A sparse, bound-respecting parametrization of velocities

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Motivation

- The choice of model space to describe the background velocity is essential.
  In particular, the model should:
  - provide an accurate representation of the real medium, while being sparse, to avoid oversampling in areas where the velocity varies only slightly.
  - accomodate for explicit bounds on the velocity (e.g. stability issues in finite difference schemes).
  - be adequate for seismic processing: in practice, the velocity should be:
    * sampled on regular grids (e.g. finite difference schemes).
    * twice continuously differentiable (high-frequency asymptotics assumption).

Proposed solution

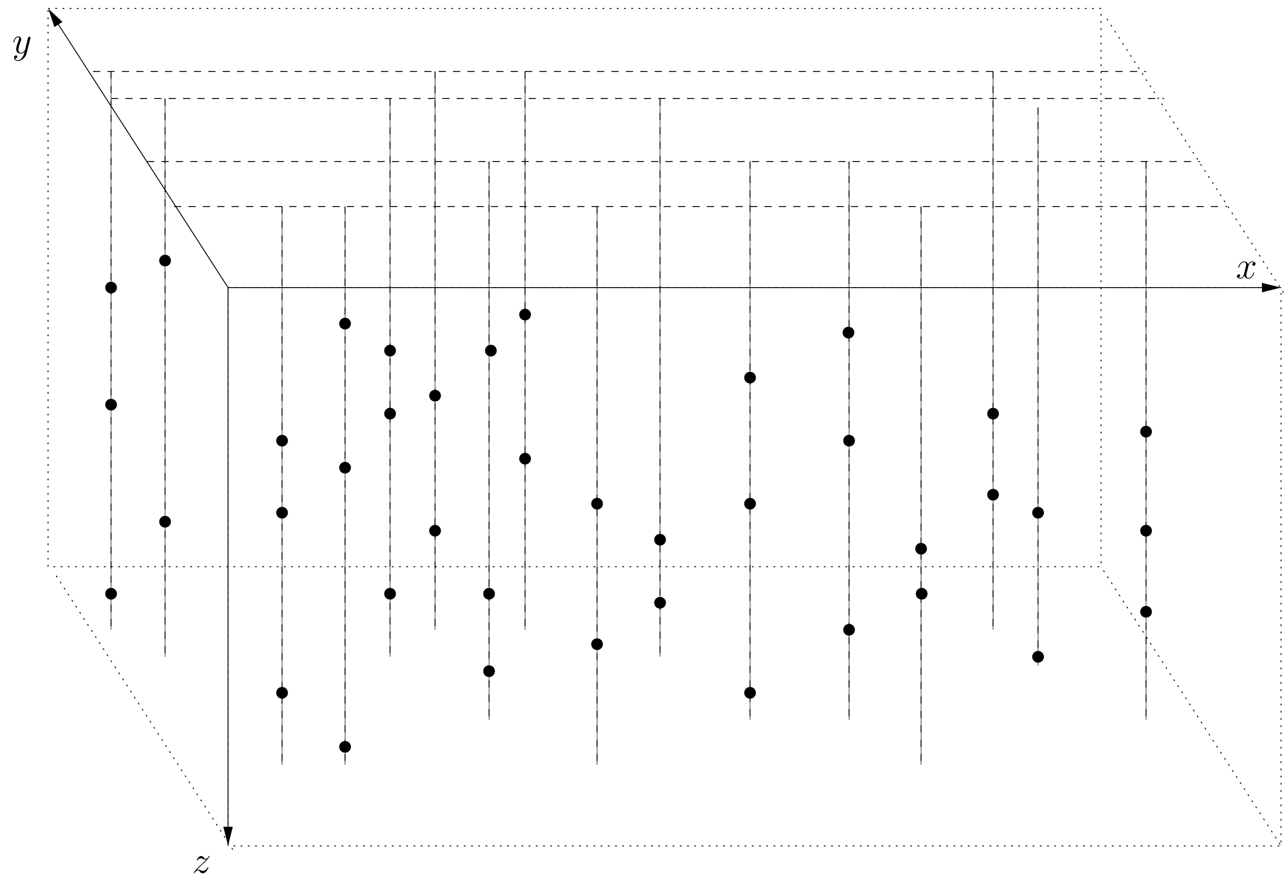
We propose a combination of:

- a parsimonious parametrization of the velocity field, by means of user-specified nodal values.
- a computationally efficient algorithm to smoothly approximate nodal velocity values on regular grids, for models of arbitrary dimensions.

Characteristics:

- the algorithm allows for used-defined smoothness.
- it also guarantees that bounds explicitly placed on the nodes are preserved by the corresponding approximants.

Model parametrization

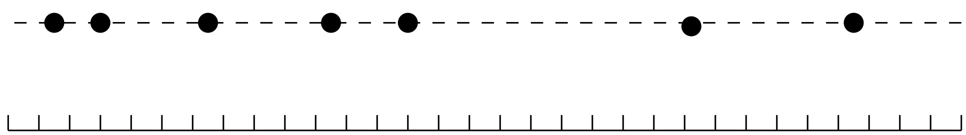


Partially Irregular Grids (PIGrids). The velocities are specified as: $v_{ijk} \equiv v(y_i, x_{ij}, z_{ijk})$.

One-dimensional smooth approximation

The building block of the algorithm consists of:

- Piecewise linear interpolation on a regular grid.



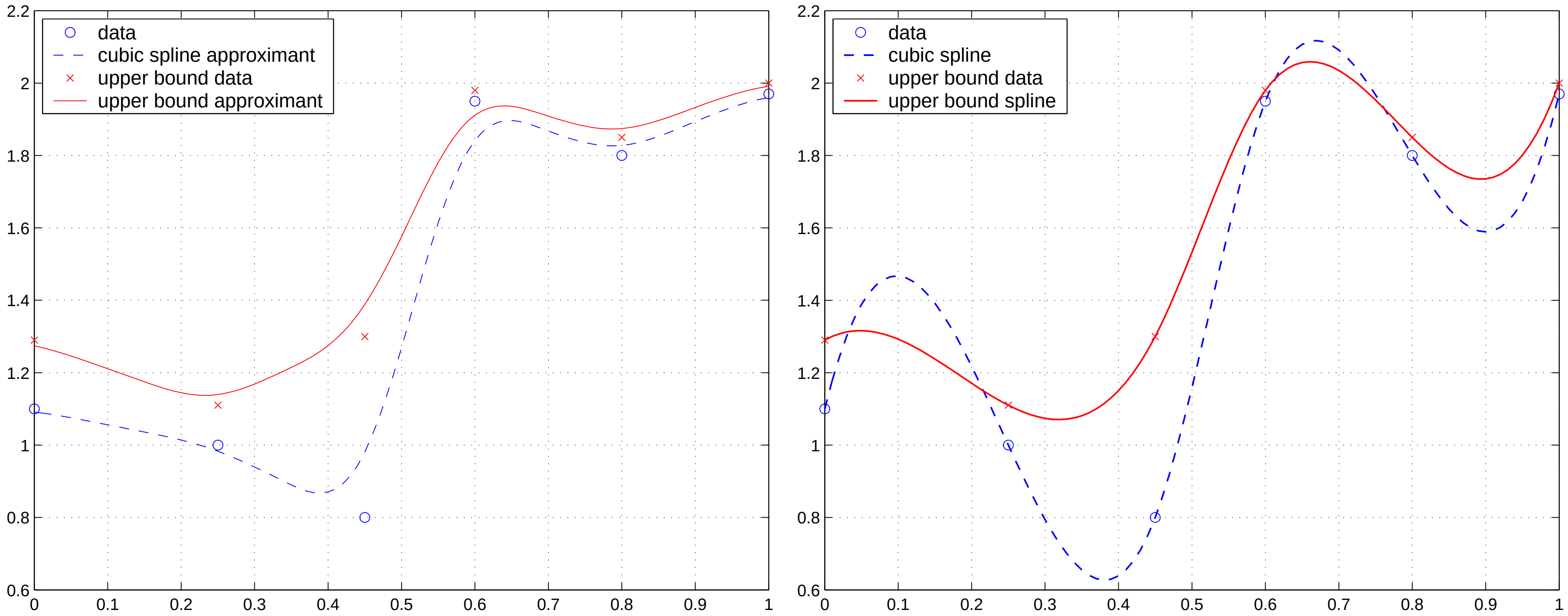
- Smoothing is accomplished via convolution with a triangular kernel of the form:

$$k(x) = \begin{cases} \frac{2}{h}\left(1 - \frac{|x|}{h/2}\right) & \text{for} \quad -\frac{h}{2} \leq x \leq \frac{h}{2} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

  where h is the smoothing width: the larger h, the wider the kernel, and the more local averaging is performed.

Explicit bounds

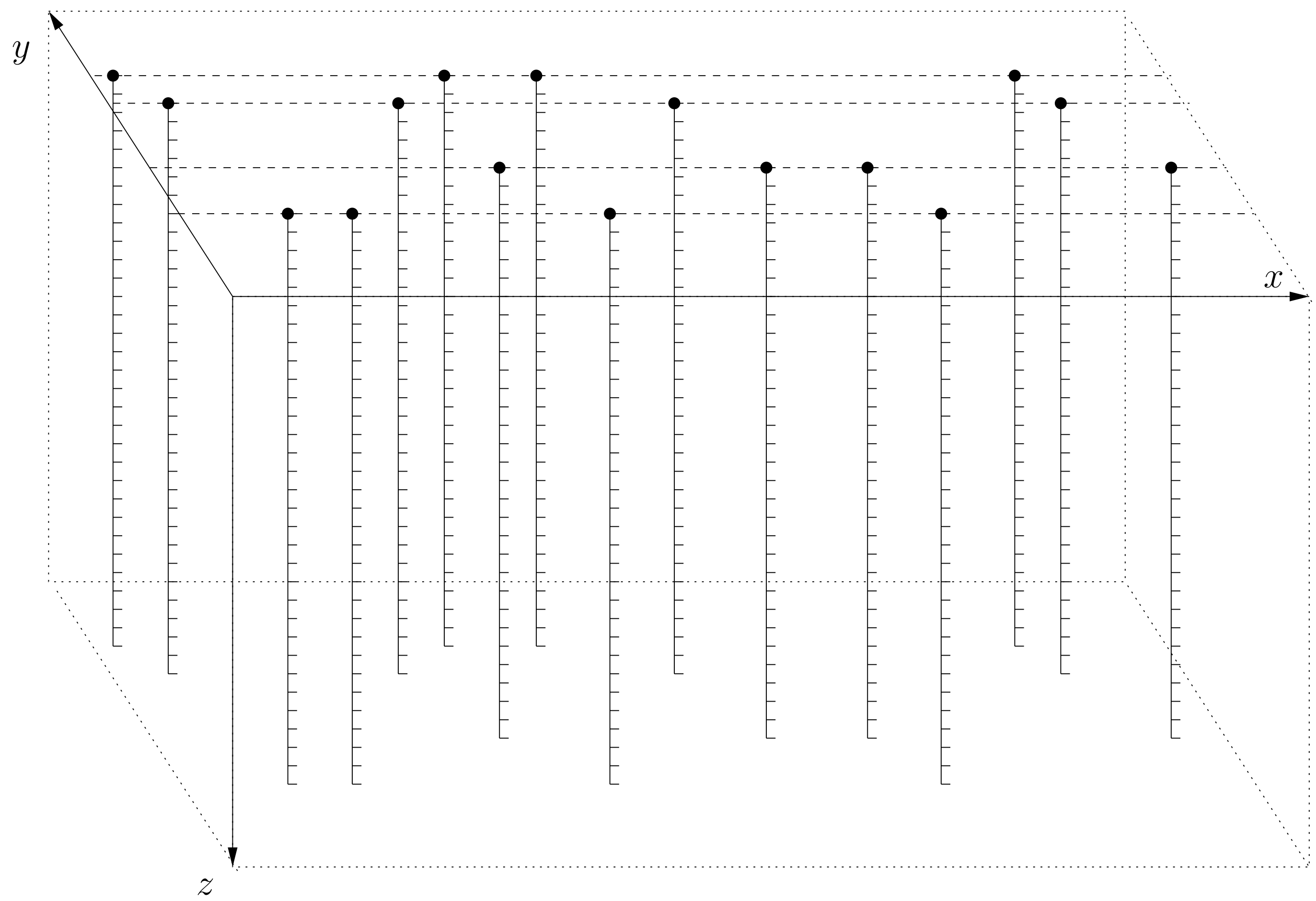
The scheme guarantees that the order is preserved, unlike cubic spline interpolation.



Extension to the multidimension case

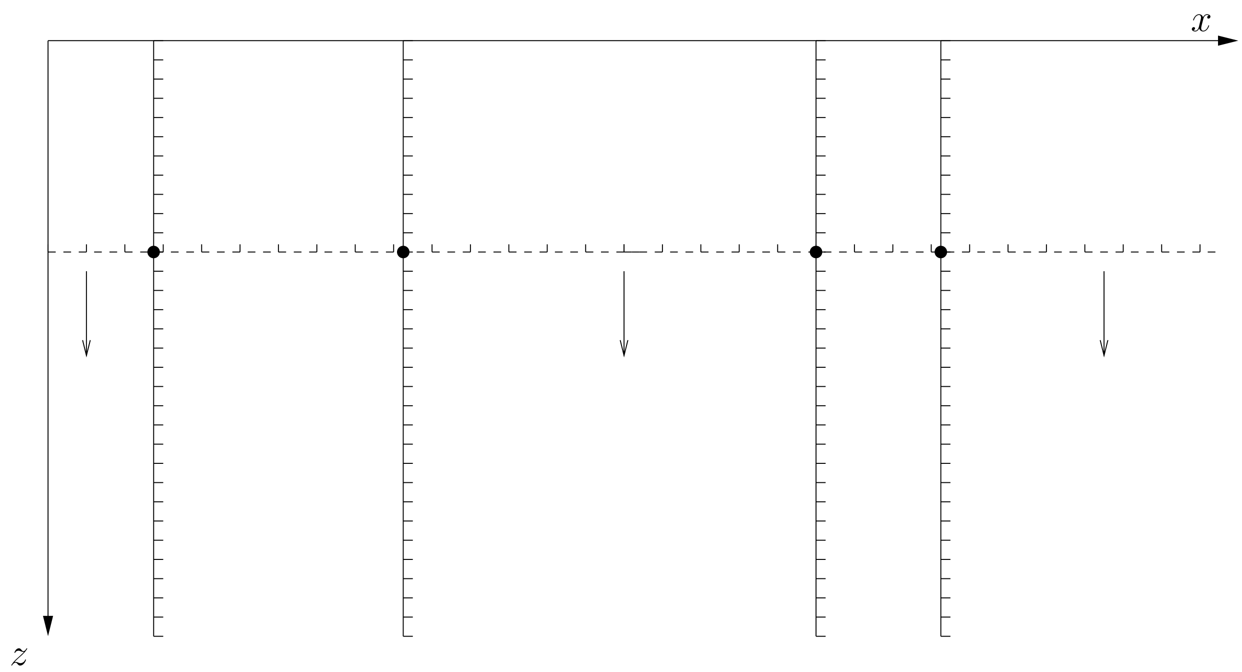
- We refer to the 1-D smooth approximation scheme as the `SMPL` operator.
- The rest of the algorithm consists of re-arranging the data into 1-D irregular samples which can then be treated by the `SMPL` operator.
- Example: consider the "raw" data shown before, and suppose the output (regular) grid is described by n_z, Δz, n_x, Δx, n_y, Δy, i.e. the number of samples and sampling interval in depth, in-line and cross-line directions, respectively.
- First step of the algorithm: apply the `SMPL` operator to each "vertical well" corresponding to a point (y_i, x_{ij}) on the surface.

Model after one step of the algorithm

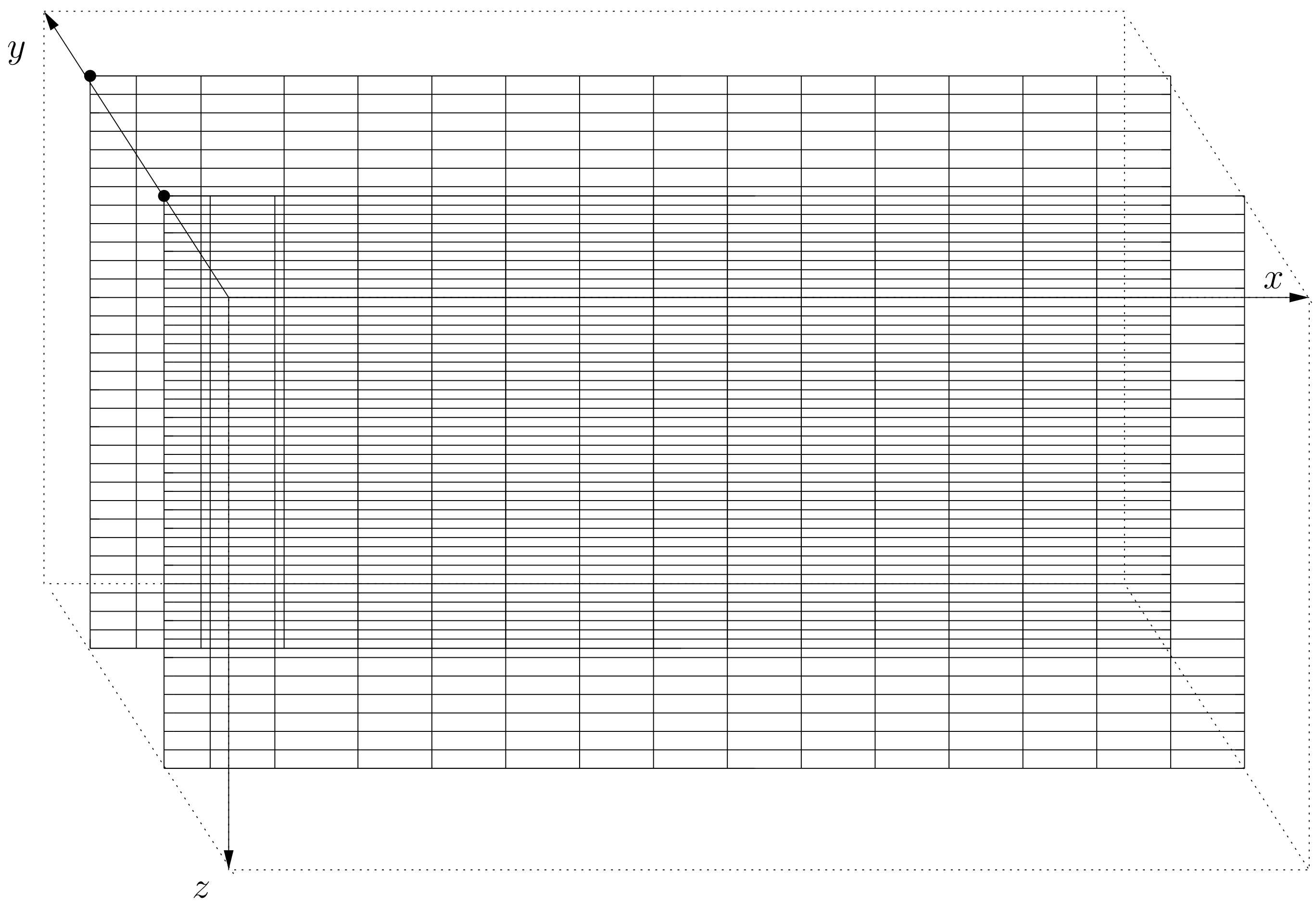


Description of the second step

This step consists of looping through the discrete z-axis, forming irregularly samples at each level, and interpolating onto the regular samples along the x-axis.



Model after two steps of the algorithm



Conclusions

- A sparse, hierarchical parametrization of the velocity which allows for a user-defined placement of nodes.
- The algorithm exploits the hierarchical structure to perform the smooth approximation efficiently, it also allows for user-controlled smoothness, and guarantees that the order is preserved.
- The algorithm is **reversible**, in the sense that the adjoint operator of the SMPL operator and of the whole scheme exist, thus allowing its use in a gradient-based optimization context.
- In particular, the parametrization and the algorithm have been successfully used for NMO-based differential semblance optimization (see next talk).