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# Interval velocity estimation via NMO-based Differential Semblance

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# Highlights

- Classical Semblance is equivalent to least squares data fitting and has local maxima.
- All stationary points of Differential Semblance are global minimizers.
- A recent approach to Differential Semblance has some numerical problems. I proposed an alternative approach to overcome these difficulties.

# The Simplest Acoustic Model

The acoustic wave equation where density is considered constant and equal to one, with a point source:

$$\frac{1}{c^2(x)}\frac{\partial^2 p}{\partial t^2}(x,t;x_s) - \nabla^2 p(x,t;x_s) = f(t)\delta(x-x_s)$$

$x$ is the position vector
$x_s$ is the position of the point source
$f(t)$ is the source time function
$c(x)$ is the particle velocity
$p(x,t;x_s)$ is the pressure

Forward map: $S[c] = p|_{Y=(x_r,t;x_s)}$ (predicted seismic data)
$x_r$ is the receiver position and $x_s$ is the source position.

Inverse problem: given observed seismic data $d$, find velocity field $c$ so that

$$S[c] \simeq d$$

The inverse problem is large scale and nonlinear.

# Linearization

Write $c = v(1+r)$, then $\delta p(x,t;x_s)$ satisfies

$$\frac{1}{v^2(x)}\frac{\partial^2 \delta p}{\partial t^2}(x,t,x_s) - \nabla^2 \delta p(x,t,x_s) = \frac{2r(x)}{v^2(x)}\frac{\partial^2 p}{\partial t^2}(x,t,x_s)$$

Linearized forward map: $F[v]r = \delta p|_{Y=(x_r,t;x_s)}$

- $v$ smooth, $r$ oscillatory $\Rightarrow F[v]r$ approximates primary reflections
- Error consists of multiple reflections.
- No mathematical results are known which justify these observations in any rigorous way.

# Convolutional model for layered media

(Theoretical derivation by Winslow 2000, based on linearization and high frequency approximation)

$$F[v]r(t,h) = f(t) *_t r(T_0(t,h))$$

$h$ is the half offset
$t_0$ is the traveltime at zero offset
$f(t)$ is the source time function
$r(t_0) = \frac{\delta v(t_0)}{v(t_0)}$
$T_0(t,h)$ is a change of variables function. It is the inverse function of $T(t_0,h)$ (Hyperbolic approximation to two-way traveltime)
Ideal case: $f(t) = \delta(t)$. Then

$$F[v]r(t,h) = r(T_0(t,h))$$

# Classical Semblance is equivalent to least squares data fitting

Turn the linearized inverse problem into a least squares problem: given CMP data $d$, find $v$, $r$ so that

$$min\ J[v,r] = \|F[v]r - d\|^2$$

$$\begin{aligned}
&= \int\int dt\ dh\ (r(T_0(t,h)) - d(t,h))^2 \\
&= \|d\|^2 + \int\int dt_0\ dh\ \frac{\partial T}{\partial t_0}(t_0,h) \times (r(t_0)^2 - 2r(t_0)d(T(t_0,h),h)) \\
&= \|d\|^2 + \int dt_0\ j(t_0)r(t_0)^2 - 2\int dt_0\ r(t_0)\int dh\ \frac{\partial T}{\partial t_0}(t_0,h) \times \\
&\quad \times d(T(t_0,h),h)
\end{aligned}$$

Then

$$J[v,r] = \| d \|^2 + < jr, r > - 2 < r, Sd >,$$

where $Sd$ is the weighted stacking

$$Sd[v](t_0) = \int dh \, \frac{\partial T}{\partial t_0}(t_0,h) d(T(t_0,h),h),$$

and

$$j[v](t_0) = \int dh \, \frac{\partial T}{\partial t_0}(t_0,h).$$

Since $Sd$, $j$ only depend on $v$, then if $v$ is fixed, we can get the optimal $r = \frac{1}{j} Sd$

$$min \ J[v,r] = \ \|d\|^2 - < \frac{1}{j} Sd, Sd >$$

$$\Longleftrightarrow max \ J_S[v] = < \frac{1}{j} Sd, Sd >$$

Then the classical semblance turns out to be equivalent to the least squares data fitting.

# Differential Semblance

Introduce nonphysical model $r(t_0, h)$. Physical model satisfies constraint $\frac{\partial r}{\partial h} = 0$.

$$
\begin{aligned}
min\ J[v, r] &= \int\int dt\ dh\ (r(T_0(t, h), h) - d(t, h))^2 \\
&= \int\int dt_0\ dh\ \frac{\partial T}{\partial t_0}(t_0, h)(r(t_0, h) - d(T(t_0, h), h))^2
\end{aligned}
$$

The objective function is very easy to minimize without constraint: $r(t_0, h) = d(T(t_0, h), h)$. Then the model is infeasible since $\left\| \frac{\partial r}{\partial h} \right\|^2 > 0$. To reduce the infeasibility: $min_v\ \left\| \frac{\partial r}{\partial h} \right\|^2$
Differential Semblance objective function is

$$
J_{DS}[v] = \left\| \frac{\partial}{\partial h} d(T(t_0, h), h) \right\|^2
$$

# Comparison between Classical Semblance and Differential Semblance

(a) Classical Semblance (Chauris, 2001):

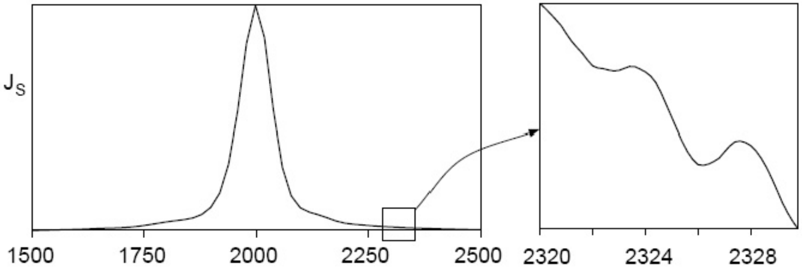


Figure: Classical Semblance cost function

(b) Differential Semblance (Chauris, 2001):

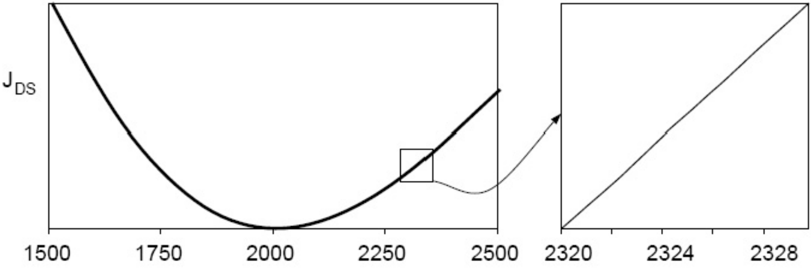


Figure: Differential Semblance cost function

# Motivation for DS

- DS
  - All stationary points of DS are global minimizers. (Symes, TR99-09)
  - DS uses gradient method to solve the optimization problem.
- OLS
  - Output Least squares objective function has local minima and these local minimizers are far from any global minimizer.
  - Gradient methods are unreliable.
  - Computational cost for global optimization methods is high.

A recent approach to DS (Jintan Li, 2007)

- Objective function:

$$J[v] = \left\| \frac{\partial}{\partial h} d(T(t_0, h), h) \right\|^2$$

- $J[v]$ and $\nabla J[v]$ have to be computed numerically. But grid points in the $t_0$ axis are not mapped to grid points in the $t$ axis.

$$d(t, h) \rightarrow d(T(t_0, h), h)$$

- Local cubic interpolation is needed to compute the oscillatory data $d$ which will cause error.

# Discretization

$t_{0j} = j\Delta t_0,\ h_i = i\Delta h,$

$$d(T(t_{0j}, h_i), h_i) \simeq d^{int}(T(t_{0j}, h_i), h_i)$$

$$\frac{\partial}{\partial h} d(T(t_{0j}, h_i), h_i) \simeq \frac{1}{\Delta h}(d^{int}(T(t_{0j}, h_{i+1}), h_{i+1}) - d^{int}(T(t_{0j}, h_i), h_i))$$

Define the discrete moveout derivative operator:

$$M[v]d(t_{0j}, h_i) = \frac{1}{\Delta h}(d^{int}(T(t_{0j}, h_{i+1}), h_{i+1}) - d^{int}(T(t_{0j}, h_i), h_i))$$

Thus the discrete objective function

$$J[v] = \sum_{ij} |M[v]d(t_{0j}, h_i)|^2$$

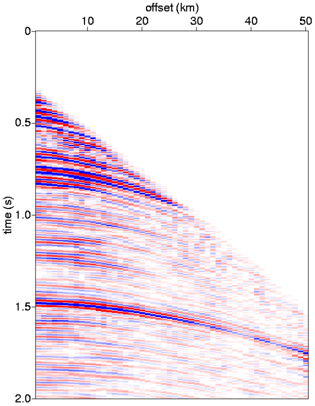


Figure: Original CDP

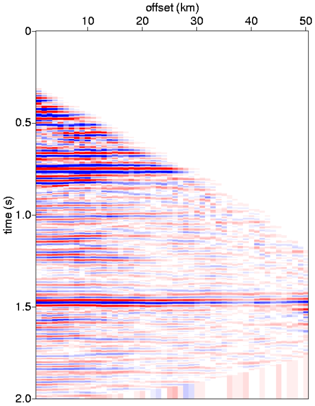


Figure: Corrected CDP

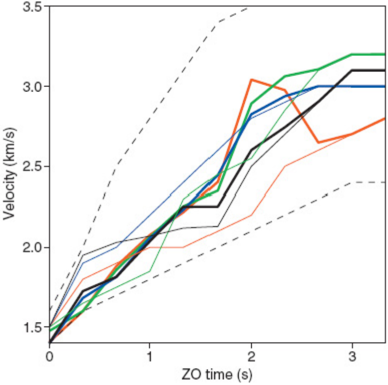


Figure: Instability of DSVA velocity estimates

Alternative approach to DS

$$J[v] = \left\| (p\frac{\partial d}{\partial t} + \frac{\partial d}{\partial h})(t, h) \right\|^2$$

where slowness

$$p(t, h) = \frac{\partial T}{\partial h}(T_0(t, h), h)$$

This approach involves interpolation of smooth function $p(t, h)$ instead of oscillatory data $d(t, h)$, then the interpolation error in $p$ is smaller than the previous approach. Then the noises in $J$ and $\nabla J$ are smaller. Thus this optimization is more stable.

Recall

$$J[v] = \left\| (p\frac{\partial d}{\partial t} + \frac{\partial d}{\partial h})(t,h) \right\|^2$$

Since $t$ is oversampled, we don't have problem in computing $\frac{\partial d}{\partial t}$.
Since offset $h$ is often undersampled, $\frac{\partial d}{\partial h}$ has to be calculated carefully.
How to deal with $\frac{\partial d}{\partial h}$?

# Proposed strategy

If $v_0 - \Delta v \leq v \leq v_0 + \Delta v$, then

$$p\frac{\partial d}{\partial t} - \frac{\partial d}{\partial h} \simeq N[v_0]d + (p - p_0)\frac{\partial d}{\partial t}$$

where operator $N$ has been defined by

$$N[v]d(t_j, h_i) = M[v]d(T_0(t_j, h_i), h_i)$$

Then

$$(p\frac{\partial d}{\partial t} - \frac{\partial d}{\partial h})(t_j, h_i) \simeq N[v_0]d(t_j, h_i) + (p - p_0)\frac{\partial d}{\partial t}(t_j, h_i)$$

This will be accurate if $f_{max} \leq f(\Delta v)$

Summary:

- CS vs. DS
- Recent approach vs. my approach

Future works:

- Justify the proposed strategy.
- Implement the algorithm

Thank you