Self Introduction of Dong Sun

- Education:
  - 2006 - present, persuing Ph.D. at CAAM, Rice University
    Advisor: Dr. W. Symes
  - Aug. 2005 - July 2006, MA in Mathematics, Washington State Univ.
  - Sep. 1998 - June 2005, BA & MS in Computational Math, Nankai University, China
    Thesis, 'A two-level method with upwind discretization for the Navier-Stokes Equations'.
- Recent Projects:
  - Current project, implement a DS method with nonlinear modeling for layered medium
  - Summer 2007, implement OLS inversion for 1-D acoustic wave
  - Summer 2006, MA Project at WSU
    'Numerical Solutions for a Coupled Parabolic Equations Arising from Induction Heating Processes', 2006 AIMS 6th Intl. Conf. on Dyn. Sys., Dif. Eqns. and Apps.
  - Summer 2008. Internship at ExxonMobil Upstream Research Comp.

The nonlinear differential semblance algorithm for plane waves in layered media

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TRIP Annual Meeting 2008

Highlights of this project

- the first implementation of Differential Semblance (DS) with nonlinear modeling
  many other successful DS implementations exist;
  all based on linearized model
- an implementation trying to overcome the obstacles caused by the lack of low frequency energy
  least-squares inversion and its variants suffer from the low-frequency lacuna (see Shin and Min(2006) for a recent example)
- reasonably accurate solution for any initial estimate (expected result)

Outline

1. Model
2. Motivation
3. Method
4. Expected result & future work
5. Appendix. Gradient computation

Layered Constant Density Acoustic Model

Wave equation for acoustic potential $u(x, z, t)$:

$$\left(\frac{1}{c^2(z)}\frac{\partial^2}{\partial t^2} - \nabla^2\right) u(x, z, t) = \omega(t)\delta(x - x_s, z - z_s),$$

plus initial, boundary conditions.

velocity $c(z)$ $(0 < c_{min} \leq c(z) \leq c_{max})$,
source time function $\omega(t)$ (band-limited).

Forward map: $s_\omega[c] = u(x, z_r, t)$ (predicted *seismogram*).

To simplify the original problem, introduce Radon Transformed (the slant-stack) field.

Plane Wave Decomposition

Radon transform:

$$U(p,z,t) = \int dx\, u(x,z,t+px)$$

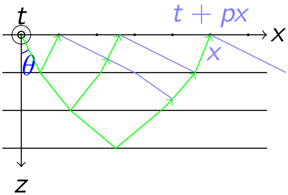
leads to a series of plane-wave problems:

$$\left(\frac{1}{v^2(p,z)}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right) U(p,z,t) = \omega(t)\delta(z-z_s),$$

where

$v[c](p,z) = \frac{c(z)}{\sqrt{1-c^2(z)p^2}}$ for $|p| \le p_{max} < 1/c_{max}$,

$S_\omega[v] := U(p,z_r,t)$.



ray parameter

$$p = \frac{\sin(\theta)}{c(z)}$$

Goal: given data D, find $v(p,z)$ so that $S_\omega[v] \simeq D$,

then solve $v = \frac{c}{\sqrt{1-c^2p^2}}$ for $c(z)$.

Output Least Squares Inversion (OLS) (1)

Given observed response D at depth $z = z_r$, estimate $v(p, z)$ from

$$\min_v J_{OLS}(v, D) := \frac{1}{2} \|S_\omega[v] - D\|^2$$

(some regularization usually needed).

Only Newton and its relatives computationally feasible

Upshot of extensive study from 80's on:

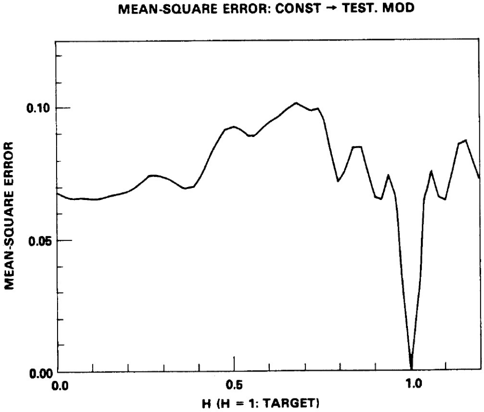
OLS reflects almost any physics of seismic wave propagation.
But, Newton-like iteration doesn't work. (Gauthier 86, Kolb 86, Santosa & Symes 89, Bunks 95, Shin and Min 06, etc.)

Output Least Squares Inversion (OLS) (2)

Important reasons for the failure

(1) J_{OLS} has lots of spurious local minima

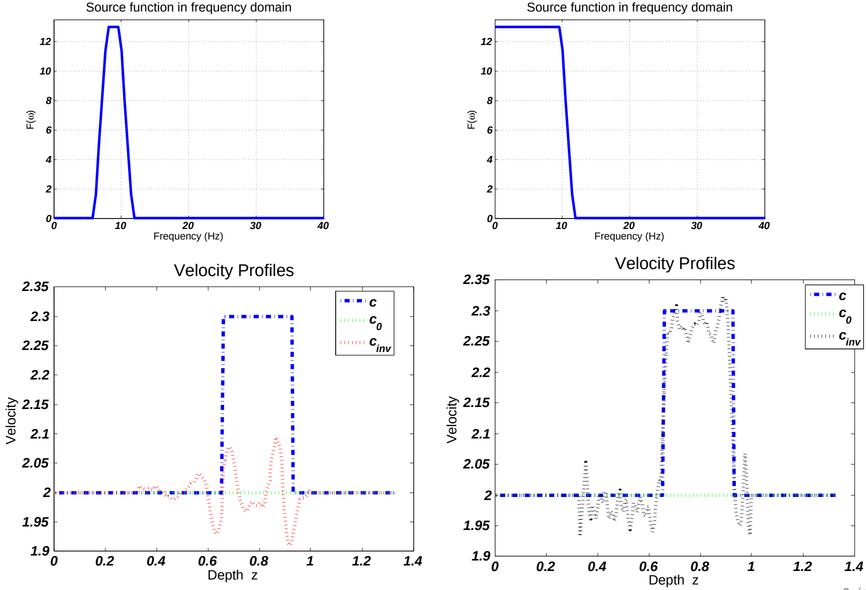
$\Longrightarrow$ Newton-like iteration stagnates at some local minimum far away from the global one.

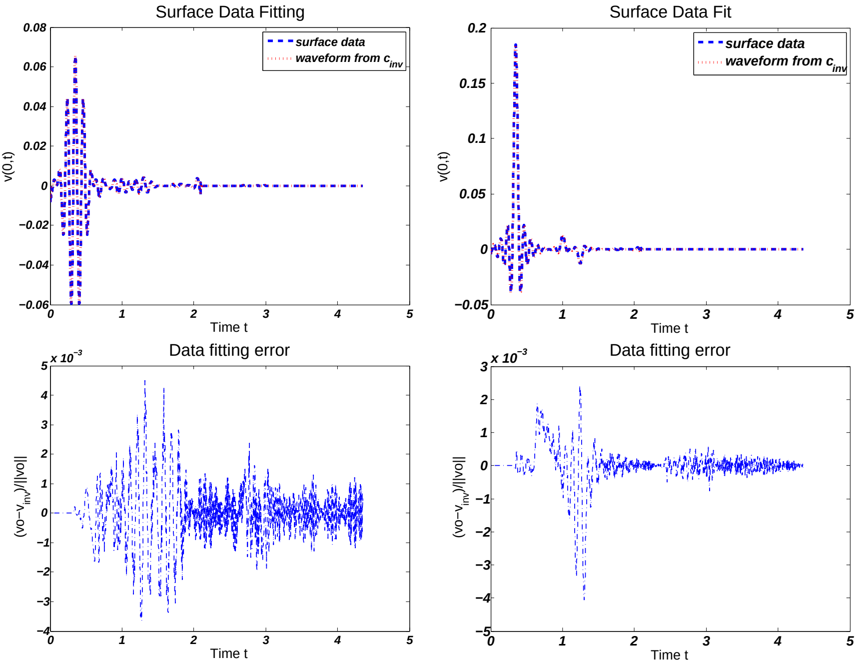


(Symes & Carazzone 92)

Output Least Squares Inversion (OLS) (3)

(2) Lack of low frequency energy. Simple examples in 1D




Surface Data Fitting
surface data
waveform from c_{inv}
v(0,t)
Time t
Surface Data Fit
surface data
waveform from c_{inv}
v(0,t)
Time t
Data fitting error
x 10^{-3}
$(vo-v_{inv})/||vo||$
Time t
Data fitting error
x 10^{-3}
$(vo-v_{inv})/||vo||$
Time t

Solution Strategy: Nonlinear DS

$\min_v \frac{1}{2} \|S_\omega[v(p,z)] - D\|^2$

Key Observation — low-frequency source and data components ω_l, d_l

$S_{\omega+\omega_l}[v] = D + d_l$

unique v

Coherency Condition: $\frac{\partial c}{\partial p} = 0$

(The earth is unique!)

New Objective:

$$\min_{d_l} \quad \left\| \frac{\partial c}{\partial p} \right\|^2$$
$$s.t. \quad \|S_{\omega+\omega_l}[v] - (D + d_l)\|^2 = 0$$

Nonlinear DS, Component I: Simulation (1)

Numerical Scheme

Solve

$$\left(\frac{1}{v^2(p,z)}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right) U(p,z,t) = \omega(t)\delta(z),$$

plus initial and boundary conditions.

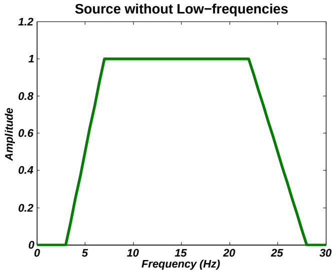
Let $W = \frac{\partial U}{\partial t}$, $\sigma = \frac{\partial U}{\partial z}$, then

$$\begin{cases} \frac{1}{v^2}\frac{\partial W}{\partial t} = \frac{\partial \sigma}{\partial z} + \omega(t)\delta(z) \\ \frac{\partial W}{\partial z} = \frac{\partial \sigma}{\partial t} \end{cases} .$$

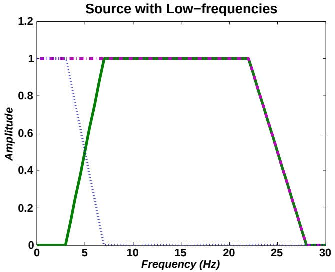
Method: Staggered Grid Finite Difference Scheme (J. Virieux 84 & 86)

Nonlinear DS, Component I: Simulation (2)

Produce Data



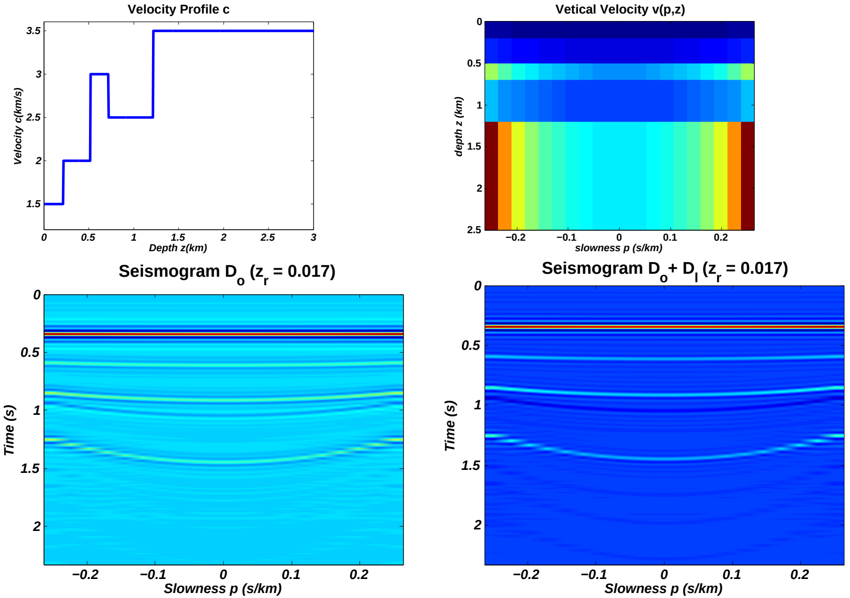
$\longrightarrow D_o$ (`Synthetic Data`)



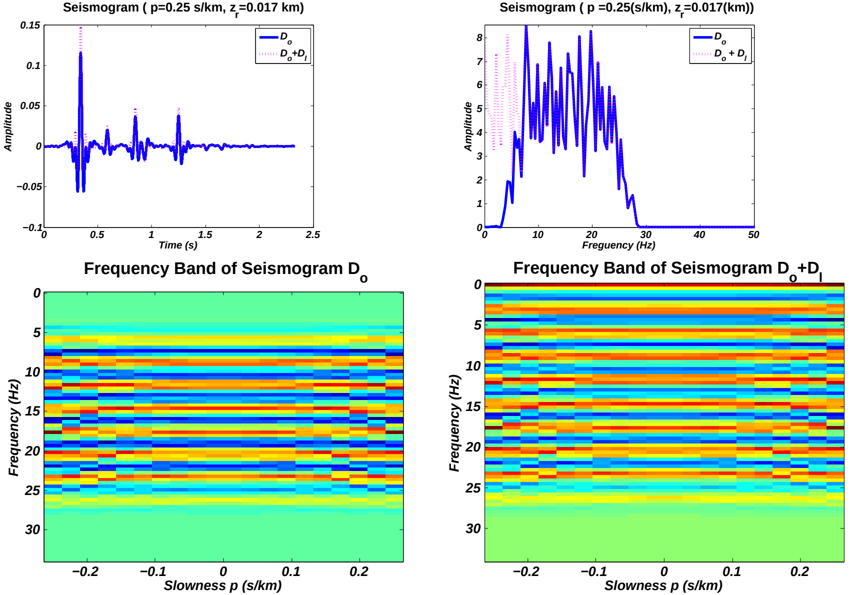
$\longrightarrow D_o + D_l$

(D_l : `low-frequency components`)

Nonlinear DS, Component I: Simulation (3)



Nonlinear DS, Component I: Simulation (4)



Nonlinear DS, Component II: Inversion

Inverse Problem: Given synthetic data $D_o \xrightarrow{DSO} v(p,z) \longrightarrow c(z)$

Differential Semblance Optimization (DSO) Problem:

$$\begin{array}{ll} \min_v & \|Q(v)\|^2 \\ s.t. & S_{\omega+\omega_l}[v] \simeq D_o + d_l, \end{array}$$

where $Q(v) := \frac{\partial c}{\partial p}$, d_l : low-frequency controls, and

$$S_{\omega+\omega_l}[v] \simeq D_o + d_l \xrightarrow{LS} \texttt{Unique } v.$$

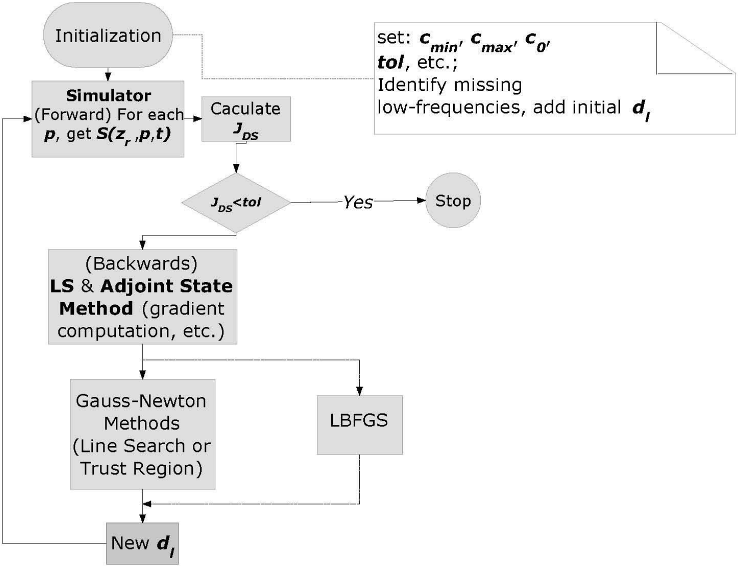
Therefor, DSO becomes

$$\min_{d_l} \|Q(v[d_l])\|^2 .$$

Need to compute : $\nabla_{d_l}\left(\|Q(v)\|^2\right)$, Hessian-vector product.
(in Appendix)

Nonlinear DS, Summary

How does this DSO work?



Expected Result and Future Work

$$\left.\begin{array}{l}\texttt{Any initial guess}\\ \texttt{Band-limited data}\end{array}\right\} \xrightarrow[\texttt{Layered medium}]{\texttt{Non-linear DSO}} \texttt{Global minimum}$$

Current Plan: finish this DS implementation

Future work : extend this implementation to more complex medium

Thank You!

Expected Result and Future Work

$$\left.\begin{array}{l}\texttt{Any initial guess}\\ \texttt{Band-limited data}\end{array}\right\} \xrightarrow[\texttt{Layered medium}]{\texttt{Non-linear DSO}} \texttt{Global minimum}$$

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Thank You!

Gradient Computation 1 – Adjoint State Method

Let

$$g(v, d_l(\eta)) := \frac{1}{2} \| S_{\omega+\omega_l}[v] - D_o - d_l \|$$

$$f(v) := \| Q(v) \|^2, \ \hat{f}(d_l(\eta)) := f(v(d_l(\eta))).$$

Then

$$\nabla_\eta \hat{f}(d_l(\eta)) = (d_l)_\eta^T v_{d_l}^T \nabla_v f(v(d_l(\eta))),$$

and

$$g_v v_{d_l} (d_l)_\eta + g_{d_l} (d_l)_\eta = 0.$$

Hence

$$v_{d_l} (d_l)_\eta = -g_v^\dagger g_{D-l}.$$

Therefor

$$\nabla_\eta \hat{f}(d_l(\eta)) = -(d_l)_\eta^T g_{d_l}^T (g_v^\dagger)^T \nabla_v f(v(d_l(\eta))).$$

Gradient Computation 2 – Adjoint State Method

Let $h(U, v) = 0$ denote the discretized wave equation, and

$$(S_{\omega+\omega_l}[v])_v = MU_v,$$

where M is the sampling operator.

Then

$$h_U U_v + h_v = 0,$$

and

$$Uv = -h_U^{-1} h_v.$$

Since

$$\begin{aligned} g_v &= \|S_\omega[v(p,z)] - D - d_l(\eta)\| \, (S_{\omega+\omega_l}[v])_v \\ g_{d_l} &= -\|S_\omega[v(p,z)] - D - d_l(\eta)\| \, I \end{aligned}$$

Then

$$v_{d_l}(d_l)_\eta = (S_{\omega+\omega_l}[v])_v^\dagger.$$

Finally

$$\nabla \hat{f}(d_l) = [(Mh_U{}^{-1} h_v)^\dagger]^T \nabla_v f(v(d_l(\eta))).$$