# Practice and Pitfalls in NMO-based Differential Semblance Velocity Analysis

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#### SUMMARY

Differential semblance provides an automated alternative to commonly used velocity analysis methods. For data exhibiting low structural relief, an implementation based on hyperbolic moveout is fast enough for 3D velocity estimation on a workstation. Application of this technique to a time processed land 3D dataset from the onshore Gulf of Mexico, demonstrates this capability. Compared to more convention methods of velocity analysis, it produced reliable results in very short order.

#### INTRODUCTION

Differential semblance velocity analysis ("DSVA") is an automatic velocity estimation technique, based on the observation that nearby traces in an adequately sampled image gather are non-aliased: their sample-wise difference is proportional to their phase difference. Thus these neighboring trace differences directly represent traveltime error information, which may be used to construct a velocity update. This idea is also used in depth-domain stereotomography (Chauris et al., 2004).

Virtually any method of constructing prestack image gathers lead to variants of DSVA. Versions based on plane-wave asymptotic acoustic and elastic layered inversion (Symes and Carazzone, 1991; Minkoff and Symes, 1997), 2D Kirchhoff migration (Chauris and Noble, 2001; Mulder and ten Kroode, 2002), and anisotropic elastic Kirchhoff migration/inversion (Foss et al., 2005) have been used to process field data on an experimental basis. However, to our knowledge, no previously reported work on this technique has explored the possibility of using DSVA in a production setting.

The work reported here represents some initial steps towards such an evaluation, using the simplest version of DSVA applicable to field reflection data. The imaging engine for this DSVA variant is hyperbolic NMO correction. NMO-based DSVA has good theoretical properties (Symes, 1999, 2001; Stolk and Symes, 2003). Synthetic and and field data tests suggest that this algorithm is at least somewhat robust against mild violations of the theoretical assumptions (Gockenbach and Symes, 1999; Li and Symes, 2005). In particular, it appears able to estimate reasonably accurate interval velocities in the presence of mild lateral velocity variation. This algorithm is fully automatic, and careful implementations throughputs upwards of 100 traces / s on modest workstations. Processor guidance is possible through specification of upper and lower interval velocities.

Since DSVA detects moveout, the presence of coherent noise, for example from multiple reflections, can strongly influence its output. Gockenbach and Symes (1999) suggested a compute-intensive extension to the basic DSVA algorithm which aids in coherent noise suppression. This report presents another, much simpler approach, originally suggested by Mulder and ten Kroode (2002). We show that this simpler method, incorporating coherent noise suppression into velocity analysis, can also be automated to some extent.

#### DESCRIPTION OF THE ALGORITHM

Both algorithm and its implementation accommodate 3D survey geometry and velocity models. For notational convenience and because the examples presented later in the paper are 2D only, we will describe the 2D specialization explicitly. The extension to 3D is straightforward.

Data sorted into CDP bins will be denoted d(m, h, t), *m* denoting midpoint, *h* half-offset. The hyperbolic traveltime approximation

$$t(t_0,h) = \sqrt{t_0^2 + \frac{4h^2}{v_{\text{RMS}}^2(m,t_0)}}$$

leads to the hyperbolic NMO transformation  $d \rightarrow r$ ,

$$r(m, h, t_0) = d(m, h, t(m, h, t_0))$$

The RMS velocity  $v_{\text{RMS}}(m, t_0)$  is in turn a function of interval velocity  $v(m, t_0)$ :  $v_{\text{RMS}} = v_{\text{RMS}}[v]$ .

In principle it is possible to scale the expression for the reflectivity by an approximate ray-trace amplitude. In practice, reasonable results are obtained by preprocess scaling, for example by AGC.

If v, hence  $v_{\text{RMS}}[v]$ , is kinematically consistent with the data, then the image gathers should be (to some approximation) independent of h, at least in terms of phase. Thus the function

$$J_{\rm DS}[v;d] \equiv \frac{1}{2} \sum_{m,h,t_0} \left| \frac{\partial r}{\partial h} \right|^2$$

should be at or near its minimum as a function of v when v is correct. Numerical optimization of this function of the parameters of v is the DSVA algorithm.

 $J_{\text{DS}}$  is smooth as a function of velocity and data, in a reasonable sense in fact it is unique (up to inessential modifications) amongst all quadratic forms in the image volume in having this property (Stolk and Symes, 2003). Moreover this NMO-based variant has at least in theory no spurious critical points: all minima are global minima, to good approximation (Symes, 1999, 2001). Therefore gradient-based optimization should be effective. The gradient is given by

$$\nabla_{\nu} J_{\text{DS}}[\nu;d] = -D_{\nu} \nu_{\text{RMS}}[\nu]^* \left[ \sum_{h} 8h^2 t(\cdot,h)^{-1} \nu_{\text{RMS}}[\nu](\cdot,\cdot)^3 \right]$$
$$\times \frac{\partial}{\partial h} (d(\cdot,h,t(\cdot,h))) \frac{\partial}{\partial h} \left( \frac{\partial d}{\partial t}(\cdot,h,t(\cdot,h)) \right)$$

In this expression,  $D_{\nu}v_{\text{RMS}}[\nu]$  denotes the Jacobian or derivative of the RMS velocity as a function of the interval velocity, and the asterisk denotes the adjoint or transpose of this operator.

Implementation of these formulae with sampled data requires that the change of variables  $t \rightarrow t_0$  be approximated by interpolation. We have found piecewise cubic interpolation to be adequately smooth for this purpose. The *t*- derivative appearing in the expression for the adjoint is conveniently approximated by differentiating the interpolant. We have used a first-order divided difference to approximate the derivative in *h*, interpreting this as a second order derivative approximate at the midpoint of the offset interval. The computation of  $d \rightarrow r$ ,  $J_{\text{DS}}$  and  $\nabla_v J_{\text{DS}}$  can be combined in one nested loop in the order (inner to outer)  $t_0, h, m$ , and computed in a single pass through the data. If the data is presorted into increasing offset within each CDP, then only one trace of workspace (to hold the previous moved out trace) is required.

# **Differential Semblance**

Midpoints and (3D, absolute) offsets are read from trace headers, and cannot be assumed to define a regular grid. In particular the implementation must include a means to extract interval velocity as a function of  $t_0$  for any midpoint *m*. We defined the interval velocity by piecewise multilinear interpolation of nodal values defined on a regular coarse grid in 1D, 2D, or 3D, with constant extrapolation to locations outside the grid. Piecewise linear interpolation has the advantage of *convexity*: if two piecewise linear velocities have ordered node values, i.e. those for the first are all less than those of the second the same will be true of the interpolants at any point. Higher order spline interpolants do not have this convexity property.

The function value and gradient computations are input to an optimization algorithm. We use the Limited Memory BFGS ("LBFGS") algorithm of Nocedal Nocedal and Wright (1999), a quasi-Newton algorithm with a reputation for reasonable reliability and convergence in a wide variety of problems. The implementation used in our work is an application of the Rice Vector Library ("RVL"), an object-oriented framework for coupling simulation and optimization Symes et al. (2005). RVL also defines appropriate interfaces through which to pass function and gradient results.

#### MULTIPLE SUPPRESSION VIA MOVEOUT DISCRIMINATION

Figure 1. shows a CDP gather taken from a land 3D Kirchhoff prestack time migrated survey on the left. DSVA produced the image gather in the left center panel, which shows the typical behavior of this algorithm when moveout conflicts exist in data: it attempts to flatten every event, but since this is impossible it compromises, undercorrecting the slow events and overcorrecting the fast ones.

The hypothesis that the slow events are coherent noise (multiple reflections) suggests a strategy for removing them: (1) dip filter the image gathers to remove all events with positive slope; (2) inverse nmocorrect this filtered data to produce a "sanitized" data set; (3) apply DSVA to the "sanitized" data. A similar concept has been suggested by Mulder and ten Kroode (2002), who did not however construct an automated cycle like that suggested here.

The right center figure shows the image gather resulting from one cycle of this process. The overall flatness is improved noticeably. The rightmost figure shows the DSVA interval velocity estimates for this CDP, with the original DSVA velocity in the dashed line, the velocity based on the "sanitized" data in the solid line. Clearly in critical places the removal of slow apparent moveout has resulted in a higher velocity estimate, most likely closer to the truth. For example the event at about 2.7 s appears flat in the unfiltered image gather. Carrying out one step of the cycle reveals that it is actually slower than a suite of nearby events which are flattened by the faster velocity resulting from removing yet slower events.

We used the Seismic Unix (Stockwell, 2001) utility sudipfilt to carry out the crucial filtering step in the above cycle. Since the undesirable dips can only be suppressed, not eliminated entirely, this process is iterative. Each step is entirely automatic. Some supervision is required, for example in selection of parameters for the moveout rejection filter. This experiment revealed another potential pitfall: this data is contaminated by high apparent velocity events which are difficult to discern before filtering away the slow events, and which are to some degree balanced by those slow events in the velocity analysis. This serendipitous balance is destroyed when the slow events are removed. To avoid gross overestimation of the velocity these very fast events must also be suppressed, and that has been done in the example presented above.

## POST-MIGRATION VELOCITY ANALYSIS

On of our primary goals was to evaluate DSVA as a method for flattening gathers in preparation for post-migration AVO analysis. Our approach to the problem was to start with CDP gathers that had been time migrated and were as well imaged as time processing would allow. We used gathers both with and without post-migration noise removal (typically Radon for multiple removal). Since the algorithm required an initial guess, a single interval velocity function v(z) was provided. In addition to the initial guess for the velocity, upper and lower bounds for the velocity were also provided.

While we were not restricted to a single function by the algorithm, we chose not to make the initial guess nor the upper and lower bounds space varying. This was made just to simplify the setup.

Our comparison is with closely spaced handpicked velocities. At the location of our example Figure 2. shows the interval velocity derived from the conventional velocity analysis. On the left of the same figure is the initial velocity and the interval velocities derived from the DSVA. Notice that while the starting velocity function was reasonable, it did not have to be close to the final estimate for a reasonable result. The DSVA output resembles the conventional result rather closely.

When the DSVA velocities are used for NMO correction, the resulting gathers are nicely flattened and again compare well with the gathers corrected with a conventionally derived velocity function (Figures 3 and 4).

The biggest difference is within the mute in the shallow section. Apparently reduction in offsets does not provide control for DSVA to completely flatten the events. Obviously the conventional approach does a better job in this zone. The results do improve if the initial velocity provided to DSVA is closer to the velocity required to flatten the shallow events.

## CONCLUSIONS

Using DSVA compares quite well with conventional velocity analysis for flattening CDP gathers in time. The method works quite well where there is full fold and a reasonable amount of moveout to the far offsets. In problem areas starting with a better initial estimate helps to produce flat events.

Because of its speed and the way it treats conflicting moveout, the DSVA has a potential role in automatic multiple removal where there is differential moveout between multiples and primaries.

With additional work, DSVA can become a very useful seismic processing tool.

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## **Differential Semblance**



Figure 1: Illustration of the invert - dipfilter - resimulate cycle for suppression of slow coherent noise. Left: input muted CDP. Left Center: result of DSVA. Right Center: result of dipfilter - inverse NMO -DSVA cycle applied to gather in Left Center. Right: velocities corresponding to Left Center (dashed) and Right Center (solid).

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Figure 2: Interval velocities derived from the analysis. On the left, in blue, is the interval velocity function from a conventional velocity analysis. On the right shows the initial velocity, red, and the resulting interval velocities from the DSVA, black.

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**Conventional Velocity Analysis** 



Figure 3: CDP gathers with NMO correction from conventional velocity analysis.

Figure 4: CDP gathers with NMO correction based on DSVA.