Interval velocity estimation via Kirchhoff-based differential semblance

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Introduction

Velocity analysis updates velocity parameters to flatten primary reflections in image gathers.

Differential semblance velocity analysis (DSVA) optimizes a differential measure of gather flatness:

- mean square of all differences between *neighboring* traces in prestack image volume (DS). (so tries to flatten everything !)

Theory:

- DS is *unique* velocity-dependent quadratic form in prestack image volume, which is *smooth in data and velocity* (Kim & WWS 98, Stolk & WWS 03)
- For version based on NMO, *all stationary points are global minima* for noise-free *primaries only* data (WWS 01)

Versions of DSVA

- Hyperbolic or ray-trace NMO correction in CMP and plane wave domains (Carazzone & S. 91, Minkoff & WWS. 97, S. 98)
- Two way reverse time (Versteeg & WWS. 93, Kern & WWS. 94)
- Prestack Kirchhoff migration (Chauris & Noble 01, Mulder & ten Kroode 01, Brandsberg-Dahl & deHoop 03)
- Shot profile and DSR migration (Shen et al. 03, Foss et al. 04)

For reliable results, all variants require

- suppression of coherent noise (multiples, unmodeled converted waves,...)
- velocity models compatible with imaging engine (usually: nonreflecting)

Part I : NMO-based DSVA

When:

- lateral heterogeneity is weak
- coherent noise (eg. multiple reflection) can be suppressed

NMO-based DSVA interval velocity estimation is:

- kinematically accurate
- robust
- fast (100+ traces $/s$ on modest workstation)

NMO-based DSVA

Simplest possible prestack imaging engine $\Rightarrow$ *fastest* but accurate only for slow lateral variation of velocity and reflector structure.

Our implementation:

- standard data structure (SEGY) for data input and diagnostic output
- standard preprocessing: filter, mute, various noise suppression methods
- flexible velocity modeling accommodating 1D, 2D, and 3D variation, but enforcing smoothness and upper and lower velocity envelopes(use ”PIGrid”, see TRIP 2005 report)
- effective numerical optimization (limited memory BFGS)

NMO and Derivatives

NMO-based DSVA requires:

- NMO transformation - can get this anywhere (eg. SU)
- *Derivative* and *adjoint derivative* of the NMO transformation with respect to velocity
- Implemented hyperbolic moveout using local polynomial interpolation, degree ≥ 3 to assure sufficiently smooth response to velocity perturbation

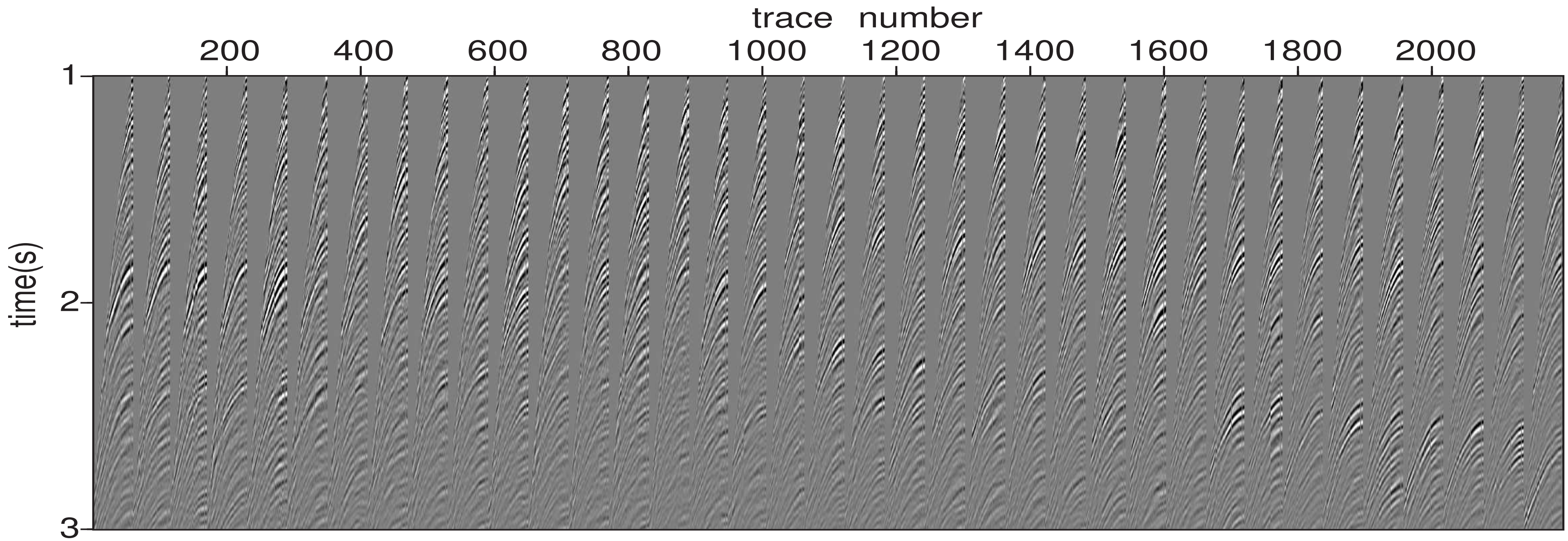
Adaptive Choice of Velocity Model Dimension

- First pass: find optimal 1D model. If the gathers are flattened adequately, end; otherwise go to second pass

- Second pass: find optimal 2D model. If the gathers are flattened adequately, end; otherwise go to third pass.

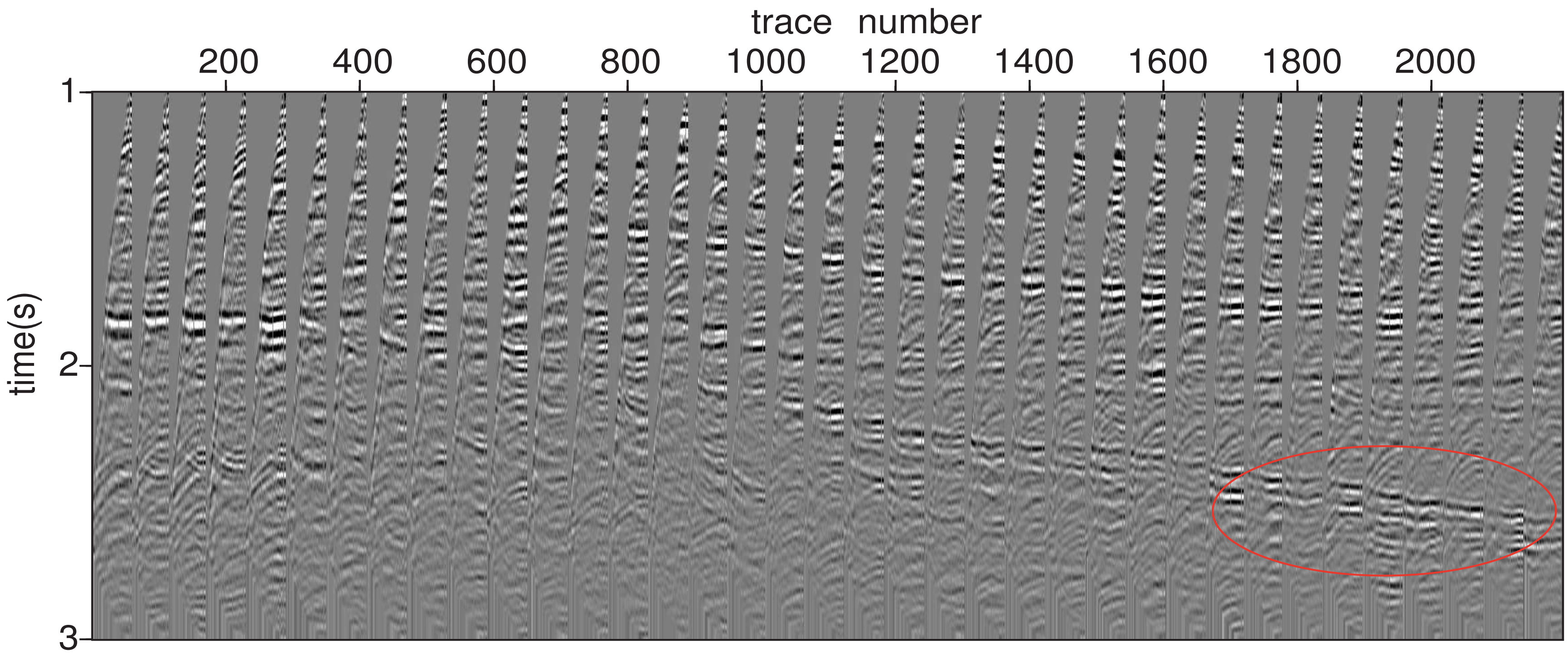
- Third pass: find optimal 3D model, end.

Example

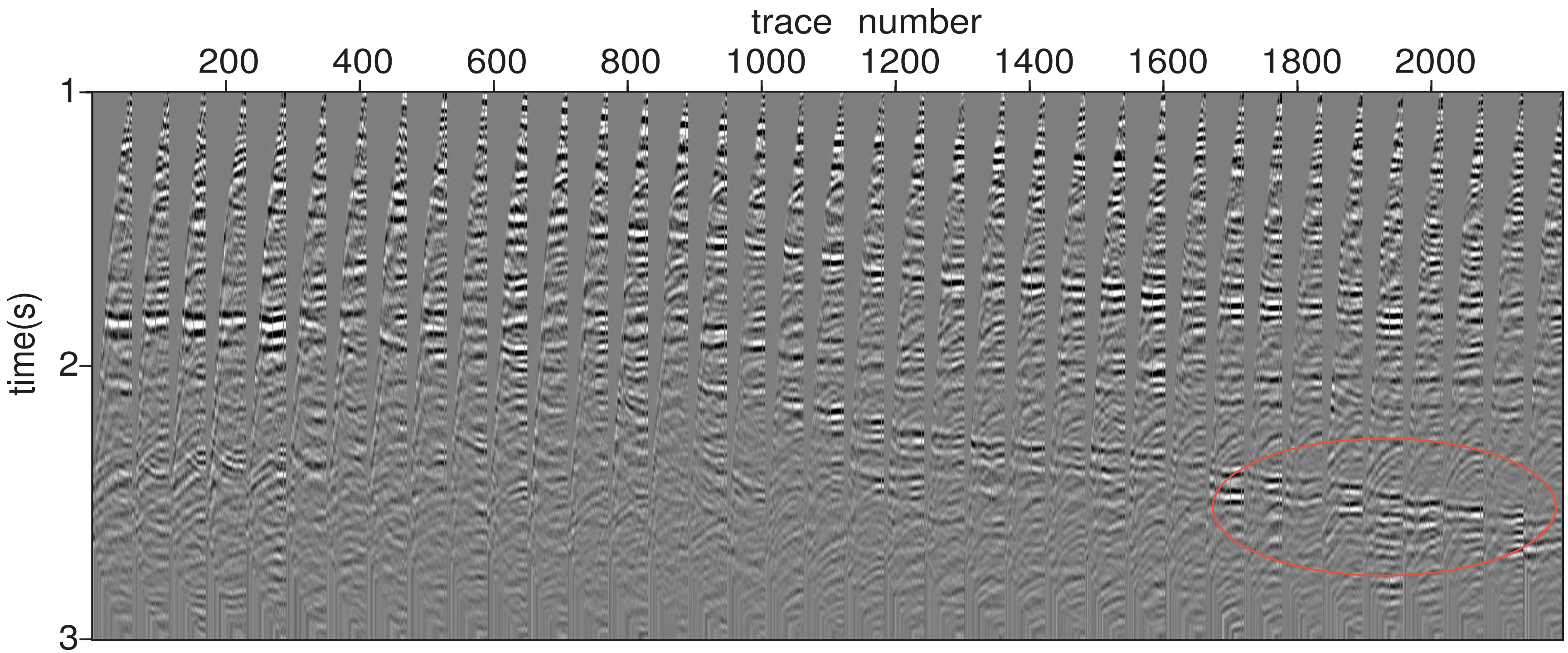


38 CMPs extracted from the **Viking Graben** data set. Preprocessing: *hyperbolic radon transform multiple suppression*, zero phase band pass filter, mute. (Keys and Foster,1996)

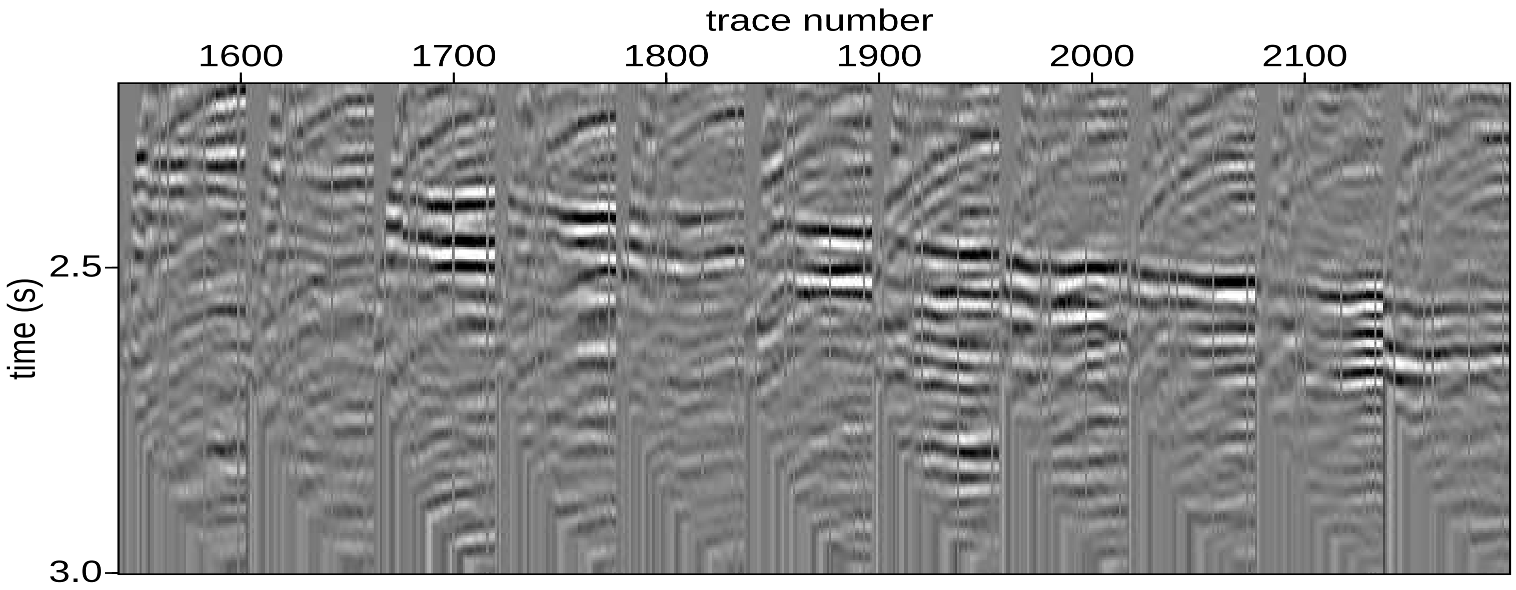
NMO corrected gathers after DSVA, $v(z)$:



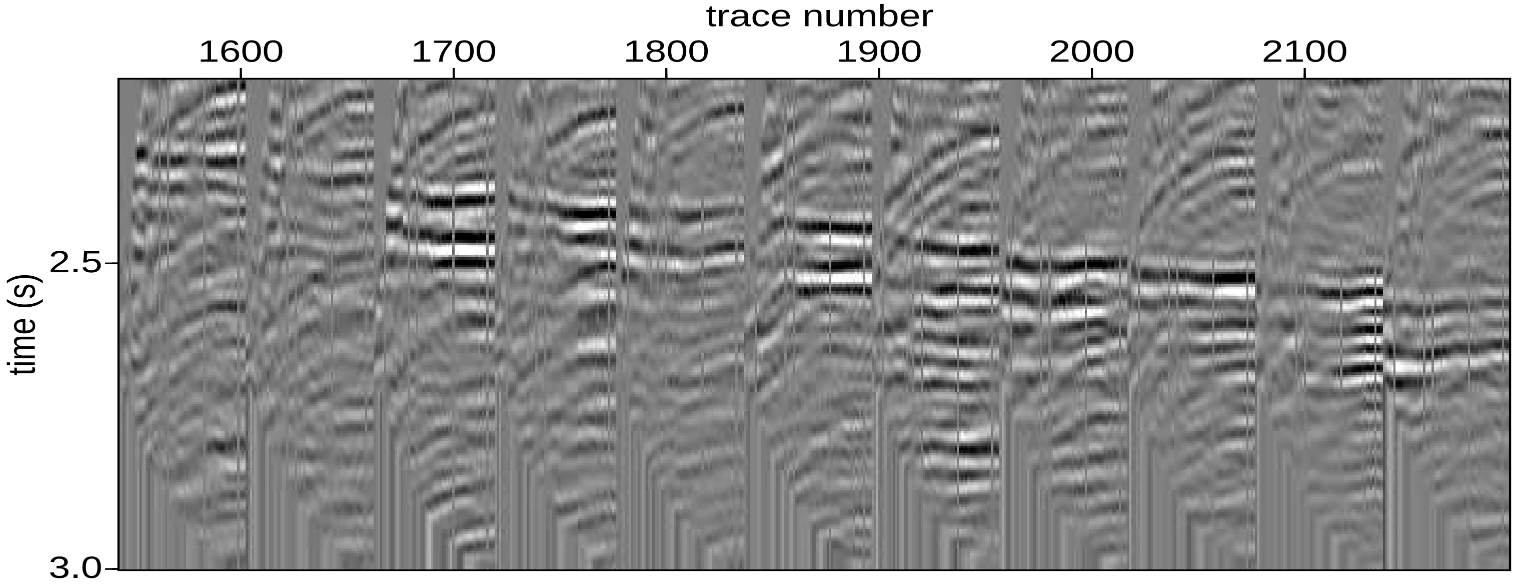
NMO corrected gathers after DSVA, $v(x,z)$:



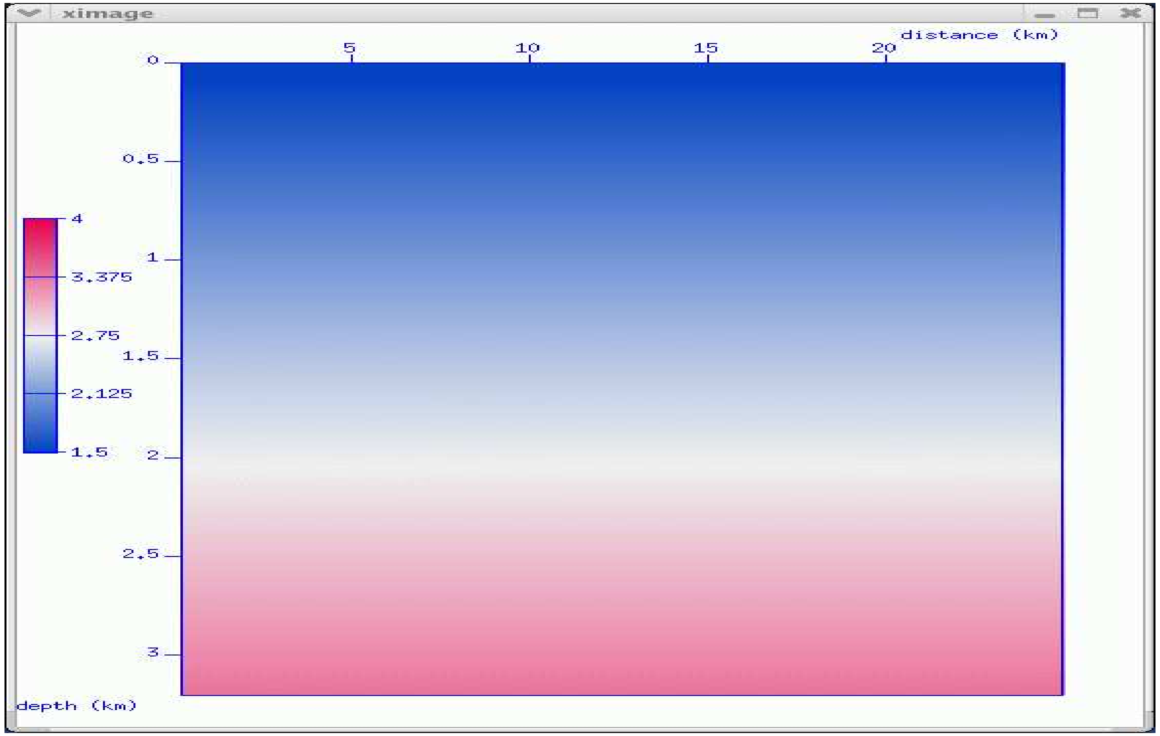
Detail: gathers from $v(z)$ DSVA



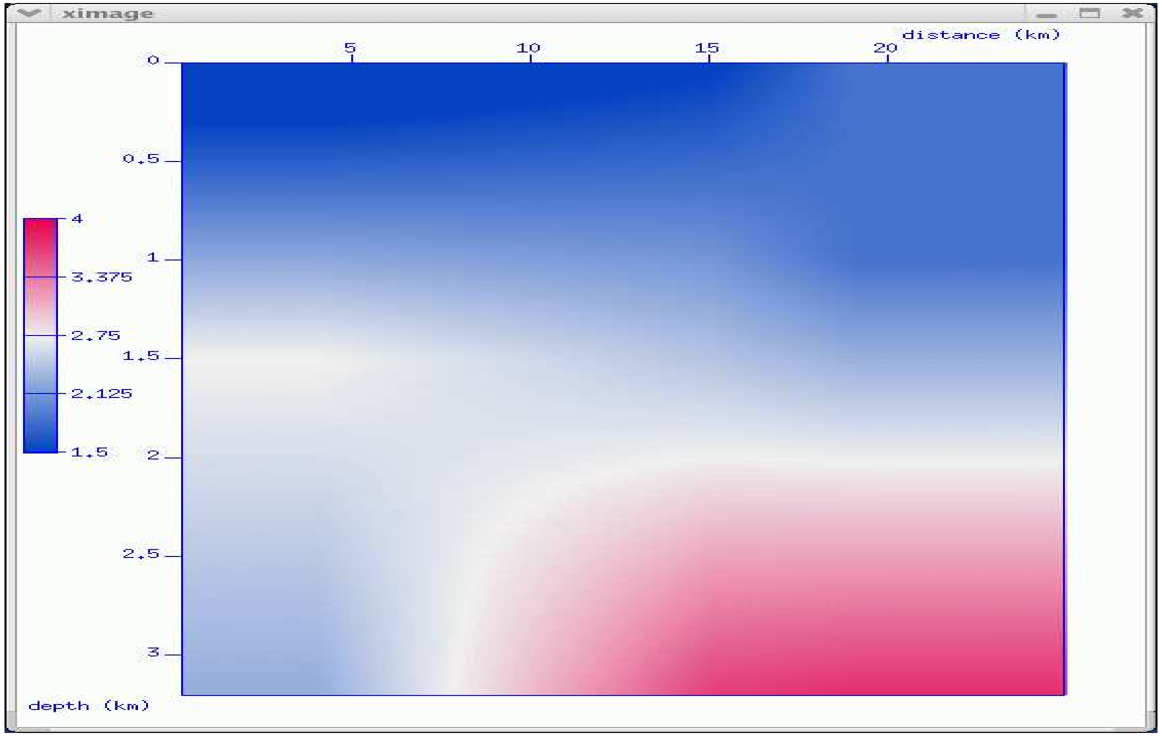
Detail: gathers from $v(x, z)$ DSVA



DSVA : $v(z)$ Interval velocities



DSVA : $v(x, z)$ Interval velocities



Conclusion for Part I

- For regions of mild lateral heterogeneity and data from which coherent noise can be suppressed, NMO-based DSVA may give useful, fast, and robust initial velocity estimates

- Coherent noise (eg. multiple reflections) degrades accuracy - for better automatic VA, such noise must be either suppressed from data or included in VA wave propagation model

Part II : Kirchhoff migration DSVA

Kirchhoff migration:

- A high-frequency approximation of the wave equation
- Image grid can be arbitrarily specified
- Can yield good and reasonable results except when the multipathing occurs
- Low computational cost

Part II: Kirchhoff migration

The image is computed by:

$$r_{[v]}(x,h) = \int dx_s d_{[v]}(x_s, x_r, T_{[v]}(x_s, x) + T_{[v]}(x, x_r)) * A_{[v]}(x_s, h...) \qquad (1)$$

$T_{[v]}(x_s, x)$: travel time from source to the reflection point;
$T_{[v]}(x, x_r)$: travel time from reflection point to receiver

$A_{[v]}(x_s, h...)$: the specified amplitude, which can vary under different circumstances

Differential semblance

The DS objective is:

$$J[v,d] = \frac{1}{2}\sum \left|\frac{\partial r}{\partial h}\right|^2 = \frac{1}{2}\sum_{i=0}^{Nh-1} \langle \frac{r_{i+1}-r_i}{\triangle h}, \frac{r_{i+1}-r_i}{\triangle h}\rangle \tag{2}$$

(image: $r_i = r(h_i, \widetilde{x})$, offset: $h_i = h_0 + i\triangle h$)

hence

$$\delta J = \sum_{i=0}^{Nh-1} \langle \frac{r_{i+1}-r_i}{\triangle h}, \frac{\delta r_{i+1}-\delta r_i}{\triangle h}\rangle \tag{3}$$

let

$$f_i = \frac{r_{i+1}-r_i}{\triangle h} \tag{4}$$

$$
\begin{aligned}
\delta J &= \sum_{i=0}^{N_h-1} \langle f_i, \frac{\delta r_{i+1}}{\triangle h} \rangle - \sum_{i=0}^{Nh-1} \langle f_i, \frac{\delta r_i}{\triangle h} \rangle \\
&= \langle f_{Nh-1}, \delta r_{Nh} \rangle - \langle f_0, \delta r_0 \rangle + \sum_{i=1}^{Nh-1} \langle \frac{f_{i-1} - f_i}{\triangle h}, \delta r_i \rangle \\
&= \sum_{i=1}^{Nh-1} \langle \frac{\frac{r_i - r_{i-1}}{\triangle h} - \frac{r_{i+1} - r_i}{\triangle h}}{\triangle h}, \delta r_i \rangle + \langle f_{Nh-1}, \delta r_{Nh} \rangle - \langle f_0, \delta r_0 \rangle \\
&= \sum_{i=1}^{N_h-1} \langle \frac{2r_i - r_{i+1} - r_{i-1}}{\triangle h^2}, \delta r_i \rangle + \langle f_{Nh-1}, \delta r_{Nh} \rangle - \langle f_0, \delta r_0 \rangle
\end{aligned}
\tag{5}
$$

here

$$
q(\widetilde{x}) = \begin{cases} f_0 & i = 0 \\ \frac{f_{i-1} - f_i}{\triangle h} & i = 1, \ldots N_h - 1 \\ f_{N_h-1} & i = N_n \end{cases} \tag{6}
$$

The derivative of the image

The derivative of the image is:

$$\delta r_{[v]}(\widetilde{x}, h) = \int dx_s \frac{\partial d}{\partial t}(x_r, x_s, T_{[v]}(x_s, \widetilde{x})_+ T_{[v]}(x_r, \widetilde{x})) * (\delta T_{[v]}(x_s, \widetilde{x}) + \delta T_{[v]}(x_r, \widetilde{x})) \tag{7}$$

Bring δr into δJ, we get:

$$\begin{aligned} \delta J = &- \sum_{i=0}^{N_h - 1} (\sum_{\widetilde{x}} \sum_{x_s} q_i(\widetilde{x}) \frac{\partial d}{\partial t}(x_r, x_s, T_s + T_r)(\delta T(x, x_s) + \delta T(x_s + 2h, x)) \\ &= - \sum_{\widetilde{x}} g(\widetilde{x})(\delta T(x, x_s) + \delta T(x_s + 2h, x)) \\ &+ \text{boundary conditions} \end{aligned} \tag{8}$$

The gradient of the objective function

Find an operator $A(y)$ such that

$$\langle A(y)g, \delta v\rangle = \langle g(\widetilde{x}), \delta T(y, \widetilde{x})\rangle \tag{9}$$

which means

$$\sum_{\widetilde{x}} (A(y)g)(\widetilde{x})\delta v(\widetilde{x}) = \sum_{\widetilde{x}} g(\widetilde{x})\delta T(y, \widetilde{x}) \tag{10}$$

do this for $y = x_s$, $y = x_s + 2h$, then the gradient ojective is:

$$\nabla J[v, d] = \sum_{\widetilde{x}, h} (A(x_s)g)(x_s, \widetilde{x}, h) + (A(x_s + 2h)g)(\widetilde{x}, x_s + 2h, h) \tag{11}$$

Velocity representation

Possibilities:

- constant velocity (test first)
- piecewise linear
- B-splines
- PIGrid (”Partially Irregular Grid”)(see 2005 TRIP report)

Travel time and amplitude calculation

Travel time calculation:

Fast sweeping method for both time and adjoint calculation (see Jianliang Qian's talk)

Amplitude calculation: (later...)

Data preparation

- Coherent noise and direct waves should be sufficiently suppressed or muted

- The data is presorted by increasing 3D offset - will work for both 2D and 3D data with the header words critically defined (sx,gx,gelev,selev,offset....) (use suazimuth & susort)

Implementation

- Define a global image grid (NX,NY,NZ,OX,OY,OZ,dx,dy,dz)
- Loop over each trace in each common offset gather
  - Compute the travel time table and the amplitude on the global image grid
  - Choose the local grid within which all the traveltime $<=$ Max data time ($nt * ns + delrt$)
  - Calculate the image on this local grid
- Update the common offset image gather
- Compute the derivative of the image gather
- Compute the objective and the adjoint derivative of the objective

Acknowledgement

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