Approximate Inverse Scattering Using Pseudodifferential Scaling

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  Completed Masters Coursework in Physics
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  High School Physics Teacher (2 years)

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Let:

- $m(x)$: The model (velocity, density,...)
- $p(x,t)$: The pressure field

Then, if S is the Forward Map:

- The Forward Problem:

$$S[m] = p|_{surface}$$

- The Inverse Problem:

$$S[m] \approx S^{obs}$$

Given S^{obs}, get $m(x)$

Nonlinear and Large Scale !

Linearization

If we have an approximation m_0 to the model, Linearization is advantageous:

- Write $m = m_0 + \delta m$
  m_0: Given reference model
  δm: First order perturbation about m_0

- Define Linearized Forward Map $F[m_0]$ (Born Modeling):

$$F[m_0]\delta m = \delta p$$

- Reduce to the Linear Subproblem

$$F[m_0]\delta m \approx S^{obs} - S[m_0] := d$$

Normal Operator

Interpreted as a least squares problem, linear subproblem yields the normal equations

$$F^*[m_0]F[m_0]\delta m = F^*[m_0]d$$

$F^*[m_0]F[m_0]$ is the Normal Operator

The problem is still *Large Scale*,order of Pflops/Pbytes $\Rightarrow$ cannot use direct methods to invert F^*F.

Properties of the normal operator have been extensively studied (Beylkin,1985; Rakesh,1988) for *smooth* m_0

- Nearly diagonal in the basis of localized monochromatic pulses (plane waves)
- Theoretical Setting: Pseudodifferential Operators

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The Eigenvalue Problem!

- Can compute $F^*[m_0]d$ and $F^*[m_0]F[m_0](\ldots)$ (migration, e.g. RTM, Born Modeling+Migration)
- Know how to diagonalize $F^*[m_0]F[m_0]$
- *Don't* know the eigenvalues
- If we knew eigenvalues $\Rightarrow$ Invert

Linear Algebra Analog

How to compute the eigenvalues?

- Given $Ax = b$, with A S.P.D $\Rightarrow A = V^T \Lambda V$.
- Given b and Ab, applying A is very expensive!
- Given V, collection of eigenvectors.
- Given an algorithm that applies $V^T \Lambda V$ cheaply.
- $$\Lambda_b = \underset{\Lambda}{\textit{argmin}} \|V^T \Lambda V b - Ab\|^2$$
- Now that we have Λ_b,

$$A^{-1} = V^T \Lambda_b^{-1} V \Rightarrow x = A^{-1} b$$

Scaling Methods

- Spatial delta function approximation of eigenvectors $\Rightarrow$ Diagonal Approximation of Hessian (Claerbout and Nichols, 1994; Rickett, 2003)
- Near Diagonal Approximation of Hessian (Guitton, 2004)
- Herrmann et al. (2007) use *curvelets* to approximate eigenvectors

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An Alternative

An alternative, relying on the *asymptotic expansion lemma*:

Lemma

$$F^*[m_0]F[m_0]\chi(x)e^{i\omega\psi(x)} = q_m(x, \omega\nabla\psi(x))\chi(x)e^{i\omega\psi(x)} + O(\omega^{m-\beta})$$

ω is the frequency, $\beta > 0$, q_m principal symbol of order m.
$\chi(x)$ compactly supported in a small ball

- $\chi(x)e^{i\omega\psi(x)}$ localized monochromatic pulse: eigenvectors
- $q_m \approx$ encodes the eigenvalues: Λ
  - $\chi(x)$ is localized
  - q_m is slowly varying

Approximation of ΨDO

- The action of the ΨDO (Bao and Symes, 1996):

$$Q_m u(x,z) \approx \int\int q_m(x,z,\xi,\eta)\hat{u}(\xi,\eta)e^{i(x\xi+z\eta)}\,d\xi\,d\eta$$

  q_m is the principal symbol, homogeneous of degree m.
  $\hat{u} = \mathcal{F}[u]$.
- Direct Algorithm $O(N^4\log(N))$ complexity!
- Writing $\xi = \omega\cos(\theta)$, $\eta = \omega\sin(\theta)$. Then,
  $q_m(x,z,\xi,\eta) = \omega^m \tilde{q}_m(x,z,\theta)$
-

$$\tilde{q}_m \approx \sum_{l=-K/2}^{l=K/2} c_l(x,z)e^{il\theta} = \sum_{l=-K/2}^{l=K/2} \omega^{-l}c_l(x,z)(\xi+i\eta)^l$$

Algorithm

$$Q_m u \approx \sum_{l=-K/2}^{l=K/2} c_l(x,z)\mathcal{F}^{-1}[\omega^{m-l}(\xi + i\eta)^l \hat{u}(\xi,\eta)]$$

1. Calculate $\hat{u} = \mathcal{F}[u]$
2. Calculate $\mathcal{F}^{-1}[\omega^{m-l}(\xi + i\eta)^l \hat{u}(\xi,\eta)]$
3. Calculate $c_l(x,z) \approx \mathcal{F}[\tilde{q}_m]$
4. Estimate $Q_m u$

Use FFT $\Rightarrow O(KN^2[\log(N) + \log(K)])$ complexity vs $O(N^4\log(N))$ complexity for the direct algorithm. K independent of N.

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Inversion

Given the migrated image $m_{mig} = F^* d$, and the remigrated image $m_{remig} = F^* F m_{mig}$

$$q_m = \underset{\tilde{q}_m \geq 0}{\text{argmin}} \|Q_m[\tilde{q}_m] m_{mig} - m_{remig}\|^2$$

Let

$$q^\dagger = \frac{1}{q_m + \varepsilon}, \qquad \varepsilon > 0$$

be the pseudoinverse of q_m and obtain an approximate inverse

$$\delta m \approx Q_m[q^\dagger] m_{mig}$$

We obtain a Scaling Method.

The Master Plan

- Implement Bao and Symes (1996) Algorithm: Discretizations, FFT...

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- Get a Masters !

Applications

- When m_0 is a good approximation:
  - Fast solution of the Linear Inverse Problem
  - Variable density acoustics
  - Linear Elasticity
- When m_0 is not a good approximation:
  - View the linear problem as a Newton step
  - Preconditioning of iterative methods

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Summary

- Normal Operator is
  - Pseudodifferential
  - Nearly diagonal in basis of localized monochromatic pulses
- Efficient Algorithm to apply a ΨDO
- Scaling Method
  - Fast and reliable solution if m_0 is a good approximation
  - Preconditioning iterative methods if m_0 is not a good approximation

THANK YOU !