hoop-Douma, Herrmann et al., Chauris.

Conjecture: bulk modulus operator K etc. should be sparse in phase space, hence admit low-cost implementation.

Extended Modeling

Second key question: how to update $\bar{v}$ while remaining in the set of data-fitting models $\bar{v}$ with $\bar{\mathcal{F}}[\bar{v}] \simeq d$?

Possible answer, analogous to approach in Dong Sun project: based on

Conjecture: suppose source is impulsive, has full bandwidth down to dc. Then $\bar{v}$ uniquely determined by data. [Not even known in enough generality for 1D! cf. Kirk Blazek's talk.]

If so, then can use low frequency data components, missing from field data, as control parameters permitting navigation of feasible set $\{\bar{v} : \bar{\mathcal{F}}[\bar{v}] \simeq d\}$ -nonlinear substitute for migration velocity = macromodel.

Extended Modeling

Another important issue: source calibration.

Patrick Lailly, Florence Delprat 01,03: nonlinear inversion (any kind!) *demands* good knowledge of source - but for extremely complex media with intense internal multiples, very difficult to secure!

Contrast: Minkoff & S 97, Winslow 99, Anno et al. 03: successful linearized inversion for source and reflectivity.

What is typical of the Earth?

Agenda

MVA and Semblance

Extended Modeling: MVA + WI

Conclusions

Conclusions

Takeaway messages of this talk:

- "Kirchhoff" and "Wave Equation" prestack migrations have different kinematic properties.
- MVA solves a "partially linearized" WI problem based on *extended modeling* - nonphysical degrees of freedom.
- MVA via waveform tomography ("differential semblance"), uses semblance condition and numerical optimization - all events constrain velocity updates, much less tendency towards local minima than least squares WI.
- Nonlinear extended scattering = framework for uniting MVA and waveform inversion.