The Rice Inversion Project

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Annual Project Review, 2002-3

Sponsors, 2003

Amerada Hess
ConocoPhillips
Landmark Graphics
Sensorwise (new!)
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Agenda

0900-1200:

(1) WWS: Reverse Time S-G Migration and Differential Semblance
(2) PS: DS Velocity Analysis via Depth Extrapolation
(3) CS: Analysis of Phase Screen Depth Extrapolation

1200-1330:

Lunch, Cohen House

1330-1430:

Wrapup session

Reverse Time S-G Migration and Differential Semblance

William W. Symes

TRIP Review
March 2003

www.trip.caam.rice.edu

Partially linearized seismic inverse problem ("velocity analysis"): given observed seismic data S^{obs}, find smooth *velocity* $v(\mathbf{x})$ oscillatory *reflectivity* $r(\mathbf{x})$, functions of $\mathbf{x} \in X$ so that

$$F[v]r \simeq S^{\mathrm{obs}}$$

Scattering operator F defined by acoustic "partially linearized" model: acoustic potential field u and its perturbation δu solve

$$\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) u = \delta(t)\delta(\mathbf{x} - \mathbf{x}_s), \; \left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) \delta u = 2r\nabla^2 u$$

plus suitable bdry and initial conditions.

$$F[v]r(\mathbf{x}_s, \mathbf{x}_r, t) = \frac{\partial \delta u}{\partial t}(\mathbf{x}_s, \mathbf{x}_r, t)$$

where source positions $= \{\mathbf{x}_s\}$, receiver positions $= \{\mathbf{x}_r\}$.

Agenda:

- How common offset and Claerbout's *survey sinking* or *shot-geophone* migration are similar, and how they are different

- How to perform shot-geophone migration as a sequence of *two-way reverse time* shot-profile ("RTSG") migrations

- How RTSG migration avoids *kinematic artifacts*

- How RTSG images arbitrary dips

- A new variant of differential semblance

Common Offset vs. Shot-Geophone.

Common features: both involve a prestack *image* or *reflectivity* volume $\bar{X}$ = many copies of subsurface X parametrized by a bin parameter $\mathbf{h}$ (half-offset)

Physical reflectivity volume produced from physical reflectivity by an *extension operator* χ.

Prestack migration operator $\bar{G}[v]$ = *adjoint* of *prestack modeling operator* $\bar{F}[v]$ (or closely related operator), parametrized by velocity function $v(\mathbf{x})$.

Reformulation of inverse problem = **velocity analysis**: given prestack data d^{obs}, find v so that $\bar{G}[v]d^{\mathrm{obs}}$ is physical, i.e. lies in the range of χ (comes from a physical reflectivity).

Common offset prestack image volume: $X =$ subsurface volume, $\Sigma_h =$ set of half-offsets in data, $\bar{X} = X \times \Sigma_h$, $\chi[r](\mathbf{x},\mathbf{h}) = r(\mathbf{x})$.

Extended forward modeling op, applied to prestack reflectivity $\bar{r}(\mathbf{x},\mathbf{h})$:

$$\bar{F}[v]\bar{r}(\mathbf{x}_s,t,\mathbf{x}_r) = \int dx\,\bar{r}(\mathbf{x},\mathbf{h}) \int ds\, g(\mathbf{x}_m+\mathbf{h},t-s;\mathbf{x})g(\mathbf{x}_m-\mathbf{h},s;\mathbf{x})$$

where $g(\mathbf{x}_s,t;\mathbf{x})$ is acoustic Green's function for source at $\mathbf{x}_s$, or close relative, and $\mathbf{x}_r$ is receiver coord, $\mathbf{x}_m = \frac{1}{2}(\mathbf{x}_r + \mathbf{x}_s)$, $\mathbf{h} = \frac{1}{2}(\mathbf{x}_r - \mathbf{x}_s)$.

If $\bar{r}$ is physical, i.e. independent of $\mathbf{h}$, then this reduces to usual integral representation ("Lippman-Schwinger equation") of Born forward modeling.

NB: note that $\bar{F}[v]$ is "block diagonal" - family of operators parametrized by $\mathbf{h}$.

$\bar{G}[v]$ = adjoint of $\bar{F}[v]$:

$$\bar{G}[v]d(\mathbf{x},\mathbf{h}) =$$

$$\int dx_s \int dt\, d(\mathbf{x}_s, t, \mathbf{x}_s + 2\mathbf{h}) \int ds\, g(\mathbf{x}_s + 2\mathbf{h}, t-s; \mathbf{x}) g(\mathbf{x}_s, s; \mathbf{x})$$

Replace g by its usual h. f. asymptotic expansion

$$g(\mathbf{x}_s, t; \mathbf{x}) \simeq A(\mathbf{x}_s, \mathbf{x})\delta(t - T(\mathbf{x}_s, \mathbf{x}))$$

and you have prestack Kirchhoff common offset migration. Add some more amplitude terms and you have Kirchhoff inversion (Beylkin 1985, Bleistein 1987).

Shot-geophone prestack image volume: Σ_d = somewhat arbitrary set of vectors near 0 ("depth half-offsets"), $\bar{X} = X \times \Sigma_d$

Physical reflectivity volumes $\chi[r](\mathbf{x},\mathbf{h}) = r(\mathbf{x})\delta(\mathbf{h})$

Prestack forward modeling op, applied to prestack reflectivity $\bar{r}(\mathbf{x},\mathbf{h})$:

$$\bar{F}[v]\bar{r}(\mathbf{x}_s,t,\mathbf{x}_r) =$$

$$\int dx \int dh\, \bar{r}(\mathbf{x},\mathbf{h}) \int ds\, g(\mathbf{x}_s,t-s;\mathbf{x}-\mathbf{h})g(\mathbf{x}_r,s;\mathbf{x}+\mathbf{h})$$

If $\bar{r}$ is physical, reduces to usual Born forward model.

Computing $\bar{G}[v]$: could produce Kirchhoff formula as in common offset case - nonstandard.

Usual adjoint computation, *après* Claerbout (1985):

(1) assume *double square root* ("DSR") hypothesis: all rays carrying significant energy are downgoing between source and reflection point or upcoming from reflection point to receiver.

(2) restrict offsets to be horizontal, i.e. $\mathbf{h} = (h_x, h_y, 0)$, and correspondingly restrict $\bar{F}$ to reflectivity volumes of the form

$$\bar{r}_z(\mathbf{x}, \mathbf{h}) = \tilde{r}_z(\mathbf{x}, h_x, h_y)\delta(h_z)$$

Restricted operator $= \tilde{F}_z[v]\tilde{r}_z$

Stolk and deHoop, TRIP 2001: up to a factor affecting amplitudes (neglected in standard implementations), (1) and (2) $\Rightarrow \bar{F}_z[v]^* d(\mathbf{x},\mathbf{h}) = w(\mathbf{x}-\mathbf{h},\mathbf{x}+\mathbf{h},0)$ where $w(\mathbf{y}_s,\mathbf{y}_r,t)$ solves 1-way wave equations in z and $\mathbf{y}_s, t$, z and $\mathbf{y}_r, t$ resp.

This is the **survey-sinking** method of Claerbout: downward continue sources, downward continue receivers *to same depth*, read off image at $t = 0$.

Standard implementations in frequency, various one-way wave equation approximations (parabolic, phase screen,...).

(Slightly different derivation: CIME notes, www.trip.caam.rice.edu)

Summary: comparison of common offset, shot-geophone migration operators

- both are adjoints of prestack modeling operators

- bin parameter is offset - restricted to surface data offsets for common offset, *unrestricted* for S-G (conventionally horizontal)

- physical prestack reflectivity volumes are different: independence from $\mathbf{h}$ vs. focussing in $\mathbf{h}$.

- Kirchhoff is available for shot-geophone (but never used!), *mandatory* for common offset

Reverse Time Shot-Geophone Migration

Based on wave equation solved by integral representation of modeling operator:

$$\bar{F}[v]\bar{r}(\mathbf{x}_r, t; \mathbf{x}_s) = \frac{\partial}{\partial t}\delta\bar{u}(\mathbf{x}, t; \mathbf{x}_s)|_{\mathbf{x}=\mathbf{x}_r}$$

where

$$\left(\frac{1}{v(\mathbf{x})^2}\frac{\partial^2}{\partial t^2} - \nabla^2_{\mathbf{x}}\right)\delta\bar{u}(\mathbf{x}, t; \mathbf{x}_s) = \int_{\mathbf{x}+2\Sigma_d} dy\,\bar{r}(\mathbf{x}, \mathbf{y})g(\mathbf{y}, t; \mathbf{x}_s)$$

(that's the same g as before, i.e. the causal Green's function).

Specify adjoint field $w(\mathbf{x}, t; \mathbf{x}_s)$ as in standard reverse time prestack migration:

$$\left(\frac{1}{v(\mathbf{x})^2}\frac{\partial^2}{\partial t^2} - \nabla^2_{\mathbf{x}}\right) w(\mathbf{x}, t; \mathbf{x}_s) = \int dx_r\, d(\mathbf{x}_r, t; \mathbf{x}_s)\delta(\mathbf{x} - \mathbf{x}_r)$$

with $w(\mathbf{x}, t; \mathbf{x}_s) = 0, t >> 0$. Then

$$\bar{G}[v]d(\mathbf{x}, \mathbf{h}) = \int dx_s \int dt\, g(\mathbf{x} + 2\mathbf{h}, t; \mathbf{x}_s) w(\mathbf{x}, t; \mathbf{x}_s)$$

i.e. exactly the same computation as for reverse time prestack, except that crosscorrelation occurs at offset $2\mathbf{h}$ rather than 0. (Equivalent: Biondi and Shan, SEG 2002).

Implementation issues:

(1) Restricted offsets: simply set $h_z = 0$ in output (this is adjoint of $\tilde{r} \mapsto \bar{r}$) to get $\tilde{G}_z[v]$.

(2) Implementation using finite difference method: no additional expense over standard reverse time prestack, except for additional loop over offsets - one correlation of g, w per offset. Expense equivalent to one additional timestep per offset sample.

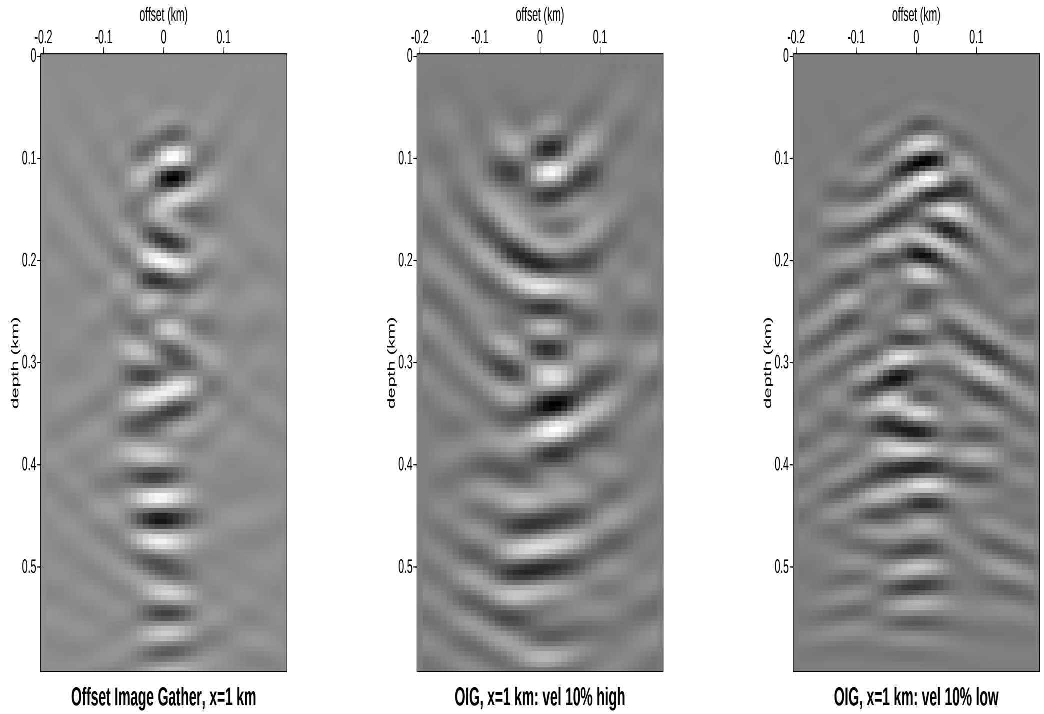
(3) For restricted offsets, eg. $h_z = 0$, simply don't compute correlations for $h_z \neq 0$.

What should be the character of the image when the velocity is correct?

Hint: for simulation of seismograms, the input reflectivity had the form $r(\mathbf{x})\delta(\mathbf{h})$.

Therefore guess that when velocity is correct, *image is concentrated near* $h = 0$.

Examples: 2D finite difference implementation of reverse time method. Correct velocity $\equiv 1$. Input reflectivity used to generate synthetic data: random! For output reflectivity (image of $\bar{F}_z[v]^*$), constrain offset to be horizontal: $\bar{r}(\mathbf{x},\mathbf{h}) = \tilde{r}_z(\mathbf{x}, h_x)\delta(h_z)$. Display CIGs (i.e. $x =$const. slices of $\tilde{r}_z$).



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Two way reverse time S-G image gathers of data from random reflectivity, constant velocity. From left to right: correct velocity, 10% high, 10% low.

Kinematics of reverse time S-G Migration

Advantage of "standard" (common shot) two way reverse time migration: images energy which violates DSR assumption (turning rays, overturned reflectors) - standard "survey-sinking" migration using depth extrapolation does not (see eg. recent *TLE* article by Lines et al.).

Same advantage acrues to reverse time shot-geophone migration (Biondi and Shan, SEG 2002).

Need to understand how *events* in data are imaged as *reflectors* in reflectivity volume $\bar{r}(\mathbf{x}, \mathbf{h})$.

Mathematics = *propagation of singularities*, following Rakesh 1988; see WWS, Stolk, Biondi TRIP 2002.

Convenient domain for expression of kinematics: source receiver parametrization

$$\bar{R}(\mathbf{y}_s, \mathbf{y}_r) = \bar{r}\left(\frac{\mathbf{y}_s + \mathbf{y}_r}{2}, \frac{\mathbf{y}_r - \mathbf{y}_s}{2}\right)$$

Events, reflectors as points in *phase space*:

Event ("element") in data: $(\mathbf{x}_s, \mathbf{x}_r, t, \omega\mathbf{p}_s, \omega\mathbf{p}_r, \omega)$

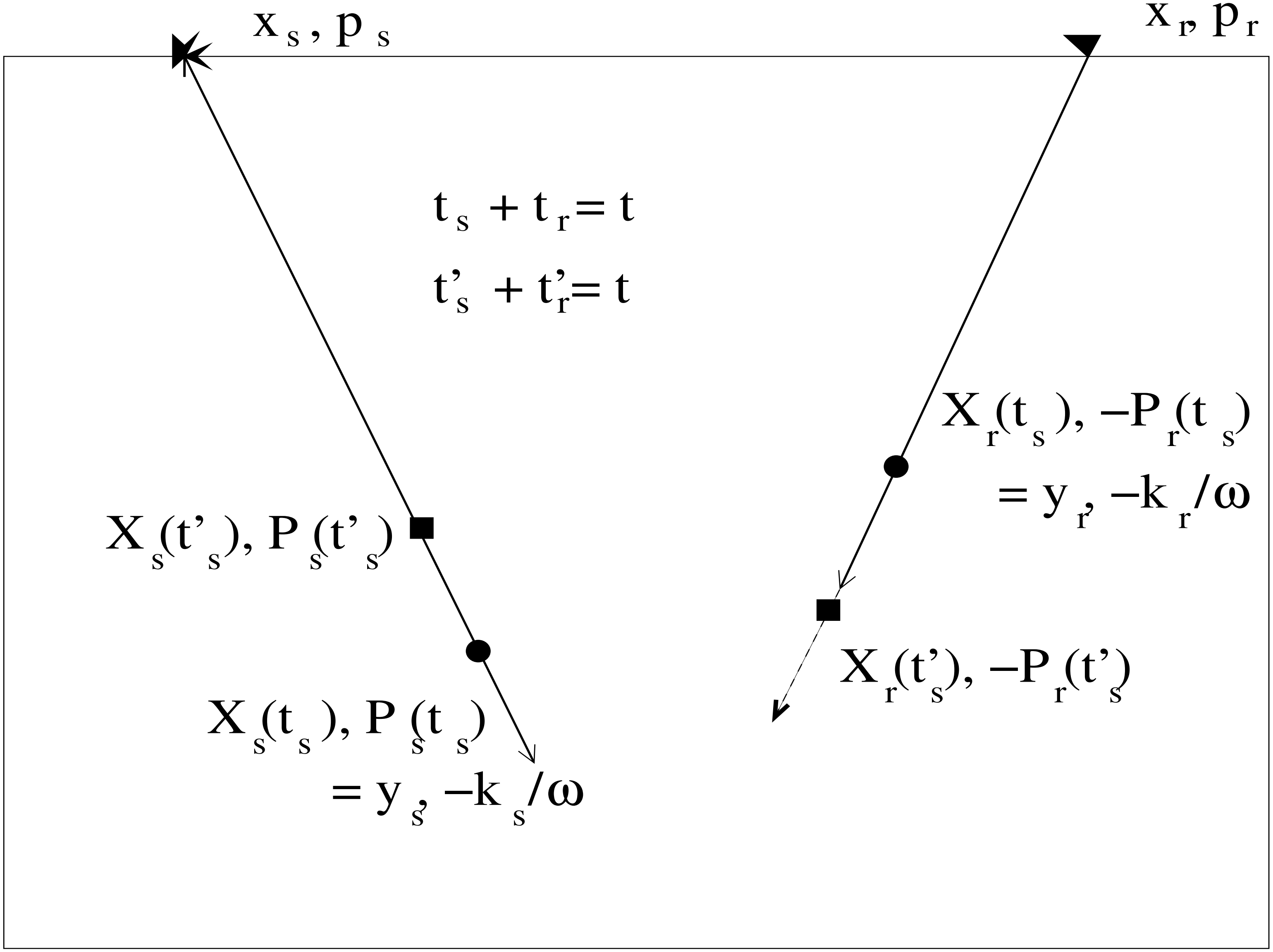
Reflector in subsurface: $(\mathbf{y}_s, \mathbf{y}_r, \mathbf{k}_s, \mathbf{k}_r)$

Imaging relation:

- source ray $(\mathbf{X}_s, \mathbf{P}_s)$, $\mathbf{X}_s(0) = \mathbf{x}_s$, $\mathbf{P}_s(0) = \mathbf{p}_s$

- receiver ray $(\mathbf{X}_r, \mathbf{P}_r)$, $\mathbf{X}_r(t) = \mathbf{x}_r$, $\mathbf{P}_r(t) = \mathbf{p}_r$

- at imaging time = time t_s along source ray, rays match reflecting element:
  - $\mathbf{X}_s(t_s) = \mathbf{y}_s$, $\omega \mathbf{P}_s(t_s) = -\mathbf{k}_s$
  - $\mathbf{X}_r(t_s) = \mathbf{y}_r$, $\omega \mathbf{P}_r(t_s) = \mathbf{k}_r$


x_s, p_s
x_r, p_r
$t_s + t_r = t$
$t'_s + t'_r = t$
$X_r(t_s), -P_r(t_s)$
$= y_r, -k_r/\omega$
$X_s(t'_s), P_s(t'_s)$
$X_r(t'_s), -P_r(t'_s)$
$X_s(t_s), P_s(t_s)$
$= y_s, -k_s/\omega$

Obvious imaging ambiguity: given data event, corresponding rays, can choose *any* t_s between 0 and t!

Convenient method to remove ambiguity (WWS, Stolk, Biondi, TRIP 2002, see also Biondi and WWS, SEP 112 for another, similar approach): *restrict offset direction*, as in original Claerbout S-G.

Horizontal offsets: $h_z = 0$, i.e.

$$\bar{r}_z(\mathbf{x}, \mathbf{h}) = \tilde{r}_z(\mathbf{x}, h_x, h_y)\delta(h_z)$$

or in source-receiver coords

$$\bar{R}_z(\mathbf{y}_s, \mathbf{y}_r) = \tilde{R}_z\left(y_{s,x}, y_{s,y}, y_{r,x}, y_{r,y}, \frac{y_{r,z} + y_{s,z}}{2}\right)\delta(y_{r,z} - y_{s,z})$$

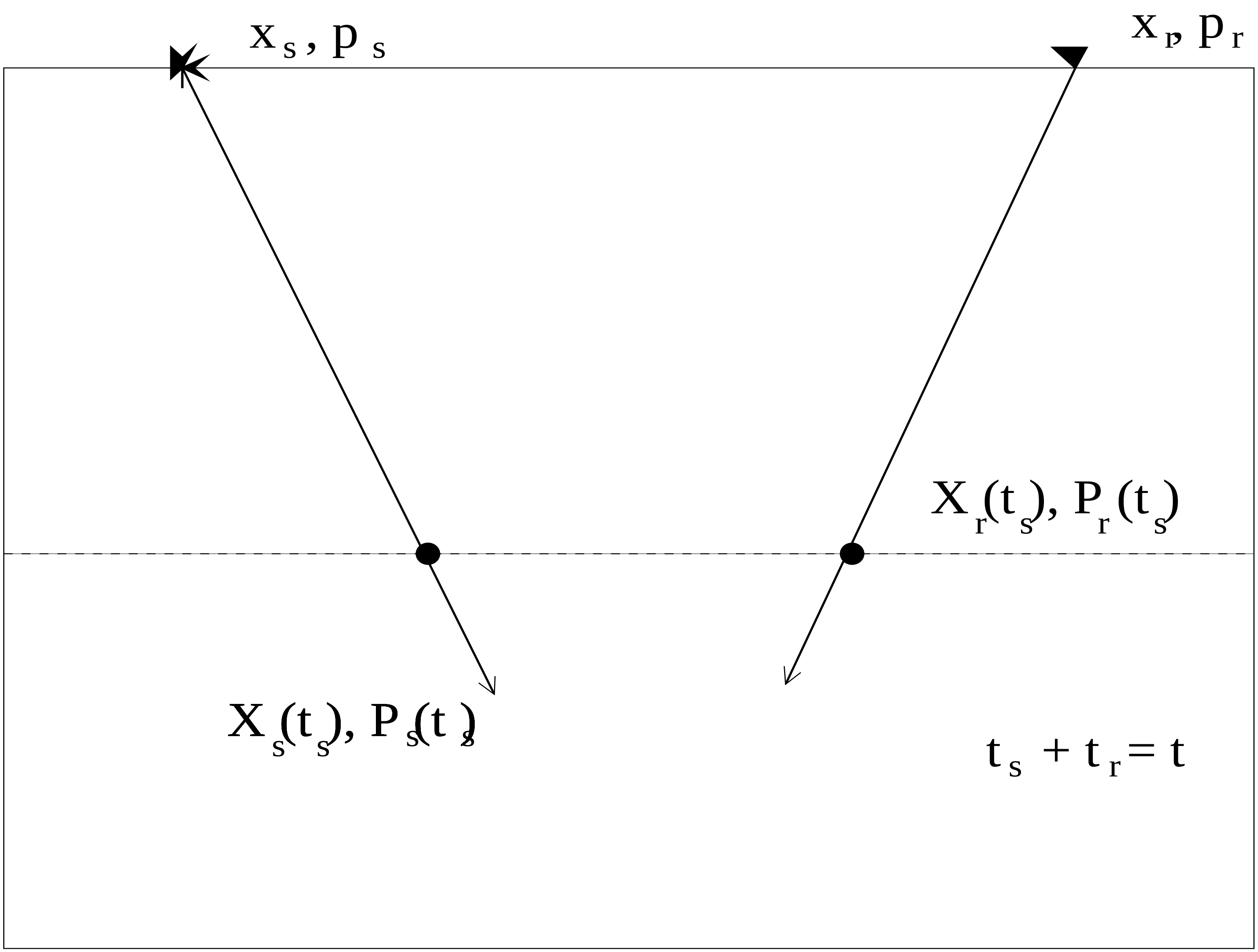
Implies phase space constraint: reflector lies in reduced phase space of $\tilde{R}$, wave vector $= (k_{s,x}, k_{s,y}, k_{r,x}, k_{r,y}, k_z)$ and z-imaging condition is $\mathbf{X}_{s,z}(t_s) = \mathbf{X}_{r,z}(t_s), \omega(\mathbf{P}_{s,z}(t_s) - \mathbf{P}_{r,z}(t_s)) = k_z$.

Stolk and deHoop, TRIP 2001: **SUPPOSE:** *DSR assumption*: all significant energy to be imaged travels on downgoing source rays ($\mathbf{P}_{s,z} > 0$) and upcoming receiver rays ($\mathbf{P}_{r,z} < 0$). **NB:** must assume to use depth extrapolation in S-G migration.

THEN: Each event is imaged in *exactly one* reflector in the horizontal offset reflectivity volume $\tilde{R}_z$, whether the velocity is correct or not.

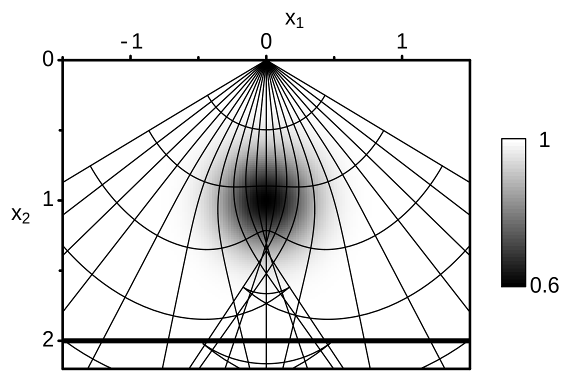
PROOF: obvious (picture).

COROLLARY: If the velocity is correct, and DSR holds, then S-G image gathers will be focussed (i.e. S-G version of semblance criterion will hold) - regardless of the complexity of the velocity field.

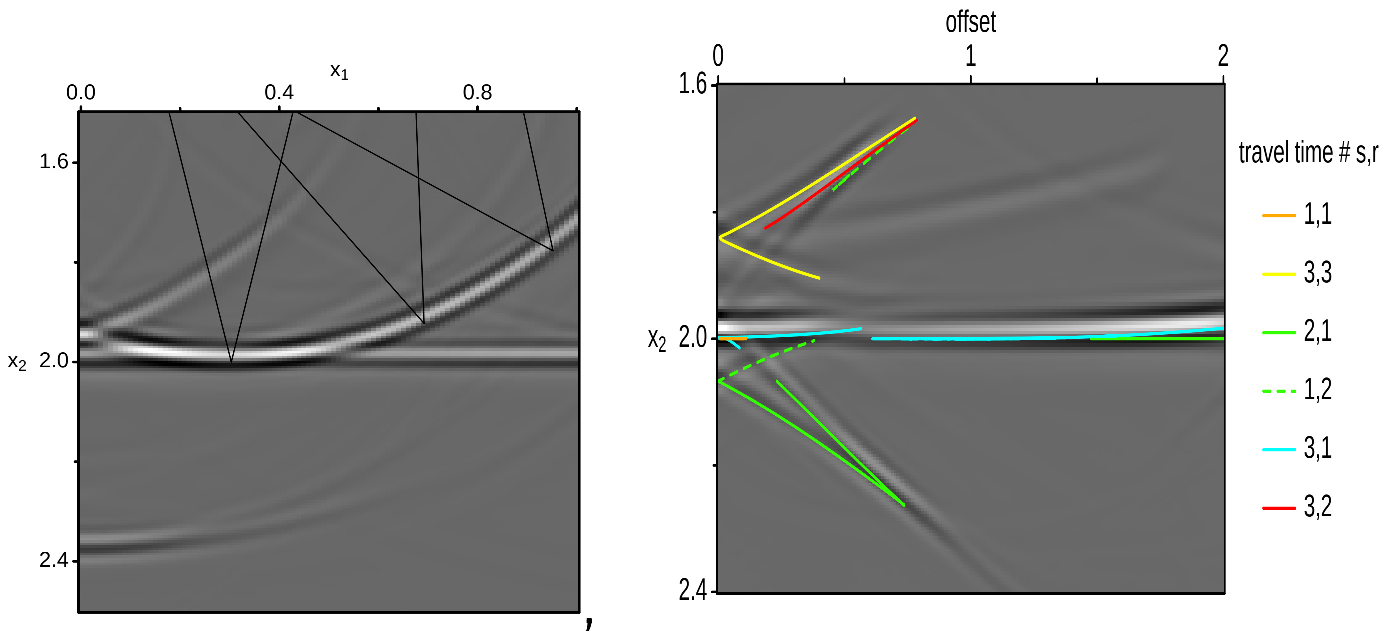


x_s, p_s
x_r, p_r
$X_r(t_s), P_r(t_s)$
$X_s(t_s), P_s(t_s)$
$t_s + t_r = t$

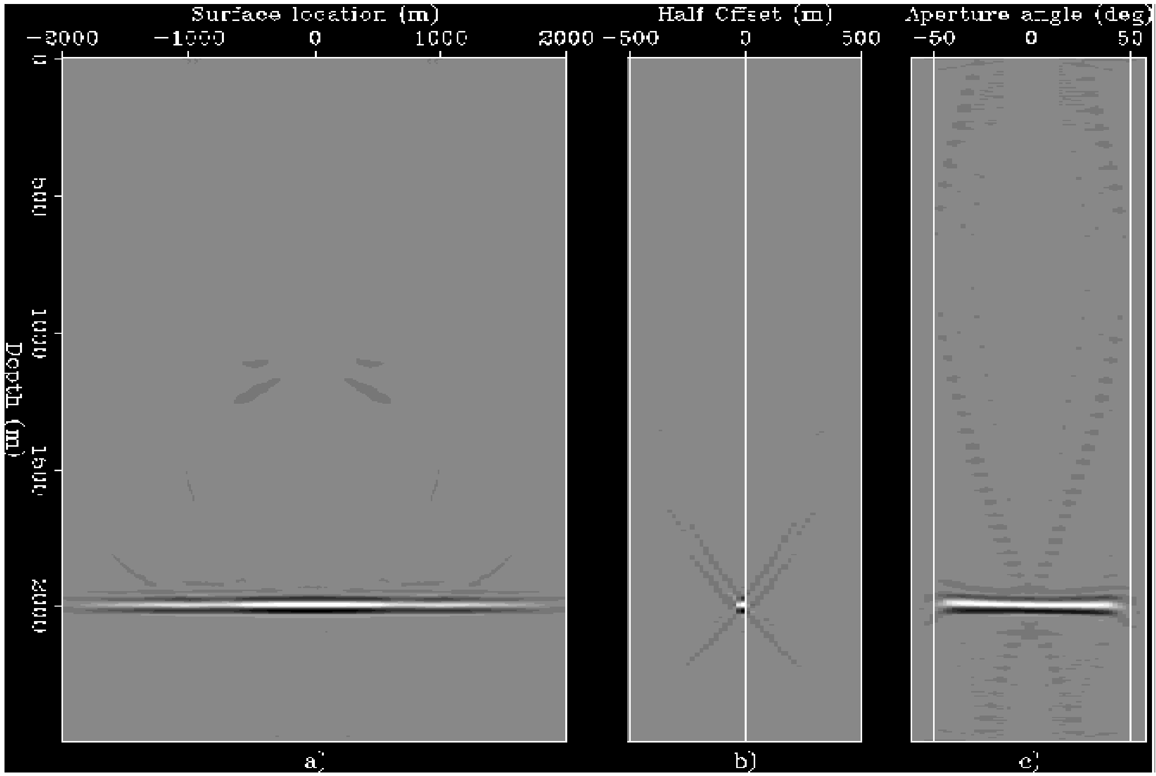
Why this is remarkable: analogous statement for common offset is *false* i.e. common offset image gathers *may not be flat* even when velocity is correct (Stolk, Stolk and WWS - TRIP 2001, after Nolan TRIP 1995 for common source).



Example: Gaussian lens over flat reflector at depth z ($r(\mathbf{x}) = \delta(x_1 - z)$, $x_1 =$ depth).



Common offset migration of lens data. **Left:** image at offset $h = 0.3$ km **Right:** CIG at $x = 1.0$ km - not smooth in h!



S-G migration of lens data. **Left:** image ($h = 0$ section) **Center:** CIG at $x = 1.0km$ **Right:** Angle CIG (Radon of CIG in h, z) [Thanks: Biondo Biondi]

Imaging arbitrary dips

DSR assumption, horizontal offset reflectivity *incompatible* with imaging reflecting elements with $k_z = 0$ (i.e. vertical reflectors): imaging condition is

$$\omega(\mathbf{P}_{s,z}(t_s) - \mathbf{P}_{r,z}(t_s)) = k_z$$

but DSR requires

$$\mathbf{P}_{s,z} > 0,\ \mathbf{P}_{r,z} < 0$$

and these are incompatible with $k_z = 0$ unless $\omega = 0$. In practice: k_z small $\Rightarrow$ low-frequency artifacts ("smearing"), see Biondi and WWS SEP 112, Biondi and Shan SEG 02.

Imaging (near-) vertical reflectors $\Rightarrow$ give up DSR, permit *vertical offsets* $\mathbf{h} = (0, h_z)$ (2D for simplicity - 3D similar), and correspondingly restrict $\bar{F}$ to reflectivity volumes of the form

$$\bar{r}_x(\mathbf{x}, \mathbf{h}) = \tilde{r}_x(\mathbf{x}, h_z)\delta(h_x)$$

Restricted operator $= \tilde{F}_x[v]\tilde{r}_x$

As before, to get adjoint $\tilde{G}_x[v]$ simply set $h_x = 0$ in output of $\bar{G}[v]$.

Two image volumes: $\tilde{G}_z[v]d$, smeared near vertical reflectors, and $\tilde{G}_x[v]d$, smeared near horizontal reflectors.

A solution (Stolk, WWS, Biondi, 2003 - see Biondi and WWS SEP 112 for another approach): introduce *dip filters* Π_x, Π_z with

$$\Pi_x(0, k_{s,z}, k_{r,z}) = 0, \ \Pi_z(k_{s,x}, k_{r,x}, 0) = 0$$

and define a total forward map on pairs of reflectivity volumes

$$\tilde{F}_t[v](\tilde{r}_x, \tilde{r}_z) = \tilde{F}_x[v](\Pi_x \tilde{r}_x) + \tilde{F}_z[v](\Pi_z \tilde{r}_z)$$

Adjoint $\tilde{G}_t[v]$ outputs *filtered* restricted offset reflectivities with smearing removed. But that is not all...

For correct velocity, images focus (source, receiver rays intersect) at $\mathbf{h} = 0$ at imaging time t_s. S-G imaging condition reduces to usual Snell's law at these points.

Because of imaging condition, rays focusing at $k_z \neq 0$ must have $\mathbf{P}_{r,z} - \mathbf{P}_{s,z} \neq 0 \Rightarrow$ depth components of source, receiver rays must *separate* immediately, i.e. $h_z = 0$ is violated for times near t_s. Leads to generalization of Stolk-deHoop theorem:

Local Focussing Theorem: If the velocity is correct, the filtered image volumes are focussed at $h_z = 0$ resp. $h_x = 0$ *within a corridor of width* h_c, i.e. $|h_x|, |h_z| < h_c$.

[Does energy focus outside the corridor? Probably. Stay tuned.]

Differential semblance

Quantifying the semblance principle: devise operator W for which $W\bar{r} \simeq 0$ is equivalent to $\bar{r}$ being physical, at least approximately.

Then minimize w.r.t. v a suitable norm

$$J[v] \equiv \frac{1}{2}\|W\bar{G}[v]d^{\mathrm{obs}}\|^2$$

Given size of these problems, want to use if possible descent-based methods, which require smoothness of objective.

Stolk and WWS TRIP 2002 (published in IP, 2003): The only operators W which work are *pseudodifferential* = compositions of differential operators and $|k|^p$ filters.

For common offset, physical = does not depend on offset, so only choice of W is

$$W = P\nabla_{\mathbf{h}}$$

with P a ΨDO of order -1. Hence name of this technique: *differential semblance*

For S-G, physical = focussed at $\mathbf{h} = 0$, hence necessarily

$$W = Ph$$

with P a ΨDO of order 0 (Stolk 2000, Stolk & deHoop 2001).

Ongoing Work

(1) implementation of DSR-based DS using one-way propagators (Shen, Stolk), demonstration of Stolk-deHoop focussing property and VA in presence of multipathing

(2) implementation of RTSG-based DS using FD WE solvers (WWS)

(3) design of noise suppression, antialiasing for these operators (Shen, WWS)

(4) further study of one-way propagators (Stolk)

(5) theoretical study of S-G based DS (WWS)